Instructions: Solve both of these problems. Your solutions must be written in \LaTeX.

Due Date: Friday, December 2.

1. This problem involves the formulas for the circumference and area of a spherical circle of radius \( r \) (see homework 7).
   
   (a) Let \( \gamma \) be any unit-speed curve on \( S^2 \). Use the formula \( \kappa_n = -\dot{\mathbf{N}} \cdot \mathbf{t} \) to show that the normal curvature of \( \gamma \) is \(-1\).
   
   (b) Let \( C \) be a spherical circle of radius \( r \) on \( S^2 \), oriented counterclockwise. Find the curvature \( \kappa \) of \( C \), and use your answer to find the geodesic curvature \( \kappa_g \) of \( C \).
   
   (c) Use the formula for the circumference of a spherical circle to compute \( \int_C \kappa_g \, ds \). Is your answer consistent with the Gauss-Bonnet Theorem?

2. Let \( P \) be the paraboloid \( z = x^2 + y^2 \), let \( p \) be the point \((1,1,2)\), and let \( \sigma: \mathbb{R}^2 \to P \) be the surface patch \( \sigma(u,v) = (u,v,u^2 + v^2) \).

   (a) Compute the matrix for the second fundamental form of \( P \) at \( p \) with respect to the basis \( \{ \sigma_u, \sigma_v \} \).

   (b) Let \( t_1, t_2 \in T_p P \) be the vectors

\[
    t_1 = \frac{1}{\sqrt{2}} (1, -1, 0) \quad \text{and} \quad t_2 = \frac{1}{3\sqrt{2}} (1, 1, 4).
\]

   Express \( t_1 \) and \( t_2 \) as linear combinations of \( \sigma_u \) and \( \sigma_v \) at the point \( p \).

   (c) Use your answers to parts (a) and (b) to compute the matrix for the second fundamental form of \( P \) at \( p \) with respect to the orthonormal basis \( \{ t_1, t_2 \} \).

   (d) What are the principal curvatures of \( P \) at the point \( p \)? What is the Gaussian curvature?