Homework 3
Math 332, Spring 2013

These problems must be written up in \LaTeX{}, and are due this Friday, February 22.

1. Let $G = \left\{ \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \mid a, b \in \mathbb{R} \text{ and } a \neq 0 \right\}$.

   (a) Use the two-step subgroup test to prove that $G$ is a subgroup of $GL(2, \mathbb{R})$.
   (b) Prove that the center of $G$ is trivial.

2. Near the end of Chapter 3, the textbook defines the **centralizer** of a group element. Specifically, if $G$ is a group and $a \in G$, the centralizer of $a$ is the set

   $$C(a) = \{ g \in G \mid ga = ag \}.$$  

   Theorem 3.6 asserts that $C(a)$ is always a subgroup of $G$. Prove this theorem using the two-step subgroup test.