Instructions: Solve both of these problems. Your solutions must be written in \LaTeX.

Due Date: Friday, November 4

1. Let \( C \) be the cylinder \( x^2 + y^2 = 1 \), and let \( f: C \to \mathbb{R}^3 \) be the function

\[
f(x, y, z) = (x \cos z, y \cos z, \sin z).
\]

Recall that the vectors \( t_1 = (-y, x, 0) \) and \( t_2 = (0, 0, 1) \) are a basis for the tangent space to \( C \) at each point.

(a) Find the \( 3 \times 2 \) matrix for \( Df \) with respect to the basis \( \{t_1, t_2\} \).
(b) Compute the set of all critical points for \( f \). Describe this set geometrically.
(c) Prove that the image of \( f \) is precisely the unit sphere \( S^2 \).
(d) Find the preimages under \( f \) of the points \((0, 0, 1), (0, 0, -1), \) and \((1, 0, 0)\). Describe these preimages geometrically.

2. Use the method of Lagrange multipliers to solve the following problems:

(a) Find the four critical points for the function \( f(x, y, z) = xy + z^2 \) on the cylinder \( x^2 + y^2 = 1 \).

(b) Find the six critical points for the function \( g(x, y, z) = xz + yz \) on the unit sphere.