DATA AND DECISIONS

HOMEWORK EXERCISES

Very Preliminary

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1 Voting Exercises

Section 1.2: Homework Exercise 1 *[Voting Methods]*

Suppose that an election has candidates A, B, C and D. There are 46 voters, who submit the following ranked ballots.

12	5	4	9	6	10
C	B	D	A	A	D
D	C	A	C	D	B
B	A	B	B	C	A
A	D	C	D	B	C.

Find the winner (or winners), if there are any winners, by each of the following methods.

SHOW YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS.

(a) Plurality Voting.

(b) Single Runoff Voting.

(c) Instant Runoff Voting.

(d) Borda Count Voting.

(e) Sequential Pairwise Voting with Fixed Agenda, with agenda A, B, C, D.

(f) Condorcet Voting.

(g) Copeland Voting.

(h) Dictatorship, with one of the voters in the right hand column the dictator.

Lippman, Math in Society, 2.4th ed., p. 54, Exercise 5

Section 1.2: Homework Exercise 2 *[Voting Methods]*

Suppose that an election has candidates A, B, C, D and E. There are 7 voters, who submit the following ranked ballots.

2	1	1	1	1	1
C	D	C	B	E	D
A	A	E	D	D	A
E	E	D	A	A	E
B	C	A	E	C	B
D	B	B	C	B	C.

Find the winner (or winners), if there are any winners, by each of the following methods.

SHOW YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS.

(a) Plurality Voting.

(b) Single Runoff Voting.

(c) Instant Runoff Voting.

(d) Borda Count Voting.

(e) Sequential Pairwise Voting with Fixed Agenda, with agenda A, B, C, D, E.

(f) Condorcet Voting.

(g) Copeland Voting.

(h) Dictatorship, with the voter in the right hand column the dictator.

Taylor-Pacelli, Mathematics and Politics, 2nd ed., Chapter 1, Exercise 3

Section 1.2: Homework Exercise 3 *[Voting Methods]*

Suppose that an election has candidates A, B, C and D. There are 4 voters, who submit the following ranked ballots.

1	1	1	1
B	C	C	A
A	A	D	D
D	B	B	B
C	D	A	C.

SHOW YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS.

(a) Find the winner using Borda Count Voting.

(b) Find the winner using a modified version of Borda Count Voting, where instead of assigning 4, 3, 2 and 1 points to each of first place, second place, third place and fourth place ranking, respective, we assign 6, 1, -1 and -6 points, respectively.

(c) Find the winner using a modified version of Borda Count Voting, where instead of assigning 4, 3, 2 and 1 points to each of first place, second place, third place and fourth place ranking, respective, we assign 1, -1, -3 and -5 points, respectively.

(d) Find the winner using a modified version of Borda Count Voting, where instead of assigning 4, 3, 2 and 1 points to each of first place, second place, third place and fourth place ranking, respective, we assign 9, 4, 1 and 0 points, respectively.

(e) Does the choice of points in the modified versions of Borda Count Voting make a difference? If it does, try to find a rule for identifying those ways of assigning points that will give the same results as standard Borda Count Voting.

Taylor-Pacelli, Mathematics and Politics, 2nd ed., Chapter 1, Exercise 9

Section 1.2: Homework Exercise 4 *[Voting Methods]*

Suppose that an election has candidates A, B, C and D. There are 3 voters, who submit the following ranked ballots:

1	1	1
A	C	B
B	A	D
D	B	C
C	D	A.

This election is to be conducted using Sequential Pairwise Voting with Fixed Agenda, with an agenda that you choose. SHOW YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS.

(a) Find an agenda so that the winner is candidate A.

(b) Find an agenda so that the winner is candidate B.

(c) Find an agenda so that the winner is candidate C.

(d) Find an agenda so that the winner is candidate D.

Taylor-Pacelli, Mathematics and Politics, 2nd ed., Chapter 1, Exercise 12

Section 1.3: Homework Exercise 1 *[Voting Methods]*

In each of the following scenarios, state which one of the fairness criteria for voting with ranked ballots is violated.

(a) An election resulted in Candidate A winning, with Candidate B coming in a close second, and candidate C being a distant third. For some reason, the election had to be held again, and C decided to drop out of the election, and then B became the winner.

(b) An election resulted in Candidate A winning, with Candidate B coming in a close second, and candidate C being a distant third. For some reason, the election had to be held again, and many people who had voted for C switched their preferences to favor A, and then B became the winner.

(c) An election resulted in Candidate A winning, with Candidate B coming in a close second, and candidate C being a distant third. In a head-to-head comparison a majority of people prefer B to each of A and C.

(d) An election resulted in Candidate A winning, with Candidate B coming in a close second, and candidate C being a distant third. In this election B had received a majority of first place votes.

Lippman, Math in Society, 2.4th ed., p. 55–56, Exercises 13–16

Section 1.3: Homework Exercise 2 *[Voting Methods]*

An election uses the following social choice method: Condorcet Voting is tried first, and if that method produces a winner, then that candidate is the winner, and if that method does not produce a winner, Borda Count Voting is used.

SHOW YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS.

(a) Does this social choice method satisfy Always a Winner Criterion?

(b) Does this social choice method satisfy Majority Criterion?

(c) Does this social choice method satisfy Condorcet Winner Criterion?

(d) Does this social choice method satisfy Pareto Criterion?

(e) Does this social choice method satisfy Monotonicity Criterion?

(f) Does this social choice method satisfy Independence of Irrelevant Alternatives Criterion?

Taylor-Pacelli, Mathematics and Politics, 2nd ed., Chapter 1, Exercise 13

Section 1.3: Homework Exercise 3 *[Voting Methods]*

Is there a social choice method that satisfies Always a Winner Criterion, Pareto Criterion, Majority Criterion and Monotonicity Criterion? If yes, state one such method; if no, explain why not.

Section 1.5: Homework Exercise 1 *[Voting Methods]*

Suppose that an election has candidates A, B, C and D. There are 35 voters, who submit the following Approval Voting ballots.

Candidate	8	7	6	3	4	2	5
A	X	X			X		X
B	X		X	X			X
C		X	X	X		X	
D	X		X		X	X	.

Find the winner using Approval Voting.

Lippman, Math in Society, 2.4th ed., p. 54, Exercise 5

Section 1.5: Homework Exercise 2 *[Voting Methods]*

All parts of this exercise relate to Approval Voting.

SHOW YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS.

(a) Does Approval Voting satisfy the Always a Winner Criterion?

(b) The Pareto Criterion for Approval Voting would be the following variant of the standard Pareto Criterion: if all voters rank approve candidate A and do not approve candidate B, then candidate B is not the winner (or tied for winner). Does Approval Voting satisfy this variant of the Pareto Criterion?

(c) The Monotonicity Criterion for Approval Voting would be the following variant of the standard Monotonicity Criterion: if candidate A is the winner (or tied for winner), and if one or more voters change their ballots by changing A from not approved to approved, then A would still be the winner. Does Approval Voting satisfy this variant of the Monotonicity Criterion?

(d) Does Approval Voting satisfy the Independence of Irrelevant Alternatives Criterion?

Section 1.6: Homework Exercise 1 *[Voting Methods]*

Suppose that an election has candidates A, B and C. There are 38 voters, who submit the following ranked ballots. The election uses Borda Count Voting.

20	18
A	B
B	A
C	C.

Explain why voters in the second column might be inclined to vote insincerely. How could it affect the outcome of the election?

Lippman, Math in Society, 2.4th ed., p. 56, Exercise 18

Section 2.1: Homework Exercise 1 *[Apportionment Methods]*

The number of salespeople assigned to work during a shift is apportioned based on the average number of customers during that shift. Apportion 25 salespeople given the information below, with the apportionment method listed.

Shift	**Average Number of Customers**
Morning	95
Midday	305
Afternoon	435
Evening	515.

Compute the apportionment using Hamilton's method.

Lippman, Math in Society, 2.4th ed., p. 91, Exercise 4

Section 2.1: Homework Exercise 2 *[Apportionment Methods]*

A small country consists of six states, whose populations are listed below. If the legislature has 200 seats, apportion the seats, with the apportionment method listed.

State	Population
A	3,411
B	2,421
C	11,586
D	4,494
E	3,126
F	4,962.

Compute the apportionment using Hamilton's method.

Lippman, Math in Society, 2.4th ed., p. 91, Exercise 8

Section 2.1: Homework Exercise 3 *[Apportionment Methods]*

A state with five counties has 50 seats in their legislature. The populations from each of the 2000 census and the 2010 census are shown below.

State	2000	2010
A	60,000	60,000
B	31,200	31,200
C	69,200	72,400
D	81,600	81,600
E	118,000	118,400.

(a) Compute the apportionment based upon the 2000 census, using Hamilton's method.

(b) Compute the apportionment based upon the 2010 census, using Hamilton's method.

(c) Which apportionment paradox do the above two calculations illustrate?

Lippman, Math in Society, 2.4th ed., p. 92, Exercise 10

Section 2.2: Homework Exercise 1 *[Apportionment Methods]*

The number of salespeople assigned to work during a shift is apportioned based on the average number of customers during that shift. Apportion 25 salespeople given the information below, with the apportionment method listed.

Shift	**Average Number of Customers**
Morning	95
Midday	305
Afternoon	435
Evening	515.

(a) Compute the apportionment using Jefferson's method.

(b) Compute the apportionment using Adams' method.

(c) Compute the apportionment using Webster's method.

(d) Compute the apportionment using the Huntington-Hill method.

Lippman, Math in Society, 2.4th ed., p. 91, Exercise 4

Section 2.2: Homework Exercise 2 *[Apportionment Methods]*

A small country consists of six states, whose populations are listed below. If the legislature has 200 seats, apportion the seats, with the apportionment method listed.

State	Population
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(a) Compute the apportionment using Jefferson's method.

(b) Compute the apportionment using Adams' method.

(c) Compute the apportionment using Webster's method.

(d) Compute the apportionment using the Huntington-Hill method.

Lippman, Math in Society, 2.4th ed., p. 91, Exercise 8

Section 2.4: Homework Exercise 1 *[Gerrymandering]*

A small state has residents who are members of the *X* party and the *O* party, and who live in the locations seen in the following figure. The state is divided in five districts, each with five voters. Draw five districts, all contiguous and with equal populations, using grid lines, such that the *O* party wins the largest possible number of the districts.

In addition to drawing the five districts, explain why it would not be possible to draw districts that would have the O party win more districts than in your drawing.

X	O	X	O	X
O	O	O	X	X
O	X	X	X	O
O	X	O	O	O
O	X	O	X	O

Section 2.4: Homework Exercise 2 *[Gerrymandering]*

A small state has residents who are members of the *X* party and the *O* party, and who live in the locations seen in the following figure. The state is divided in five districts, each with five voters. Draw five districts, all contiguous and with equal populations, using grid lines, such that the *X* party wins the largest possible number of the districts.

In addition to drawing the five districts, explain why it would not be possible to draw districts that would have the X party win more districts than in your drawing.

X	O	X	O	X
O	O	O	X	X
O	X	X	X	O
O	X	O	O	O
O	X	O	X	O

Section 2.4: Homework Exercise 3 *[Gerrymandering]*

A small state has residents who are members of the *X* party and the *O* party, and who live in the locations seen in the following figure. The state is divided in five districts, each with five voters. Draw five districts, all contiguous and with equal populations, using grid lines, that is proportional to the voters.

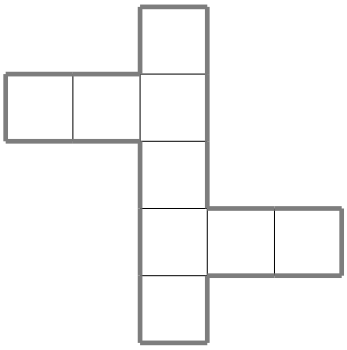
X	O	X	O	X
O	O	O	X	X
O	X	X	X	O
O	X	O	O	O
O	X	O	X	O

Section 2.4: Homework Exercise 4 *[Gerrymandering]*

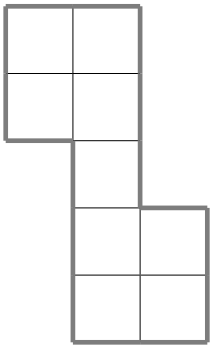
For each of the two regions shown below, find the following three numbers.

1. The grid Polsby-Popper ratio.
2. The grid Reock ratio.
3. The grid convex hull ratio.

(a)



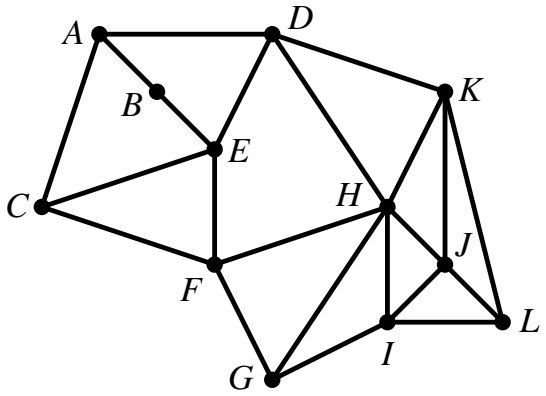
(b)



2 Networks Exercises

Section 3.2: Homework Exercise 1 *[Graph Basics]*

All parts of this problem refer to the following graph.



(a) List the degree of each of the vertices of the graph.

(b) What is the maximum degree among all the vertices of the graph?

(c) What is the minimum degree among all the vertices of the graph?

(d) Is the graph connected?

(e) Give an example of a connected subgraph of the graph that has five vertices.

(f) Give an example of a disconnected subgraph of the graph that has five vertices.

Section 3.2: Homework Exercise 2 *[Graph Basics]*

For each part of this problem, draw the graph that is listed.

(a) P_7

(b) C_5

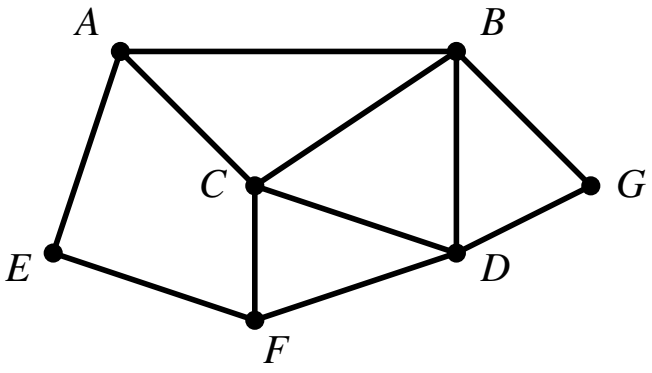
(c) K_6

(d) $K_{4,3}$

Section 3.4: Homework Exercise 1 *[Euler Paths and Euler Circuits]*

For the graph shown below, either find an Euler path or Euler circuit, or explain why there is no Euler path or Euler circuit.

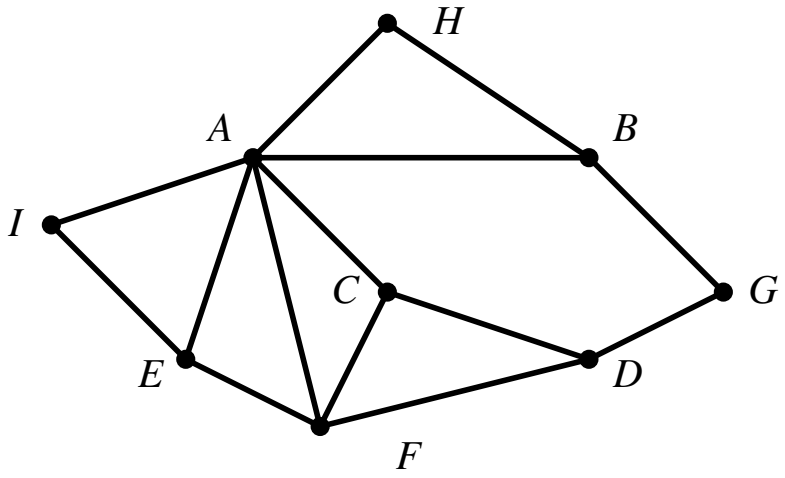
If you are showing that there is an Euler path or Euler circuit, label the edges of the graph using numbers 1, 2, 3, etc., in the order that they are traversed in the Euler path or Euler circuit.



Section 3.4: Homework Exercise 2 *[Euler Paths and Euler Circuits]*

For the graph shown below, either find an Euler path or Euler circuit, or explain why there is no Euler path or Euler circuit.

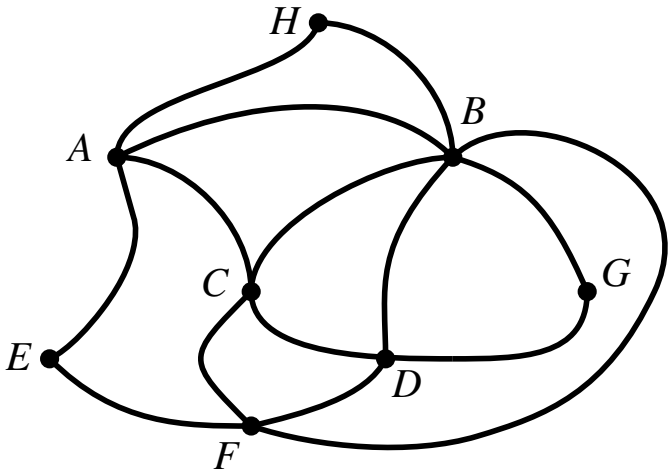
If you are showing that there is an Euler path or Euler circuit, label the edges of the graph using numbers 1, 2, 3, etc., in the order that they are traversed in the Euler path or Euler circuit.



Section 3.4: Homework Exercise 3 *[Euler Paths and Euler Circuits]*

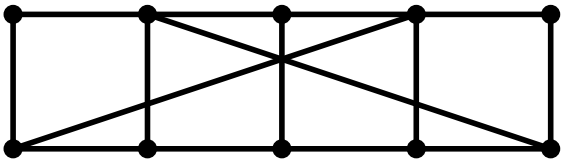
For the graph shown below, either find an Euler path or Euler circuit, or explain why there is no Euler path or Euler circuit.

If you are showing that there is an Euler path or Euler circuit, label the edges of the graph using numbers 1, 2, 3, etc., in the order that they are traversed in the Euler path or Euler circuit.



Section 3.5: Homework Exercise 1 *[Vertex Coloring]*

All parts of this exercise refer to the graph shown below.



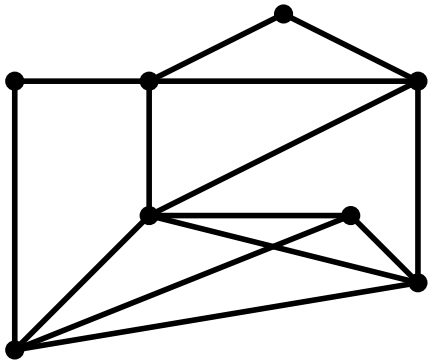
(a) Color the graph (using numbers) with the fewest possible colors.

(b) Explain how you know that your coloring of the graph uses the fewest possible colors.

(c) What is the chromatic number of the graph?

Section 3.5: Homework Exercise 2 *[Vertex Coloring]*

All parts of this exercise refer to the graph shown below.



(a) Color the graph (using numbers) with the fewest possible colors.

(b) Explain how you know that your coloring of the graph uses the fewest possible colors.

(c) What is the chromatic number of the graph?

Section 3.5: Homework Exercise 3 *[Vertex Coloring]*

The Federal Communications Commission (FCC) makes sure that the broadcast from one radio station does not interfere with the broadcast from any other radio station, which is accomplished by requiring that stations within transmitting range of each other must use different frequencies. Suppose that the FCC approves a new law where stations within 500 miles of each other must be assigned different frequencies. Consider seven stations, denoted A, B, C, D, E and F, where the distances between the stations in miles are given in the following grid. The FCC wants to assign a frequency to each station so that no two stations interfere with each other, and also wants to assign the fewest possible number of frequencies.

	A	B	C	D	E	F	G
A	–	450	550	700	600	850	900
B	450	–	500	300	250	600	750
C	550	500	–	100	530	800	900
D	700	300	100	–	470	650	700
E	600	250	530	470	–	350	490
F	850	600	800	650	350	–	530
G	900	750	900	700	490	530	–

(a) Make a graph to represent the above situation.

(b) Use vertex coloring to determine the smallest number of different frequencies so that no two stations interfere with each other.

(c) Explain how you know that your answer is indeed the smallest number of frequencies that are needed.

Section 3.5: Homework Exercise 4 *[Vertex Coloring]*

All parts of this problem refer to the following map. The goal of this problem is to determine the smallest number of colors needed to color the map so that if two states share a border they have different colors.

(a) Make a graph to represent the above map.

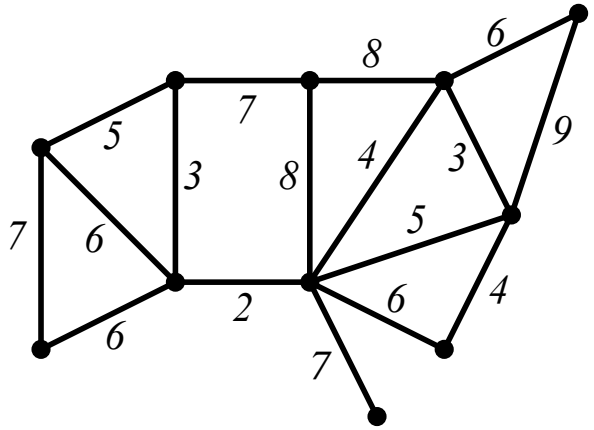
(b) Use vertex coloring to determine the smallest number of colors needed to color the map so that if two states share a border they have different colors.

(c) Explain how you know that your answer is indeed the smallest number of colors that are needed.

Smithers, Graph Theory Notes, p. 46, Exercise 3(b)

Section 3.6: Homework Exercise 1 *[Minimum Spanning Trees]*

Both parts of this exercise refer to the graph shown below.



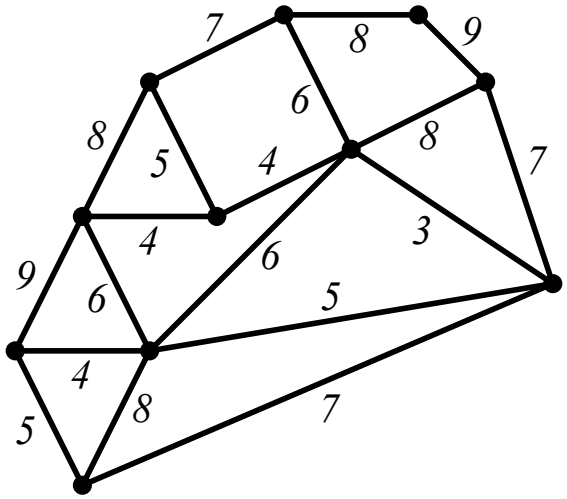
(a) Use Prim's Algorithm to find a minimum spanning tree for the graph.

Show (by shading or otherwise marking) the edges that are in the minimum spanning tree.

(b) What is the length of the minimum spanning tree that you found?

Section 3.6: Homework Exercise 2 *[Minimum Spanning Trees]*

Both parts of this exercise refer to the graph shown below.



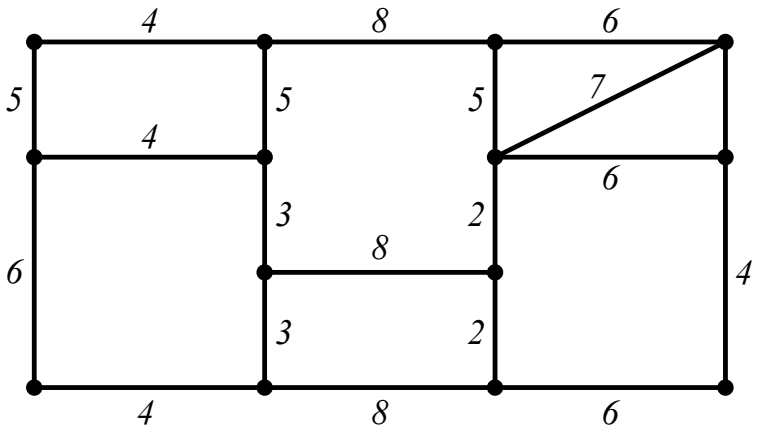
(a) Use Prim's Algorithm to find a minimum spanning tree for the graph.

Show (by shading or otherwise marking) the edges that are in the minimum spanning tree.

(b) What is the length of the minimum spanning tree that you found?

Section 3.6: Homework Exercise 3 *[Minimum Spanning Trees]*

The computers in each of the offices at Earl of March High School need to be linked by cable. The map below shows the cost of each link in hundreds of dollars.



(a) Use Prim's Algorithm to determine the least costly way of linking the 14 offices.

(b) What is the minimum cost of linking the 14 offices?

Section 3.6: Homework Exercise 4 *[Minimum Spanning Trees]*

A local restaurant has opened an outdoor patio for the summer. The owner wants to hang nine festive light fixtures at designated locations on the overhead latticework. Because of the layout of the patio and the latticework, it is not possible to install wiring between every pair of lights. The grid below shows the distances in feet between lights that can be linked directly. The owner wants to use the minimum amount of wire to get all nine lights connected.

	A	B	C	D	E	F	G	H	I
A	–	16	–	–	15	19	–	–	–
B	16	–	16	12	–	–	–	–	–
C	–	16	–	–	–	–	12	–	–
D	–	12	–	–	–	–	10	–	–
E	15	–	–	–	–	7	–	–	–
F	19	–	–	–	7	–	–	–	–
G	–	–	12	10	–	–	–	18	–
H	–	–	–	–	–	–	18	–	8
I	–	–	–	–	–	–	–	8	–

(a) Make a graph to represent the above situation.

(b) Use Prim's Algorithm to determine the most efficient way to connect all nine lights.

(c) What is the minimum amount of wire needed to connect all nine lights?

Smithers, Graph Theory Notes, p. 77, Exercise 1

Section 4.1: Homework Exercise 1 *[Summation Notation]*

The four parts of this exercise are not related.
Do not actually calculate the numerical value of the sums in this exercise.

(a) Rewrite the following in summation notation.

$$(10\cdot 1 + 8) + (10\cdot 2 + 8) + (10\cdot 3 + 8) + (10\cdot 4 + 8) + (10\cdot 5 + 8).$$

(b) Rewrite the following in summation notation.

$$\sqrt{3} + \sqrt{4} + \sqrt{5} + \cdots + \sqrt{40}.$$

(c) Write out all the terms of the following sum.

$$\sum_{i=1}^{4} \frac{i}{2}.$$

(d) Write out all the terms of the following sum.

$$\sum_{k=3}^{8} (k^2 + 3).$$

Section 4.1: Homework Exercise 2 *[Summation Notation]*

Let $x_1, x_2, \ldots, x_6$ be defined to be $6, 3, -2, 5, 4, 1$, respectively.

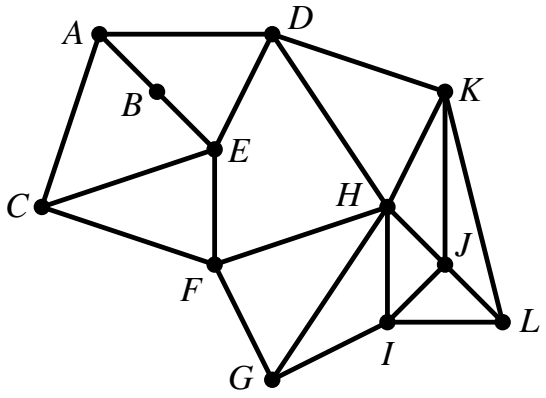
Calculate the values of the following sums.

(a) $\displaystyle\sum_{p=1}^{4} x_p$.

(b) $\displaystyle\sum_{i=2}^{6} (x_i)^2$.

Section 4.2: Homework Exercise 1 *[Average Degree]*

All parts of this problem refer to the following graph.



(a) List the degree of each of the vertices of the graph.

(b) Find the average degree of the vertices of the graph.

(c) Are there more vertices with degree below the average or above the average, or is there the same number of each?

Section 4.2: Homework Exercise 2 *[Average Degree]*

All parts of this problem refer to the following map.

(a) Make a graph to represent the above map.

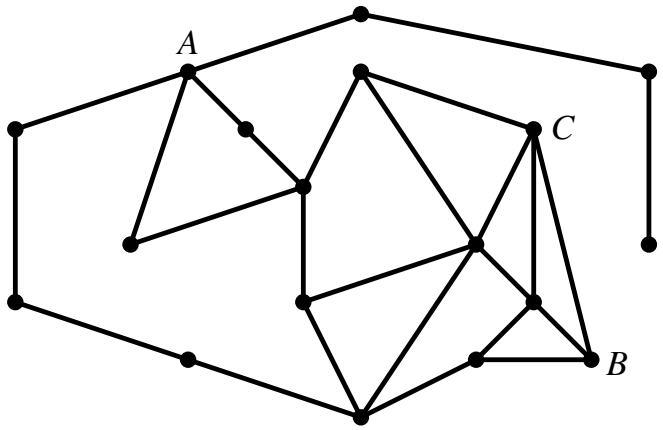
(b) List the degree of each of the vertices of the graph.

(c) Find the average degree of the vertices of the graph.

(d) Are there more vertices with degree below the average or above the average, or is there the same number of each?

Section 4.3: Homework Exercise 1 *[Breadth-First Search]*

All parts of this problem refer to the following graph.



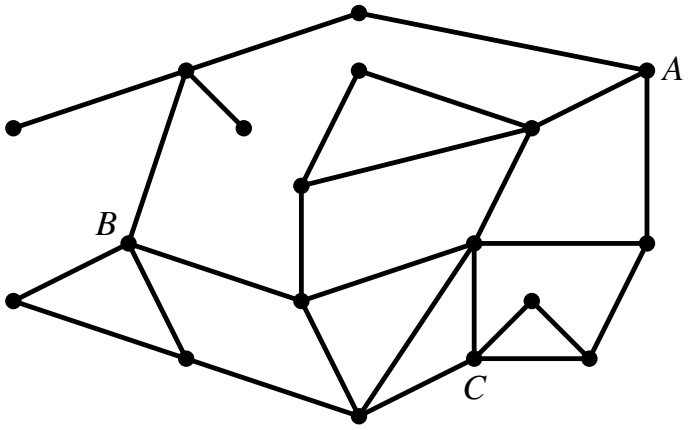
(a) Label all the vertices of the graph using Breadth-First Search starting at vertex A.

(b) Use your results from Part (a) to find the distance from A to each of the vertices B and C.

(c) Use your results from Part (a) to find a spanning tree for the graph.

Section 4.3: Homework Exercise 2 *[Breadth-First Search]*

All parts of this problem refer to the following graph.



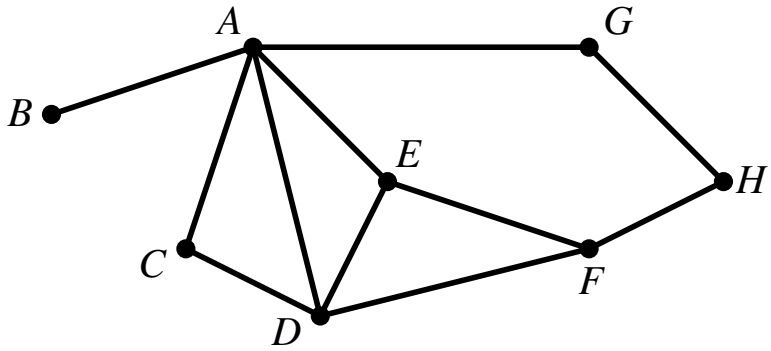
(a) Label all the vertices of the graph using Breadth-First Search starting at vertex A.

(b) Use your results from Part (a) to find the distance from A to each of the vertices B and C.

(c) Use your results from Part (a) to find a spanning tree for the graph.

Section 4.3: Homework Exercise 3 *[Breadth-First Search]*

All parts of this problem refer to the following graph.



(a) Find the distance from each vertex of the graph to every other vertex of the graph by using Breadth-First Search starting at each vertex of the graph.

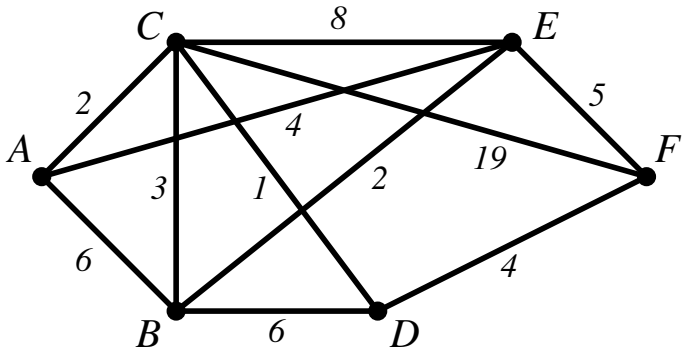
(b) Find the diameter of the graph.

(c) Find the average distance between the vertices of the graph.

Hint: When you find the average distance between the vertices of the graph, make sure you count the distance between each pair of vertices only once.

Section 4.4: Homework Exercise 1 *[Dijkstra's Algorithm]*

All parts of this exercise refer to the following weighted graph.



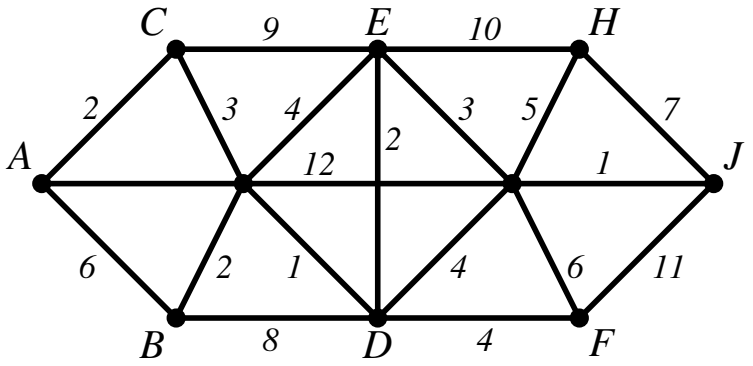
(a) Apply Dijkstra's Algorithm to this graph, where the starting vertex is A.

(b) State the weighted distance in the graph from A to each of the other vertices.

(c) State what the shortest path from A to F is. Give your answer by listing the vertices encountered on this shortest path, starting with vertex A and ending in vertex F.

Section 4.4: Homework Exercise 2 *[Dijkstra's Algorithm]*

All parts of this exercise refer to the following weighted graph.



(a) Apply Dijkstra's Algorithm to this graph, where the starting vertex is A.

(b) State the weighted distance in the graph from A to each of the other vertices.

(c) State what the shortest path from A to G is. Give your answer by listing the vertices encountered on this shortest path, starting with vertex A and ending in vertex G.

Section 4.4: Homework Exercise 3 *[Dijkstra's Algorithm]*

A local restaurant has opened an outdoor patio for the summer. The owner wants to hang nine festive light fixtures at designated locations on the overhead latticework. Because of the layout of the patio and the latticework, it is not possible to install wiring between every pair of lights. The grid below shows the distances in feet between lights that can be linked directly. The owner wants to use the minimum amount of wire to get all nine lights connected.

	A	B	C	D	E	F	G	H	I
A	–	16	–	3	15	15	–	–	–
B	16	–	16	12	–	–	–	–	–
C	–	16	–	–	–	4	12	–	–
D	3	12	–	–	–	–	10	7	–
E	15	–	–	–	–	7	–	–	–
F	15	–	4	–	7	–	–	–	–
G	–	–	12	10	–	–	–	18	–
H	–	–	–	7	–	–	18	–	8
I	–	–	–	–	–	–	–	8	–

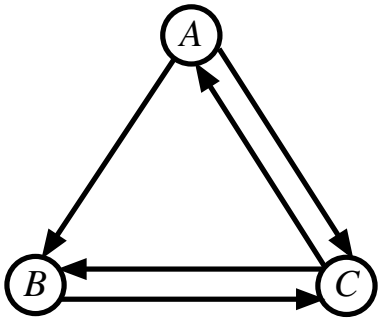
(a) Make a graph to represent the above situation.

(b) Use Dijkstra's Algorithm to find the shortest distance of wire connecting light A to each of the other lights.

Smithers, Graph Theory Notes, p. 77, Exercise 1

Section 4.5: Homework Exercise 1 *[PageRank]*

The goal of this exercise is to do the first few steps for finding the PageRank for the following directed graph. Give all your answers to three decimal places.



(a) Find the initial scores for each vertex.

(b) Redistribute the scores.

(c) Find the modified scores using $p = 0.85$.

(d) Redistribute the scores again.

(e) Find the modified scores again using $p = 0.85$.

Section 5.1: Homework Exercise 1 *[Matrices]*

Let

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 5 & -2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix}.$$

For each of the following, calculate it if possible, or say why it is not possible.

(a) $5A$.

(b) $A + B$.

(c) $A + C$.

(d) $3A - 2B$.

Section 5.2: Homework Exercise 1 *[Matrices]*

Let

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 5 & -2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} \quad \text{and} \quad W = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

For each of the following, calculate it if possible, or say why it is not possible.

(a) AW.

(b) CW.

(c) AB.

(d) BA.

(e) AC.

(f) CA.

(g) A^2.

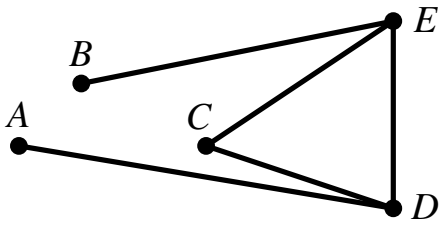
(h) C^2.

(i) $A^2 - 3A$.

Section 5.3: Homework Exercise 1 *[Application of Matrices to Graphs]*

All parts of this exercise refer to the graph shown below.

Give all your answers to three decimal places.



(a) Find the adjacency matrix M.

(b) Compute M^2.

(c) Verify that the diagonal entries of M^2 represent the degrees of the vertices of the graph?

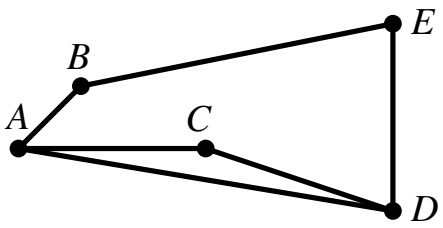
(d) Compute M^3.

(e) Verify that the non-zero diagonal entries of M^3 correspond to the vertices that are in a clique (or cliques)?

Section 5.3: Homework Exercise 2 *[Application of Matrices to Graphs]*

All parts of this exercise refer to the graph shown below.

Give all your answers to three decimal places.



(a) Find the adjacency matrix M.

(b) Compute M^2.

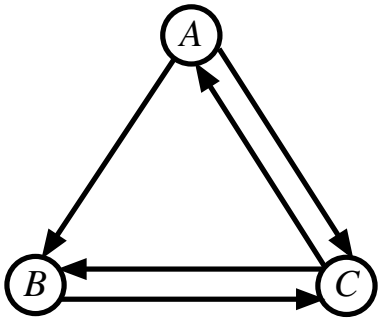
(c) Verify that the diagonal entries of M^2 represent the degrees of the vertices of the graph?

(d) Compute M^3.

(e) Verify that the non-zero diagonal entries of M^3 correspond to the vertices that are in a clique (or cliques)?

Section 5.4: Homework Exercise 1 *[PageRank]*

The goal of this exercise is to do the first few steps for finding the PageRank for the following directed graph, using matrices. Give all your answers to three decimal places.



(a) Find the initial vector of values v.

(b) Find the transition matrix A.

(c) Let $p = 0.85$. Find the modified transition matrix M.

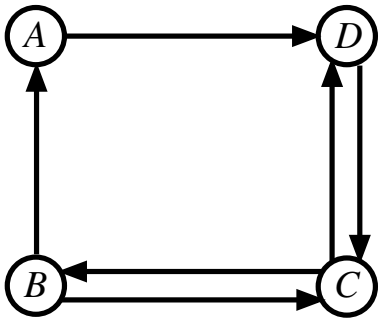
(d) Compute Mv.

(e) Compute M^2v.

(f) Compute M^3v.

Section 5.4: Homework Exercise 2 *[PageRank]*

The goal of this exercise is to do the first few steps for finding the PageRank for the following directed graph, using matrices. Give all your answers to three decimal places.



(a) Find the initial vector of values v.

(b) Find the transition matrix A.

(c) Let $p = 0.75$. Find the modified transition matrix M.

(d) Compute Mv.

(e) Compute M^2v.

(f) Compute M^3v.

3 Statistics Exercises

Section 7.2: Homework Exercise 1 *[Frequency Tables and Histograms]*

Both parts of this exercise refer to the following data.

$$18, 6, 16, 13, 1, 11, 7, 8, 12, 6, 19, 5, 2, 3, 9, 17, 9, 17.$$

(a) Construct a frequency table for the above data set, using bins of the form 1–5, 6–10, and so on.

(b) Construct a histogram for the above data set, using the same bins used for the frequency table.

Section 7.2: Homework Exercise 2 *[Frequency Tables and Histograms]*

Both parts of this exercise refer to the following data. In an animal shelter, a record was made of the weight in ounces of newborn beagle puppies, which was as follows.

6.0, 8.3, 9.1, 5.2, 8.0, 6.8, 8.3, 5.7, 9.5, 7.9, 6.2, 9.8, 5.6, 6.4, 9.9, 8.0, 8.7, 9.0, 7.4, 6.1, 6.5, 8.3, 6.7, 7.5, 6.8

(a) Construct a frequency table for the above data set, using bins of the form 5.0–5.9, 6.0–6.9, and so on.

(b) Construct a histogram for the above data set, using the same bins used for the frequency table.

Section 7.3: Homework Exercise 1 *[Statistics Basics]*

All parts of this exercise refer to the numbers

$$4, 10, 1, 3, 1, 5.$$

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the mean of these numbers.

(b) Find the median of these numbers.

(c) Find the mode of these numbers.

Section 7.3: Homework Exercise 2 *[Statistics Basics]*

All parts of this exercise refer to the numbers

$$5, 3, -5, 1, -4, 3, 4.$$

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the mean of these numbers.

(b) Find the median of these numbers.

(c) Find the mode of these numbers.

Section 7.3: Homework Exercise 3 *[Statistics Basics]*

All parts of this exercise refer to the numbers given in the frequency table

value	frequency
1	2
2	3
4	1
6	2
8	2.

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the mean of these numbers.

(b) Find the median of these numbers.

(c) Find the mode of these numbers.

Section 7.4: Homework Exercise 1 *[Statistics Basics]*

All parts of this exercise refer to the numbers

$$4, 10, 1, 3, 1, 5.$$

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the range of these numbers.

(b) Find the variance of these numbers.

(c) Find the standard deviation of these numbers.

Section 7.4: Homework Exercise 2 *[Statistics Basics]*

All parts of this exercise refer to the numbers

$$5, 3, -5, 1, -4, 3, 4.$$

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the range of these numbers.

(b) Find the variance of these numbers.

(c) Find the standard deviation of these numbers.

Section 7.4: Homework Exercise 3 *[Statistics Basics]*

All parts of this exercise refer to the numbers given in the frequency table

value	frequency
1	2
2	3
4	1
6	2
8	2.

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the range of these numbers.

(b) Find the variance of these numbers.

(c) Find the standard deviation of these numbers.

Section 7.5: Homework Exercise 1 *[Statistics Basics on Google Sheets]*

For this exercise you will need to access a Google Sheets spreadsheet that is available at the class website.

The purpose of this exercise is to calculate basic statistical information from actual data on a spreadsheet, and to practice a few other basic spreadsheet skills.

(a) Go to the class website, and click on the link to "Oscars (Google Sheets)," which is found under the heading Statistics Materials.

(b) When the Google Sheets opens, it will be "view only."

(c) In the "File" menu, click on "Make a Copy."

(d) Add your name in the title of the copy (but leave the original title too).

(e) Your copy of the spreadsheet will be editable. Answer the questions listed in the spreadsheet.

(f) Share your copy of the spreadsheet with the instructor, by clicking on the "Share" command on the right side of the menu, and inserting the instructor's email address.

Section 8.2: Homework Exercise 1 *[Probability]*

For all parts of this exercise, a coin is tossed four times.

(a) What is the probability of getting four heads?

(b) What is the probability of getting first three heads and then one tail?

(c) What is the probability of getting three heads and one tail in any possible order?

(d) Suppose you toss the coin three times and get a head each time. What is the probability of getting a tail the fourth time you toss the coin?

Section 8.2: Homework Exercise 2 *[Probability]*

For all parts of this exercise, three dice are rolled, one after the other; the numbers on the dice are recorded separately, and are not added.

(a) What is the probability of getting a 2 on each of the three dice?

(b) What is the probability of getting a 2 on the first die, a 4 on the second die, and a 6 on the third die?

(c) What is the probability of getting even numbers on all three dice?

(d) What is the probability of getting an even number on each of the first two dice and any number on the third die?

(e) What is the probability of getting a 2 on each of the three dice, or getting a 4 on each of the three dice?

(f) What is the probability of getting an even number on each of the first two dice and any number on the third die, or getting any number on the first die and an even number on the second and third dice?

Section 8.2: Homework Exercise 3 *[Probability]*

For all parts of this exercise, a card is drawn from a standard deck of cards (with no jokers), the card is replaced, the deck is shuffled, and a card is drawn again.

(a) What is the probability of getting a red card both times a card is drawn?

(b) What is the probability of getting a queen the first time a card is drawn and a red card the second time a card is drawn?

(c) What is the probability of getting a 10 both times a card is drawn, or getting an ace both times a card is drawn?

(d) What is the probability of getting a red card both times a card is drawn, or getting a king both times a card is drawn?

Section 8.2: Homework Exercise 4 *[Probability]*

For all parts of this exercise, two dice are rolled, and the two numbers are added.

(a) What is the probability of getting a sum of 5?

(b) What is the probability of getting a sum of 2 or 10?

(c) What is the probability of getting a sum that is less than 5?

Section 8.2: Homework Exercise 5 *[Gathering Data on Google Sheets]*

For this exercise, you will need 10 coins, and you will need to access a Google Sheets spreadsheet that is available at the class website.

The purpose of this exercise is to gather some actual data, for use in class.

(a) Go to the class website, and click on the link to "Coin Flipping Data," which is found under the heading Statistics Materials.

(b) When the Google Sheets opens, it will be "view only."

(c) In the "File" menu, click on "Make a Copy."

(d) Add your name in the title of the copy (but leave the original title too).

(e) Your copy of the spreadsheet will be editable.

(f) Take 10 coins.

(g) Flip all 10 coins at once.

(h) Count the number of heads.

(i) Record the number of heads.

(j) Repeat the trial a total of 15 times.

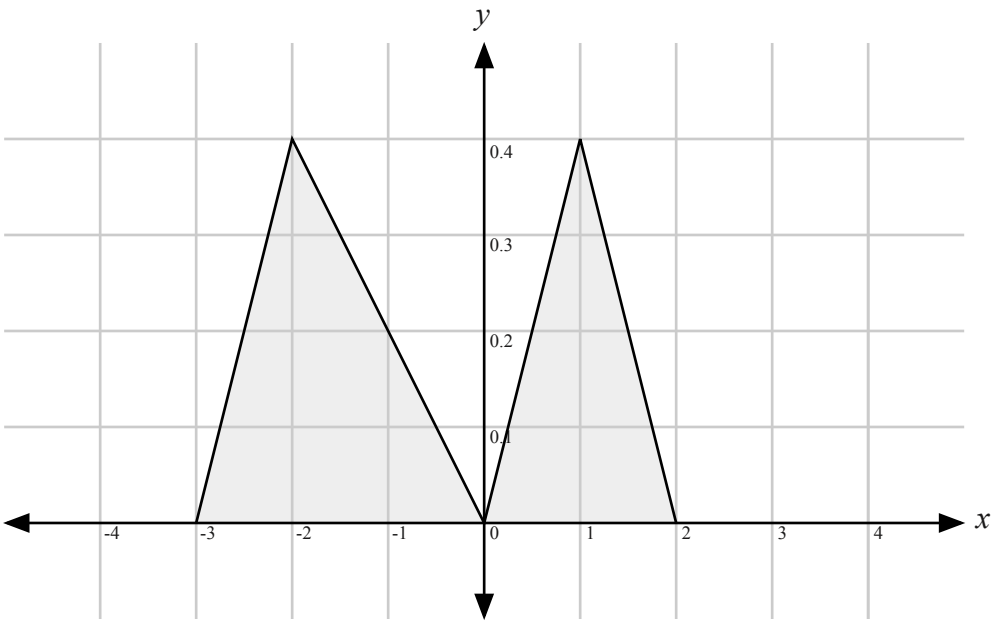
(k) In your copy of the spreadsheet, enter your name in the cell to the right of the instructor's name.

(l) Enter your results in the column below your name.

(m) Share your copy of the spreadsheet with the instructor, by clicking on the "Share" command on the right side of the menu, and inserting the instructor's email address.

Section 8.3: Homework Exercise 1 *[Probability Density Function]*

All parts of this exericse refer to the probability density function seen in the following figure.



(a) Find the probability $P(0 < X < 2)$.

(b) Find the probability $P(0 < X < 3)$.

(c) Find the probability $P(X < -2)$.

(d) Find the probability $P(-2 < X < 1)$.

Section 9.2: Homework Exercise 1 *[Normal Distribution: Z-Score to P-Value]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the probability $P(Z < 1.62)$.

(b) Find the probability $P(-0.43 < Z)$.

(c) Find the probability $P(0.35 < Z < 2.06)$.

Section 9.2: Homework Exercise 2 *[Normal Distribution: Z-Score to P-Value]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the probability $P(Z < -0.28)$.

(b) Find the probability $P(1.04 < Z)$.

(c) Find the probability $P(-1.57 < Z < 0.76)$.

Section 9.3: Homework Exercise 1 *[Normal Distribution: P-Value to Z-Score]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the number z^* such that $P(Z < z^*) = 0.0060$.

(b) Find the number z^* such that $P(Z < z^*) = 0.7055$.

(c) Find the number z^* such that $P(z^* < Z) = 0.9664$.

(d) Find the number z^* such that $P(z^* < Z) = 0.0074$.

Section 9.3: Homework Exercise 2 *[Normal Distribution: P-Value to Z-Score]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the number z^* such that $P(Z < z^*) = 0.8463$.

(b) Find the number z^* such that $P(Z < z^*) = 0.0515$.

(c) Find the number z^* such that $P(z^* < Z) = 0.0405$.

(d) Find the number z^* such that $P(z^* < Z) = 0.8943$.

Section 9.3: Homework Exercise 3 *[Normal Distribution: P-Value to Z-Score]*

(a) Find the left 96% interval of the standard normal distribution.

(b) Find the middle 96% interval of the standard normal distribution.

(c) Find the right 96% interval of the standard normal distribution.

Section 9.4: Homework Exercise 1 *[Normal Distribution: Z-Score to P-Value]*

The length of gestation for piglets is normally distributed with mean 114 days and standard deviation 0.75 day. Find the probability that a litter will be born within one day before the mean and one day after the mean.

Shafer-Zhang, Beginning Statistics, Section 5.3, Exercise 12

Section 9.4: Homework Exercise 2 *[Normal Distribution: Z-Score to P-Value]*

The weight of pucks made by a particular process is normally distributed with mean 5.75 ounces and standard deviation 0.11 ounce. A regulation hockey puck must weigh between 5.5 and 6 ounces. Find the probability that a puck made by this process will meet the weight requirement.

Shafer-Zhang, Beginning Statistics, Section 5.3, Exercise 16

Section 9.4: Homework Exercise 3 *[Normal Distribution: Z-Score to P-Value]*

The birth weight of full-term babies born in a certain region is normally distributed with mean 7.125 lbs. and standard deviation 1.29 lbs. Find the probability that a randomly selected newborn will weigh less than 5.5 lbs.

Section 9.5: Homework Exercise 1 *[Normal Distribution: P-Value to Z-Score]*

The height of adult women in the U.S. is normally distributed with mean height 63.8 in. and standard deviation 2.8 in.

(a) Find the height that is the 10^{th} percentile.

(b) Find the height that is the 80^{th} percentile.

Shafer-Zhang, Beginning Statistics, Section 5.4, Exercise 12

Section 9.5: Homework Exercise 2 *[Normal Distribution: P-Value to Z-Score]*

The finishing time among all high school boys in a particular track event in a certain state is normally distributed with mean 5 minutes 17 seconds and standard deviation 12 seconds.

Hints: (1) the winners in the race have the smallest times, not the largest times; (2) convert all times to seconds.

(a) The qualifying time in this event for participation in the state meet is to be set so that only the fastest 5% of all runners qualify. Find the qualifying time.

(b) In the western region of the state the times of all boys running in this event are normally distributed with standard deviation 12 seconds, but with mean 5 minutes 22 seconds. Find the proportion of boys from this region who qualify to run in this event in the state meet.

Section 9.5: Homework Exercise 3 *[Normal Distribution: P-Value to Z-Score]*

The life of a new type of light bulb is normally distributed with an estimated mean life of $1,321$ hours and standard deviation of 106 hours. The manufacturer will advertise the lifetime of the bulb using the largest value for which it is expected that 90% of the bulbs will last at least that long. Find that advertised value.

Shafer-Zhang, Beginning Statistics, Section 5.4, Exercise 18

Section 9.6: Homework Exercise 1 *[Central Limit Theorem]*

Scores on a common final exam in a large multiple-section course are normally distributed with mean 72.7 and standard deviation 13.1.

(a) Find the probability that the score on a randomly selected exam is between 70 and 80.

(b) Find the probability that the mean score of 38 randomly selected exams is between 70 and 80.

Shafer-Zhang, Beginning Statistics, Section 6.2, Exercise 21

Section 9.6: Homework Exercise 2 *[Central Limit Theorem]*

Many sharks enter a state of tonic immobility when inverted. Suppose that in a particular species of sharks the time a shark remains in a state of tonic immobility when inverted is normally distributed with mean 11.2 minutes and standard deviation 1.1 minutes.

(a) If a biologist induces a state of tonic immobility in this type of shark in order to study it, find the probability that the shark will remain in this state for between 11 and 12 minutes.

(b) When a biologist wishes to estimate the mean time that such sharks stay immobile by inducing tonic immobility in each of a sample of 12 sharks, find the probability that mean time of immobility in the sample will be between 11 and 12 minutes.

Shafer-Zhang, Beginning Statistics, Section 6.2, Exercise 18

Section 10.2: Homework Exercise 1 *[Confidence Intervals]*

Four hundred randomly selected working adults in a certain state, including those who worked at home, were asked the distance from their home to their workplace. The average distance was 8.84 miles with standard deviation 2.7 miles. Construct a 99% confidence interval for the mean distance from home to work for all residents of this state.

Shafer-Zhang, Beginning Statistics, Section 7.1, Exercise 8

Section 10.2: Homework Exercise 2 *[Confidence Intervals]*

A corporation monitors time spent by office workers browsing the web on their computers. In a sample of computer records of 50 workers, the average amount of time spent browsing in an eight-hour work day was 27.8 minutes with standard deviation 8.2 minutes. Construct a 99.5% confidence interval for the mean time spent by all office workers in browsing the web in an eight-hour day.

Shafer-Zhang, Beginning Statistics, Section 7.1, Exercise 10

Section 10.2: Homework Exercise 3 *[Confidence Intervals]*

The designer of a garbage truck that lifts roll-out containers must estimate the mean weight the truck will lift at each collection point. A random sample of 325 containers of garbage on current collection routes has mean 75.3 lbs. and standard deviation 12.8 lbs. Construct a 98% confidence interval for the mean weight the trucks must lift each time.

Shafer-Zhang, Beginning Statistics, Section 7.1, Exercise 14

Section 10.2: Homework Exercise 4 *[Confidence Intervals]*

For this exercise you will need to access a Google Sheets spreadsheet that is available at the class website.

The Cat Lovers Society of a certain town wants to find the average weight of adult pet cats in their location. A survey of randomly chosen pet owners is conducted, and the weights of 50 cats are collected.

(a) Go to the class website, and click on the link to "Cats (Google Sheets)," which is found under the heading Statistics Materials.

(b) When the Google Sheets opens, it will be "view only."

(c) In the "File" menu, click on "Make a Copy."

(d) Your copy of the spreadsheet will be editable.

(e) The weights of the 50 randomly chosen cats are found in the spreadsheet Cats.

(f) Using the data in the spreadsheet, construct a 94% confidence interval for the mean weight of adult pet cats in this town.

Section 10.3: Homework Exercise 1 *[Sample Size]*

A real estate agent wishes to estimate, to within \$1.50, the mean retail cost per square foot of newly built homes in her town, with 90% confidence. Based upon previous data, she estimates the standard deviation of such costs at \$6.00. Estimate the minimum size sample required.

Section 10.3: Homework Exercise 2 *[Sample Size]*

A retailer wishes to estimate, to within 15 seconds, the mean duration of telephone calls taken at its call center, with 99.5% confidence. The shortest call is 0.5 minutes, and the longest call is 8 minutes. Estimate the minimum size sample required.

Hint: Express all the information in the same units.

Shafer-Zhang, Beginning Statistics, Section 7.4, Exercise 12

Section 11.1: Homework Exercise 1 *[Hypothesis Testing]*

State the null hypothesis and alternative hypothesis for each of the following situations.

Specifically, identify the number μ_0, write the null hypothesis as one of $\mu = \mu_0$ or $\mu \leq \mu_0$ or $\mu \geq \mu_0$, and then write the corresponding alternative hypothesis.

(a) The average time workers spent commuting to work in Verona five years ago was 38.2 minutes. The Verona Chamber of Commerce asserts that the average is less now.

(b) The mean salary for all men in a certain profession is $\$58,291$. A public interest group thinks that the mean salary for women in the same profession is different.

(c) The accepted figure for the caffeine content of an 8-ounce cup of coffee is 133 mg. A dietitian believes that the average amount of caffeine in coffee served in a local restaurants is higher.

(d) The average yield per acre for all types of corn in a recent year was 161.9 bushels. An economist believes that the average yield per acre is different this year.

(e) A fishing association asserts that the average age of all self-described fly fishermen is 42.8 years. A sociologist suspects that it is higher.

Shafer-Zhang, Beginning Statistics, Section 8.1, Exercise 2

Section 11.2: Homework Exercise 1 *[Hypothesis Testing]*

A medical laboratory claims that the mean turn-around time for performance of a battery of tests on blood samples is 1.88 business days. The manager of a large medical practice believes that the actual mean is larger. A random sample of 45 blood samples yielded mean 2.09 and sample standard deviation 0.13 day. Is the manager justified in her claim, at the 10% level of significance?

Shafer-Zhang, Beginning Statistics, Section 8.2, Exercise 14

Section 11.2: Homework Exercise 2 *[Hypothesis Testing]*

A magazine publisher tells potential advertisers that the mean household income of its regular readership is $61,500. An advertising agency wishes to test this claim against their suspicion that the mean is smaller. A sample of 40 randomly selected regular readers yields mean income $59,800 with standard deviation $5,850. Determine whether the data provides sufficient evidence, at the 5% level of significance, that the magazine publisher is correct in saying that the mean household income of its regular readership is $61,500 (or more).

Shafer-Zhang, Beginning Statistics, Section 8.2, Exercise 16

Section 11.2: Homework Exercise 3 *[Hypothesis Testing]*

For this exercise you will need to access a Google Sheets spreadsheet that is available at the class website.

A survey of students in a certain high school determined that the average length of the songs on their smartphones was 3.32 minutes. Susan, a student at that school, thinks that her songs have an average length that is larger than the school average. Having taken a statistics class, Susan decides to test if that's true at the 1% level of significance. She randomly selects 40 songs on her smartphone, and writes down their lengths.

(a) Go to the class website, and click on the link to "Song Length (Google Sheets)," which is found under the heading Statistics Materials.

(b) When the Google Sheets opens, it will be "view only."

(c) In the "File" menu, click on "Make a Copy."

(d) Your copy of the spreadsheet will be editable.

(e) The lengths of the 40 randomly chosen songs on Susan's smartphone are found in the spreadsheet Song Length.

(f) Using the data in the spreadsheet, carry out the hypothesis testing and determine if Susan can justify claiming that her songs have an average length that is longer than the average of the students in her school.

Section 11.3: Homework Exercise 1 *[Hypothesis Testing]*

A drug company claims that its new headache medicine causes fewer side effects than the existing headache medicines. A doctor tests the new headache medicine on a sample of patients to find out if the claim is true at a given level of significance.

(a) In this situation, what would a type 1 error be?

(b) In this situation, what would a type 2 error be?

(c) In this situation, which type of error would be worse?