Homework 6
Math 332, Spring 2013

These problems must be written up in LaTeX, and are due this Friday, March 15.

1. Let $G$ be a group. Let $H$ be a subgroup of $G$, let $x \in G$, and let

$$xHx^{-1} = \{xhx^{-1} \mid h \in H\}.$$  

(a) Prove that $xHx^{-1}$ is a subgroup of $G$.

(b) Prove that $xHx^{-1}$ is abelian if and only if $H$ is abelian.

2. Let $G$ be the group of symmetries of a triangular prism:

Make a list of the 12 elements of $G$, organized into conjugacy classes.

3. Let $n \geq 3$.

(a) If $\sigma, \tau \in S_n$ are 2-cycles, prove that $\sigma \tau$ can be expressed as a product of 3-cycles.

(b) Prove that every element of $A_n$ can be expressed as a product of 3-cycles.

4. Let $\alpha = (1\ 2\ 3)(4\ 5\ 6)$, and let $C(\alpha)$ denote the centralizer of $\alpha$ in $S_6$. List the 18 elements of $C(\alpha)$. (*Hint:* If $\beta \in S_6$, then $\alpha \beta = \beta \alpha$ if and only if $\beta \alpha \beta^{-1} = \alpha$.)