Exam Practice Problems
Math 352, Fall 2011

1. Let \( \gamma: \mathbb{R} \to \mathbb{R}^3 \) be the curve \( \gamma(t) = (t^2 + t, \sin t, e^t) \). Compute the Frenet vectors \( t, n, b \) at the point \((0,0,1)\).

2. Let \( \gamma: \mathbb{R} \to \mathbb{R}^2 \) be the curve defined by
   \[
   \gamma(t) = (2e^t \cos t, 2e^t \sin t, e^t).
   \]
   (a) Find the arc-length of \( \gamma \) from the point \((2,0,1)\) to the point \((-2e^\pi, 0, e^\pi)\).
   (b) Find a unit-speed reparametrization of \( \gamma \).

3. Suppose that a unit-speed curve \( \gamma: \mathbb{R} \to \mathbb{R}^2 \) has signed curvature \( \kappa_s(s) = s^2 \). Given that \( \dot{\gamma}(0) = (0,1) \), find a formula for \( \dot{\gamma}(s) \).

4. Let \( \gamma: [0, 2\pi] \to \mathbb{R} \) be the following curve:

   \[
   \gamma(t) = (\cos t + \cos 2t, \sin t - \sin 2t)
   \]

   Let \( \kappa_s \) denote the signed curvature of \( \gamma \).
   (a) Compute \( \kappa_s \) at the point \((2,0)\).
   (b) Find the equation of the osculating circle for \( \gamma \) at the point \((2,0)\).
   (c) What is the value of \( \int_\gamma \kappa_s \, ds \)?

5. Let \( \gamma \) be a simple closed curve in the plane. Given that the total length of \( \gamma \) is 10, use Green’s Theorem to find the maximum possible value of \( \int_\gamma 2y \, dx + 5x \, dy \).
6. Let \( \gamma : \mathbb{R} \to \mathbb{R}^3 \) be a unit-speed curve, and suppose that \( \|\gamma(t)\| = 1 \) for all \( t \in \mathbb{R} \). Prove that \( \gamma(t) \cdot \dot{\gamma}(t) = -1 \) for all \( t \in \mathbb{R} \).

7. A circle of radius 6 is rolling inside a fixed circle of radius 16:

The fixed circle is centered at the origin, and the smaller circle is initially centered at the point \((0, -10)\). A point \(P\) lies on the circumference of the smaller circle, and initially has coordinates \((-6, -10)\). Find the coordinates of \(P\) when the center of the small circle reaches the point \((10, 0)\).

8. Let \( f : \mathbb{R}^2 \to \mathbb{R} \) be a smooth function, and let \( \gamma(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t)) \) be a curve that lies entirely on the graph \( z = f(x, y) \). Given that \( \gamma(0) = (1, 2, 3) \), \( f_x(1, 2) = 9 \), \( f_y(1, 2) = 4 \), \( \dot{\gamma}_1(t) = 2 \), and \( \dot{\gamma}_2(t) = -3 \), find the unit tangent vector to \( \gamma \) at the point \((1, 2, 3)\).

9. A unit-speed curve \( \gamma : \mathbb{R} \to \mathbb{R}^3 \) has constant curvature 1 and constant torsion 0. Given that \( \gamma(0) = (2, 0, 0) \), \( \dot{\gamma}(0) = (0, 0, 1) \), and \( \ddot{\gamma}(0) = (1, 0, 0) \), compute \( \gamma(\pi/2) \).

10. The following figure shows a circle in \( \mathbb{R}^3 \), as well as two perpendicular diameters:

Find a constant-speed parametrization \( \gamma \) for this circle satisfying \( \gamma(0) = (5, 7, 8) \) and \( \gamma(\pi/2) = (2, 7, 5) \).