1. (a) \( \lim_{n \to \infty} \frac{3^n}{2n^2 + n!} = \lim_{n \to \infty} \frac{3^n}{n!} = 0 \)

(b) Since \( \sqrt{4n^2 + 3n} \sim \sqrt{4n^2} = 2n \), we have
\[
\lim_{n \to \infty} \frac{1 + \sqrt{4n^2 + 3n}}{n + 5} = \lim_{n \to \infty} \frac{2n}{n} = 2
\]

(c) It is true that \((n^3 + 6)^2 \sim (n^3)^2 = n^6\), but this does not mean that \((n^3 + 6)^2 - n^6 \sim 0\).
Instead, we have
\[
(n^3 + 6)^2 - n^6 = (n^6 + 12n^3 + 36) - n^6 = 12n^3 + 36 \sim 12n^3,
\]
so
\[
\lim_{n \to \infty} \frac{(n^3 + 6)^2 - n^6}{3n^3 + \ln n} = \lim_{n \to \infty} \frac{12n^3}{3n^3} = 4
\]

(d) Since \(\ln(1+x) = x - \frac{1}{2}x^2 + \cdots\), we have \(\ln(1+x^3) = x^3 - \frac{1}{2}x^6 + \cdots\), so
\[
\lim_{x \to 0} \frac{\ln(1+x^3) - x^3}{x^6} = \lim_{x \to 0} \frac{(x^3 - \frac{1}{2}x^6 + \cdots) - x^3}{x^6} = \lim_{x \to 0} \frac{-\frac{1}{2}x^6 + \cdots}{x^6} = -\frac{1}{2}
\]

2. (a) Using the given recursive formula:
- \(a_2 = \frac{a_1 + 1}{2} = \frac{0 + 1}{2} = \frac{1}{2}\)
- \(a_3 = \frac{a_2 + 1}{2} = \frac{\frac{1}{2} + 1}{2} = \frac{3}{4}\)
- \(a_4 = \frac{a_3 + 1}{2} = \frac{\frac{3}{4} + 1}{2} = \frac{7}{8}\)
- \(a_5 = \frac{a_4 + 1}{2} = \frac{\frac{7}{8} + 1}{2} = \frac{15}{16}\)

(b) Based on the results from part (a), it looks like \(a_n = \frac{2^n - 1}{2^{n-1}}\)

(c) This is the geometric series \(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\), whose sum is 2.
3. (a) This is a geometric series with \( r = -\frac{1}{4} \), so the sum is \( \frac{1}{1 - (-\frac{1}{4})} = \frac{4}{5} \).

(b) The trick here is to investigate the partial sums. The first few terms of the series are

\[
\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots
\]

so the first few partial sums are 1/2, 2/3, 3/4, and 4/5. From this it is clear that the sum of the series is 1.

(c) Recall that

\[
\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots.
\]

Plugging in \( x = 1 \) gives

\[
1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots = \tan^{-1}(1) = \frac{\pi}{4}
\]

Then

\[
\frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \cdots = 1 - \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) = 1 - \frac{\pi}{4}
\]

(d) Recall that \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \), so

\[
e^2 = \sum_{n=0}^{\infty} \frac{2^n}{n!} = 1 + \sum_{n=1}^{\infty} \frac{2^n}{n!}
\]

and hence \( \sum_{n=1}^{\infty} \frac{2^n}{n!} = e^2 - 1 \).

4. We have \( \int_{3}^{\infty} \left( \frac{1}{x^2} + \frac{2}{x^3} + \frac{3}{x^4} + \frac{4}{x^5} + \cdots \right) \, dx = \left[ -\frac{1}{x} - \frac{1}{x^2} - \frac{1}{x^3} - \frac{1}{x^4} - \cdots \right]_{3}^{\infty} \)

\[
= 0 - \left( -\frac{1}{3} - \frac{1}{9} - \frac{1}{27} - \frac{1}{81} - \cdots \right) = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots. \]

This last series is a geometric series with common ratio \( r = 1/3 \), so the sum is \( \frac{1/3}{1 - (1/3)} = \frac{1}{2} \).
5. (a) Since \( \frac{1}{n^2 + 3} \sim \frac{1}{n^2} \) and \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) converges, this series \( \text{converges} \) as well.

(b) Since \( \frac{1}{2n+1} \sim \frac{1}{2n} \) and \( \sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} \) diverges, this series \( \text{diverges} \) as well.

(c) Using the root test, \( r = \frac{1}{2} < 1 \), so this series \( \text{converges} \).

(d) This series \( \text{converges} \) by the alternating series test.

(e) We have

\[
\int_{2}^{\infty} \frac{1}{x(\ln x)^3} \, dx = \left[ -\frac{1}{2(\ln x)^2} \right]_{2}^{\infty} = \frac{1}{2(\ln 2)^2}.
\]

In particular, the improper integral \( \int_{2}^{\infty} \frac{1}{x(\ln x)^3} \, dx \) converges, and hence the series \( \text{converges} \) as well by the integral test.

(f) Since \( n! \gg 2^n \), the terms of this series don’t approach zero, so the series \( \text{diverges} \).

(g) Since \( \lim_{n \to \infty} \frac{n^2}{\sqrt{n^4 + 1}} = \lim_{n \to \infty} \frac{n^2}{\sqrt{n^4}} = \lim_{n \to \infty} \frac{n^2}{n^2} = 1 \), the terms of this series don’t approach zero, so the series \( \text{diverges} \).

(h) The denominator here approaches 1 as \( n \to \infty \), which means that the terms themselves don’t approach zero, and hence this series \( \text{diverges} \).

(i) Using the log rule \( \ln(a^b) = b \ln a \), we have

\[
\sum_{n=1}^{\infty} \frac{1}{n \ln(2n)} = \sum_{n=1}^{\infty} \frac{1}{n^2 \ln 2} = \frac{1}{\ln 2} \sum_{n=1}^{\infty} \frac{1}{n^2},
\]

so this series \( \text{converges} \).
6. Let’s tally up the squares. We have:

- One large square with area 1,
- Three smaller squares attached to this, each with area \((1/3)^2 = 1/9\),
- Nine smaller squares attached to this, each with area \((1/9)^2 = 1/81\),
- Twenty-seven smaller squares attached to this, each with area \((1/27)^2 = 1/729\), etc.

Thus the total area is:

\[
1 + 3 \times \frac{1}{9} + 9 \times \frac{1}{81} + 27 \times \frac{1}{729} + \cdots = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots.
\]

The series on the right is a geometric series with common ratio \(r = 1/3\), so the sum is

\[
\frac{1}{1 - (1/3)} = \frac{3}{2}.
\]

7. (a) Writing out the terms of this series gives \(x^3 + x^7 + x^{11} + x^{15} + \cdots\). This is a geometric series with common ratio \(r = x^4\), so the sum is

\[
\frac{x^3}{1 - x^4}.
\]

(b) This series has \(r = 3|x|\). Thus it converges for \(3|x| < 1\), or equivalently \(|x| < 1/3\), so the radius of convergence is \(1/3\).

(c) We have \(\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}\), so

\[
\tan^{-1}(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} (2x)^{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{2n+1} x^{2n+1}
\]

and hence \(x^2 \tan^{-1}(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{2n+1} x^{2n+3}\)

(d) We have \(f'(x) = \frac{1}{2} (x+25)^{-1/2}\) and \(f''(x) = -\frac{1}{4} (x+25)^{-3/2}\), so

\[
f(0) = 5, \quad f'(0) = \frac{1}{10}, \quad \text{and} \quad f''(0) = -\frac{1}{500},
\]

and hence the first three terms are

\[
f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 = 5 + \frac{1}{10}x - \frac{1}{1000}x^2.
\]
8. (a) The angle is
\[
\cos^{-1}\left(\frac{\langle -4,3,5 \rangle \cdot \langle 3,4,-5 \rangle}{|\langle -4,3,5 \rangle||\langle 3,4,-5 \rangle|}\right) = \cos^{-1}\left(\frac{-25}{\sqrt{50}\sqrt{50}}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ.
\]

(b) The hard part here is figuring out how the four points are arranged around the parallelogram. Here is a picture of the situation:

Thus the area is \(\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5\).

(c) Since \(\langle 1,-1,3 \rangle \cdot \langle 2,5,1 \rangle = 0\), the vectors \(\langle 1,-1,3 \rangle\) and \(\langle 2,5,1 \rangle\) are orthogonal.

(d) The normal vector to the plane is \(\langle 3,2,4 \rangle\), so a vector is parallel to the plane if and only if it is orthogonal to \(\langle 3,2,4 \rangle\). Then the vector in question must be orthogonal to both \(\langle 3,2,4 \rangle\) and \(\langle 2,-1,1 \rangle\), so taking the cross product should yield the answer:

\[
\langle 3,2,4 \rangle \times \langle 2,-1,1 \rangle = \begin{vmatrix} i & j & k \\ 3 & 2 & 4 \\ 2 & -1 & 1 \end{vmatrix} = \langle 6,5,-7 \rangle.
\]

The vector \(\langle -6,-5,7 \rangle\) is also a correct answer, as is any other nonzero scalar multiple of \(\langle 6,5,-7 \rangle\).
9. (a) We can find the normal vector to this triangular face by taking the cross product of the
two vectors shown in the following picture.

Thus the normal vector is $\langle -1, 1, 0 \rangle \times \langle -1, 0, 1 \rangle = \langle 1, 1, 1 \rangle$.

(b) This should be the same as the angle between $\langle 1, 1, 1 \rangle$ and $\langle 0, 0, 1 \rangle$, which is
$$
\cos^{-1} \left( \frac{\langle 1, 1, 1 \rangle \cdot \langle 0, 0, 1 \rangle}{|\langle 1, 1, 1 \rangle||\langle 0, 0, 1 \rangle|} \right) = \cos^{-1} \left( \frac{1}{\sqrt{3}} \right) \approx 55^{\circ}
$$

10. (a) The point $(12, 0, 0)$ lies on the first plane, and the point $(30, 0, 0)$ lies on the second plane,
so the vector that stretches between them is $\langle 18, 0, 0 \rangle$. The unit vector $\frac{1}{9} \langle 1, 8, 4 \rangle$ is normal
to both planes, so taking the dot product should yield the distance:
$$
\frac{1}{9} \langle 1, 8, 4 \rangle \cdot \langle 18, 0, 0 \rangle = 2
$$

(b) The first line is parallel to the vector $\langle 0, 1, 2 \rangle$, and the second line is parallel to the vector
$\langle -1, 4, 1 \rangle$. The normal vector for the plane must be perpendicular to both of these, so we
take the cross product:
$$
\langle 0, 1, 2 \rangle \times \langle -1, 4, 1 \rangle = \begin{vmatrix}
i & j & k \\
0 & 1 & 2 \\
-1 & 4 & 1 \\
\end{vmatrix} = \langle -7, -2, 1 \rangle
$$
The point $(2, 1, 3)$ lies on the first line and hence on the plane, so the equation for the
plane is $-7x - 2y + z = -13$. (Of course, the equation $7x + 2y - z = 13$ is also correct,
as is any equivalent equation.)
(c) The point \( (3,0,0) \) lies on this plane, and the vector from \( (3,0,0) \) to \( (2,1,1) \) is \( \langle -1,1,1 \rangle \).

The unit normal vector to the plane is 

\[
\mathbf{u} = \frac{1}{7} \langle 2,3,6 \rangle = \langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \rangle,
\]

so the point \( (2,1,1) \) is a distance of 1 from the plane in the direction of \( \mathbf{u} \). Then 

\[
(2,1,1) - \mathbf{u} = \left( \frac{12}{7}, \frac{4}{7}, \frac{1}{7} \right)
\]

is the closest point on the plane.

11. The vector from \( (3,3) \) to \( (8,13) \) is \( \langle 5,10 \rangle \), which is parallel to \( \langle 1,2 \rangle \). Turning clockwise 90° gives \( \langle 2,-1 \rangle \), which must be parallel to the vector from \( (3,3) \) to \( P \). The unit vector in this direction is 

\[
\frac{1}{\sqrt{5}} \langle 2,-1 \rangle.
\]

Now, the vector from \( (8,13) \) to \( (10,7) \) is \( \langle 2,-6 \rangle \), and 

\[
\frac{1}{\sqrt{5}} \langle 2,-1 \rangle \cdot \langle 2,-6 \rangle = \frac{10}{\sqrt{5}} = 2\sqrt{5}.
\]

So the distance from \( (3,3) \) to \( P \) is \( 2\sqrt{5} \). Then 

\[
P = (3,3) + 2\sqrt{5} \left( \frac{1}{\sqrt{5}} \langle 2,-1 \rangle \right) = (3,3) + \langle 4,-2 \rangle = (7,1)
\]

12. The given line is parallel to the vector \( \langle -2,1,1 \rangle \), and the given plane is normal to the vector \( \langle 1,2,1 \rangle \). Both of these must be parallel to the desired plane, so we can take their cross product to get a normal vector for the desired plane:

\[
\langle 3,-2,1 \rangle \times \langle 1,2,1 \rangle = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & -2 & 1 \\
1 & 2 & 1 \\
\end{vmatrix} = \langle -4,-2,8 \rangle = -2\langle 2,1,-4 \rangle.
\]

The point \( (2,3,1) \) lies on the line and hence on the desired plane, so the equation for the desired plane is 

\[
2x + y - 4z = 3
\]

(Of course, any equivalent equation would also be a correct answer.)