These problems must be written up in \LaTeX, and are due this Friday, March 8.

1. (a) List the possible cycle structures for an element of $S_6$.
   
   (b) Make a table showing the number of elements of $S_6$ with each cycle structure.

2. Let $n \in \mathbb{N}$, and let $G$ be a subgroup of $S_n$. If $i \in \{1, 2, \ldots, n\}$, the \textbf{stabilizer} of $i$ in $G$ is the set
   \[ \text{stab}_G(i) = \{ \alpha \in G \mid \alpha(i) = i \}. \]
   
   Prove that stab$_G(i)$ is a subgroup of $G$.

3. If $n \geq 2$, prove that all of the 2-cycles in $S_n$ are conjugate to one another.

4. In a \textbf{perfect riffle shuffle}, a deck of 52 cards is cut into two halves, which are then merged in an interleaving fashion:

   \[
   \begin{array}{c}
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \end{array}
   \end{array}
   \]

   Note that the top card of the deck remains on top after the shuffle.

   (a) Write an element $\sigma \in S_{52}$ that represents a perfect riffle shuffle of a 52-card deck. Express $\sigma$ both as a table and as a product of disjoint cycles.

   (b) What is $\sigma^8$? In simple terms, what does this say about perfect riffle shuffles?