Instructions: Solve all four of these problems. Your solutions must be written in \LaTeX.

Due Date: Friday, September 23

1. Let $e(t), f(t), g(t)$ be a rotating, right-handed frame of orthogonal unit vectors. Show that there exists a vector $\omega(t)$ (the rotation vector) such that

$$\dot{e} = \omega \times e, \quad \dot{f} = \omega \times f, \quad \text{and} \quad \dot{g} = \omega \times g.$$ 

(*Hint: Use the fact that the matrix of coefficients for $\dot{e}, \dot{f}, \text{and} \dot{g}$ with respect to the frame is skew-symmetric.*)

2. A frame $e(t), f(t), g(t)$ is rotating counterclockwise around the positive $z$-axis at a rate of 5 radians/sec. The initial positions of the vectors are $e(0) = (1, 0, 0), f(0) = (0, 1, 0)$, and $g(0) = (0, 0, 1)$.

(a) Find formulas for $e(t), f(t), \text{and} g(t)$.

(b) Express $\dot{e}, \dot{f}, \text{and} \dot{g}$ as linear combinations of $e, f, \text{and} g$. What is the rotation vector $\omega(t)$?

3. A frame $e(t), f(t), g(t)$ is rotating counterclockwise around the vector $(1, 1, 1)$ at a rate of 1 radian/sec. The initial positions of the vectors are $e(0) = (1, 0, 0), f(0) = (0, 1, 0)$, and $g(0) = (0, 0, 1)$

(a) Find a formula for $e(t)$. (*Hint: Start by finding $e$ when $t = 2\pi/3, t = 4\pi/3, t = \pi, \text{and} t = \pi/2$.*

(b) Find formulas for $f(t)$ and $g(t)$.

(c) Express $\dot{e}, \dot{f}, \text{and} \dot{g}$ as linear combinations of $e, f, \text{and} g$. What is the rotation vector $\omega(t)$?

4. Let $\gamma : \mathbb{R} \to \mathbb{R}^3$ be a unit-speed curve, and let $t(s), n(s), b(s)$ be the corresponding Frenet frame.

(a) What is the rotation vector $\omega$ for the Frenet frame? Express your answer as a linear combination of $t, n, \text{and} b$.

(b) Show that $\dot{t} \times t = \kappa^2 \omega$. 