DATA AND DECISIONS

IN-CLASS EXERCISES

Very Preliminary

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$\boxed{1}$ Voting Exercises

Section 1.2: In-Class Exercise 1 *[Voting Methods]*

Suppose that an election has candidates A, B, C, D and E. There are 7 voters, who submit the following ranked ballots.

1	2	1	1	1	1
A	A	C	C	B	E
B	D	B	D	C	C
C	B	D	B	D	D
D	E	E	A	A	B
E	C	A	E	E	A.

The goal of this exercise is to find different methods of voting that lead to different winners using the above ranked ballots. For each question, either give a voting method that leads to the desired winner, or state why you think there is no such method.

(a) Can you find a voting method that would have A be the winner using the above ranked ballots?

(b) Can you find a voting method that would have B be the winner using the above ranked ballots?

(c) Can you find a voting method that would have C be the winner using the above ranked ballots?

(d) Can you find a voting method that would have D be the winner using the above ranked ballots?

(e) Can you find a voting method that would have E be the winner using the above ranked ballots?

Taylor-Pacelli, Mathematics and Politics, 2nd ed., p. 8

Section 1.2: In-Class Exercise 2 *[Voting Methods]*

A non-profit agency is electing a new chair of the board. The votes are shown below.

9	19	11	8
Atkins	Cortez	Burke	Atkins
Cortez	Burke	Cortez	Burke
Burke	Atkins	Atkins	Cortez.

Find the winner (or winners), if there are any winners, by each of the following methods.

SHOW YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS.

(a) Plurality Voting.

(b) Single Runoff Voting.

(c) Instant Runoff Voting.

(d) Borda Count Voting.

(e) Sequential Pairwise Voting with Fixed Agenda, with agenda Atkins, Burke, Cortez.

(f) Condorcet Voting.

(g) Copeland Voting.

(h) Dictatorship, with one of the voters in the right hand column the dictator.

Lippman, Math in Society, 2.4th ed., p. 53, Exercise 4

Section 1.2: In-Class Exercise 3 *[Voting Methods]*

The homeowners association is deciding a new set of neighborhood standards for architecture, yard maintenance, etc. Four options, denoted A, B, C and D, have been proposed. The votes are shown below.

8	9	11	7	7	5
B	A	D	A	B	C
C	D	B	B	A	D
A	C	C	D	C	A
D	B	A	C	D	B.

Find the winner (or winners), if there are any winners, by each of the following methods.

SHOW YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS.

(a) Plurality Voting.

(b) Single Runoff Voting.

(c) Instant Runoff Voting.

(d) Borda Count Voting.

(e) Sequential Pairwise Voting with Fixed Agenda, with agenda A, B, C, D.

(f) Condorcet Voting.

(g) Copeland Voting.

(h) Dictatorship, with one of the voters in the right hand column the dictator.

Lippman, Math in Society, 2.4th ed., p. 54, Exercise 6

Section 2.1: In-Class Exercise 1 *[Apportionment Methods]*

A college offers tutoring in Mathematics, English, Chemistry, and Biology. The number of students enrolled in each subject is listed below. If the college can only afford to hire 15 tutors, determine how many tutors should be assigned to each subject.

Class	**Number of Students**
Mathematics	330
English	265
Chemistry	130
Biology	70.

Compute the apportionment using Hamilton's method.

[Lippman, Math in Society, 2.4th ed., p. 91, Exercise 1

Section 2.1: In-Class Exercise 2 *[Apportionment Methods]*

An extremely small country consists of five states, whose populations are listed below. If the legislature has 119 seats, apportion the seats, with the apportionment method listed.

State	Population
A	810
B	473
C	292
D	594
E	211.

Compute the apportionment using Hamilton's method.

Lippman, Math in Society, 2.4th ed., p. 91, Exercise 7

Section 2.2: In-Class Exercise 1 *[Apportionment Methods]*

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Class	**Number of Students**
Mathematics	330
English	265
Chemistry	130
Biology	70.

(a) Compute the apportionment using Jefferson's method.

(b) Compute the apportionment using Adams' method.

(c) Compute the apportionment using Webster's method.

(d) Compute the apportionment using the Huntington-Hill method.

[Lippman, Math in Society, 2.4th ed., p. 91, Exercise 1

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(d) Compute the apportionment using the Huntington-Hill method.

Lippman, Math in Society, 2.4th ed., p. 91, Exercise 7

Section 2.4: In-Class Exercise 1 *[Gerrymandering]*

A small state has residents who are members of the *X* party and the *O* party, and who live in the locations seen in the following figure. The state is divided in five districts, each with five voters. Draw five districts, all contiguous and with equal populations, using grid lines, such that the *O* party wins the largest possible number of the districts.

In addition to drawing the five districts, explain why it would not be possible to draw districts that would have the O party win more districts than in your drawing.

X	O	O	X	X
O	X	X	O	O
X	O	O	O	O
O	X	X	X	O
O	X	O	O	O

Section 2.4: In-Class Exercise 2 *[Gerrymandering]*

A small state has residents who are members of the X party and the O party, and who live in the locations seen in the following figure. The state is divided in five districts, each with five voters. Draw five districts, all contiguous and with equal populations, using grid lines, such that the X party wins the largest possible number of the districts.

In addition to drawing the five districts, explain why it would not be possible to draw districts that would have the X party win more districts than in your drawing.

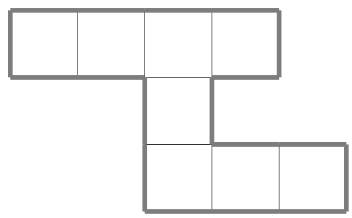
X	O	O	X	X
O	X	X	O	O
X	O	O	O	O
O	X	X	X	O
O	X	O	O	O

Section 2.4: In-Class Exercise 3 *[Gerrymandering]*

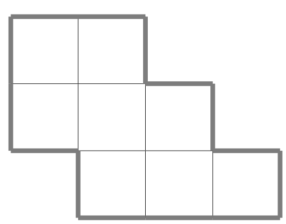
For each of the two regions shown below, find the following three numbers.

1. The grid Polsby-Popper ratio.
2. The grid Reock ratio.
3. The grid convex hull ratio.

(a)



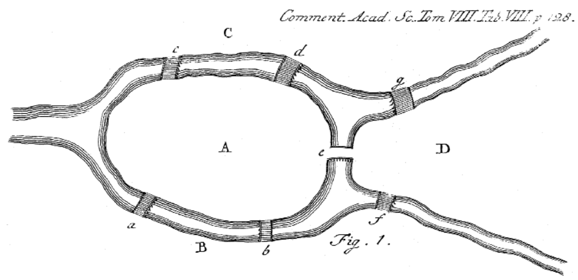
(b)



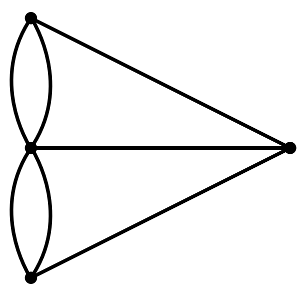
2 Networks Exercises

Section 3.1: In-Class Exercise 1 *[Bridges of Königsberg]*

In the 18th century, the mathematician Leonhard Euler solved a puzzle asked by the residents of the city of Königsberg (now in Russia and called Kaliningrad). At the time, the city had seven bridges, which connected an island with various banks of the river Pregel; Euler's sketch of the river and the seven bridges (labeled in lower case as a, b, c, d, e, f and g) is seen below. The puzzle is whether or not it was possible to start somewhere in the city, and find a path that crosses every bridge once and only once; it doesn't matter if you ended up where you started or not.



To solve this puzzle, it is easier if the problem is simplified by representing the bridges via a graph, shown below, where each vertex represents a land mass, and each edge represents a bridge. The question then becomes whether or not it was possible to start at some vertex, and find a path that crosses every edge once and only once; it doesn't matter if you ended up where you started or not.



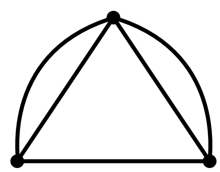
Try to solve the Bridges of Königsberg problem; use the graph rather than the original figure. If you think that a path can be found, show it, and if you think a path cannot be found, explain why not.

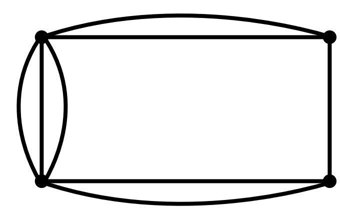
Section 3.1: In-Class Exercise 2 *[Bridges of Königsberg]*

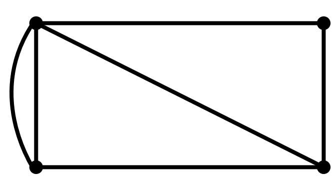
Try applying the ideas you found for the Bridges of Königsberg to each of the following graphs.

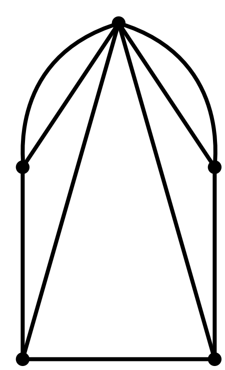
That is for each graph, is it or is it not possible to start at some vertex, and find a path that crosses every edge once and only once; it doesn't matter if you ended up where you started or not.

Hints: (1) for some graphs a path that crosses every edge once and only once can be found, and for other graphs there is no path that crosses every edge once and only once; (2) the best way to proceed is to try to find criteria that guarantee the existence of the desired type of path.







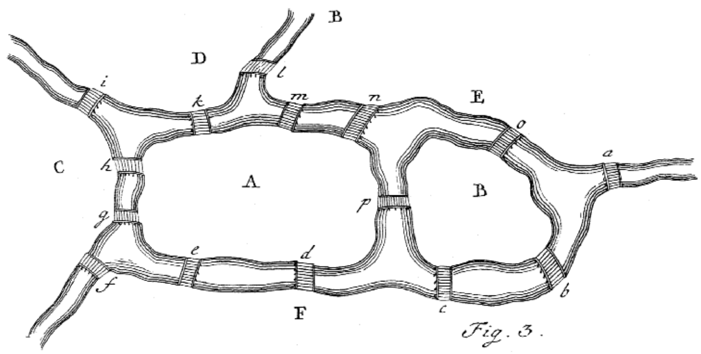


Section 3.1: In-Class Exercise 3 *[Bridges of Königsberg]*

Shown below is another sketch by Euler, this one with 15 bridges. Again, we ask whether or not it was possible to start at some vertex, and find a path that crosses every edge once and only once.

If you think that a path can be found, show it, and if you think a path cannot be found, explain why not.

Hint: First convert this problem into a graph, similarly to what was done above for Bridges of Königsberg.

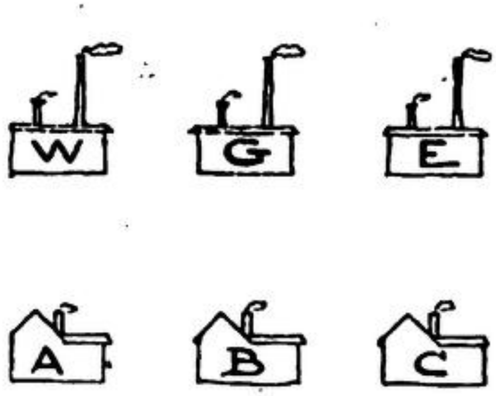


Section 3.1: In-Class Exercise 4 *[Three Utilities]*

Here is a reproduction of a puzzle from the Strand Magazine in 1913, written by puzzle author Henry Dudeney.

146.—WATER, GAS, AND ELECTRICITY.

THERE are some half-dozen puzzles, as old as the hills, that are perpetually cropping up, and there is hardly a month in the year that does not bring inquiries as to their solution. Occasionally one of these, that one had hoped was an extinct volcano, bursts into eruption in a surprising manner. For some quite unknown reason I have lately received an extraordinary number of letters (four of them from the United States) respecting the ancient puzzle that I have called "Water, Gas, and Electricity." It is much older than electric lighting, or even gas, but the new dress brings it up to date. The puzzle is to lay on water, gas, and electricity, from W, G, and E, to each of the three houses, A, B, and C, without any pipe crossing another. Take your pencil and draw lines showing how this should be done. You will soon find yourself landed in difficulties. My answer next month must serve as a reply to my many correspondents.

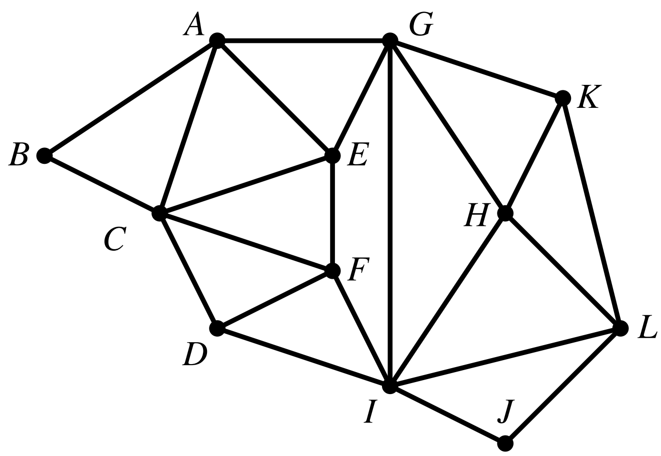


If you think it is possible to solve the puzzle, show your solution, and if you think there is no solution, explain why not.

TheStrandMagazine1913bVol.XlviJul-dec_0119.jpg

Section 3.2: In-Class Exercise 1 *[Graph Basics]*

All parts of this problem refer to the following graph.



(a) List the degree of each of the vertices of the graph.

(b) What is the maximum degree among all the vertices of the graph?

(c) What is the minimum degree among all the vertices of the graph?

(d) Is the graph connected?

(e) Give an example of a connected subgraph of the graph that has five vertices.

(f) Give an example of a disconnected subgraph of the graph that has five vertices.

Section 3.2: In-Class Exercise 2 *[Graph Basics]*

For each part of this problem, draw the graph that is listed.

(a) P_3

(b) C_4

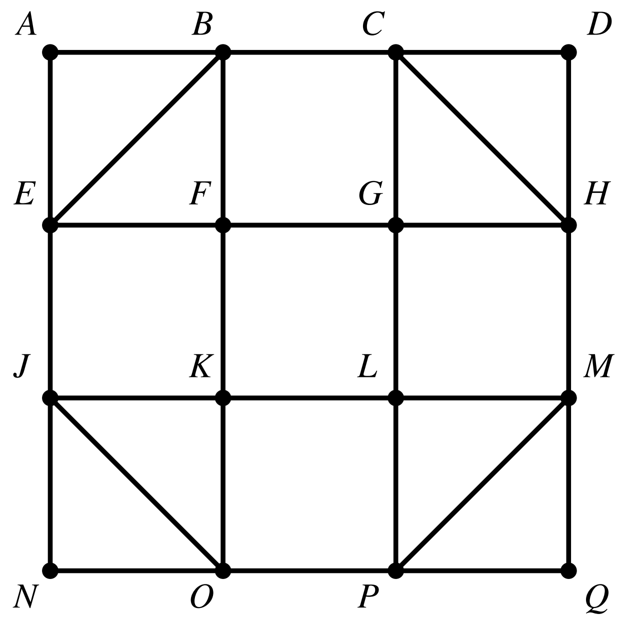
(c) K_4

(d) $K_{2,3}$

Section 3.4: In-Class Exercise 1 *[Euler Paths and Euler Circuits]*

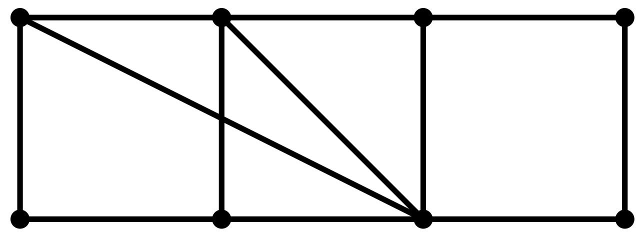
For the graph shown below, either find an Euler path or Euler circuit, or explain why there is no Euler path or Euler circuit.

If you are showing that there is an Euler path or Euler circuit, label the edges of the graph using numbers 1, 2, 3, etc., in the order that they are traversed in the Euler path or Euler circuit.



Section 3.5: In-Class Exercise 1 *[Vertex Coloring]*

All parts of this exercise refer to the graph shown below.



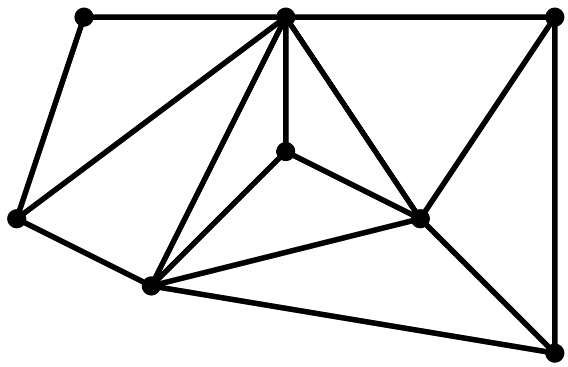
(a) Color the graph (using numbers) with the fewest possible colors.

(b) Explain how you know that your coloring of the graph uses the fewest possible colors.

(c) What is the chromatic number of the graph?

Section 3.5: In-Class Exercise 2 *[Vertex Coloring]*

All parts of this exercise refer to the graph shown below.



(a) Color the graph (using numbers) with the fewest possible colors.

(b) Explain how you know that your coloring of the graph uses the fewest possible colors.

(c) What is the chromatic number of the graph?

Section 3.5: In-Class Exercise 3 *[Vertex Coloring]*

All parts of this problem refer to the following list of chemicals, numbered 1–7, together with a list of other chemicals with which each cannot be stored. The goal of this problem is to determine the smallest number of different storage facilities that are needed in order to store all seven chemicals.

Chemical	Cannot be Stored With
1	$2, 5, 7$
2	$1, 3, 5, 4$
3	$2, 4, 6$
4	$2, 3, 7$
5	$1, 2, 6, 7$
6	$5, 3$
7	$1, 4, 5$

(a) Make a graph to represent the above situation.

(b) Use vertex coloring to determine the smallest number of different storage facilities that are needed in order to store all seven chemicals.

(c) Explain how you know that your answer is indeed the smallest number of storage facilities that are needed.

Smithers, Graph Theory Notes, p. 44, Exercise 1

Section 3.5: In-Class Exercise 4 *[Vertex Coloring]*

All parts of this problem refer to the following map. The goal of this problem is to determine the smallest number of colors needed to color the map so that if two states share a border they have different colors.

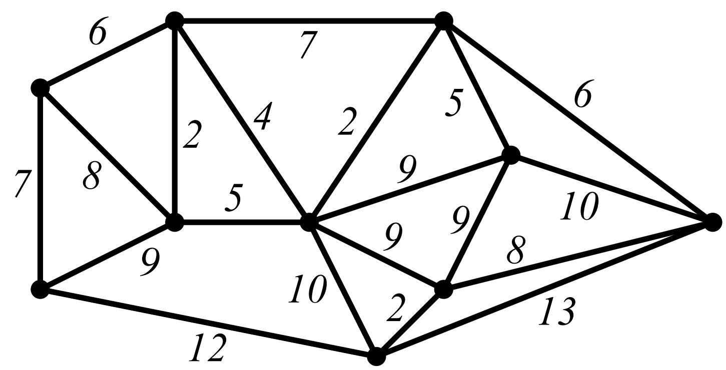
(a) Make a graph to represent the above map.

(b) Use vertex coloring to determine the smallest number of colors needed to color the map so that if two states share a border they have different colors.

(c) Explain how you know that your answer is indeed the smallest number of colors that are needed.

Section 3.6: In-Class Exercise 1 *[Minimum Spanning Trees]*

Both parts of this exercise refer to the graph shown below.



(a) Use Prim's Algorithm to find a minimum spanning tree for the graph.

Show (by shading or otherwise marking) the edges that are in the minimum spanning tree.

(b) What is the length of the minimum spanning tree that you found?

Section 3.6: In-Class Exercise 2 *[Minimum Spanning Trees]*

There are seven small towns in Madison County that are connected to each other by gravel roads, as in the following diagram. The grid below shows the distances in miles between the towns. The county wants to pave some of the roads so that people can get from town to town on paved roads, either directly or indirectly. However, Madison County is on a tight budget so the total number of miles paved needs to be minimum.

	A	B	C	D	E	F	G
A	–	10	–	–	–	26	20
B	10	–	25	13	–	15	–
C	–	25	–	18	–	–	–
D	–	13	18	–	19	–	–
E	–	–	–	19	–	14	25
F	26	15	–	–	14	–	18
G	20	–	–	–	25	18	–

(a) Make a graph to represent the above situation.

(b) Use Prim's Algorithm to determine the most efficient way to connect all seven towns.

(c) What is the minimum number of miles of paved road needed to connect all seven towns?

Smithers, Graph Theory Notes, pp. 77–78, Exercise 2

Section 4.1: In-Class Exercise 1 *[Summation Notation]*

The four parts of this exercise are not related.
Do not actually calculate the numerical value of the sums in this exercise.

(a) Rewrite the following in summation notation.

$$\frac{1}{7}+\frac{2}{7}+\frac{3}{7}+\frac{4}{7}+\frac{5}{7}+\frac{6}{7}.$$

(b) Rewrite the following in summation notation.

$$(5^2+1)+(6^2+1)+(7^2+1)+\cdots+(30^2+1).$$

(c) Write out all the terms of the following sum.

$$\sum_{i=1}^{5} 3i.$$

(d) Write out all the terms of the following sum.

$$\sum_{r=2}^{7} r^4.$$

Section 4.1: In-Class Exercise 2 *[Summation Notation]*

Let $x_1, x_2, \ldots, x_5$ be defined to be $2, -1, 4, -3, 6$, respectively.

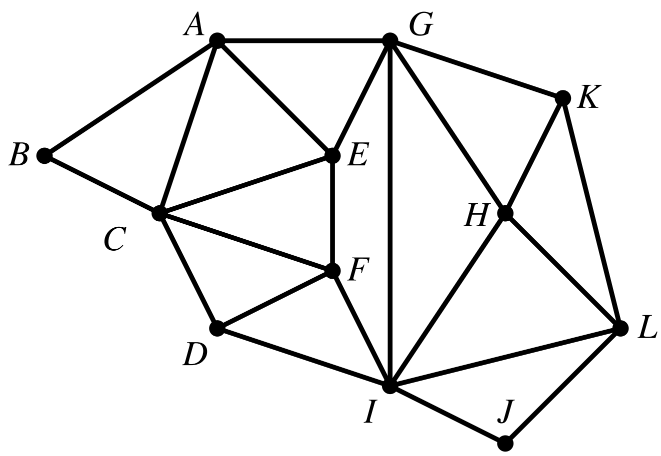
Calculate the values of the following sums.

(a) $\sum_{t=1}^{4} x_t$.

(b) $\sum_{i=2}^{5}(4x_i + 2)$.

Section 4.2: In-Class Exercise 1 *[Average Degree]*

All parts of this problem refer to the following graph.



(a) List the degree of each of the vertices of the graph.

(b) Find the average degree of the vertices of the graph.

(c) Are there more vertices with degree below the average or above the average, or is there the same number of each?

Section 4.2: In-Class Exercise 2 *[Average Degree]*

All parts of this problem refer to the following map.

(a) Make a graph to represent the above map.

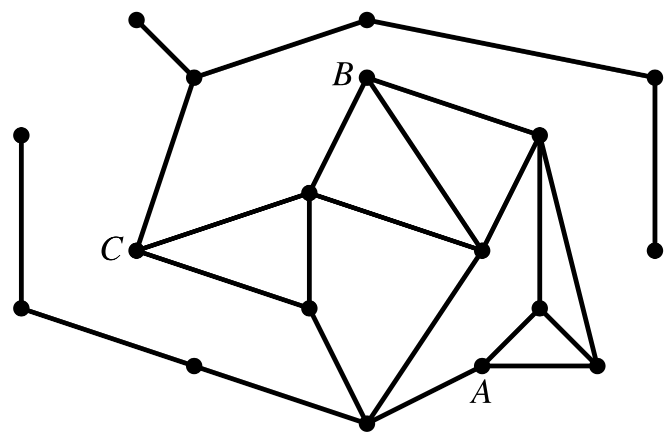
(b) List the degree of each of the vertices of the graph.

(c) Find the average degree of the vertices of the graph.

(d) Are there more vertices with degree below the average or above the average, or is there the same number of each?

Section 4.3: In-Class Exercise 1 *[Breadth-First Search]*

Both parts of this problem refer to the following graph.

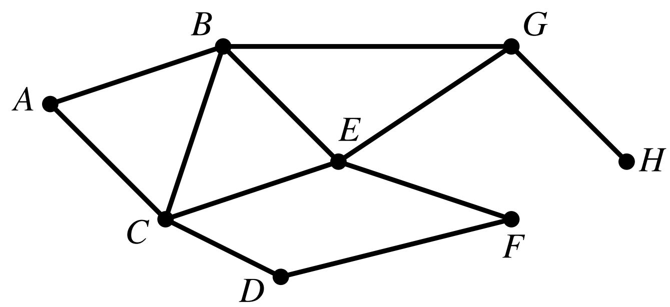


(a) Label all the vertices of the graph using Breadth-First Search starting at vertex A.

(b) Use your results from Part (a) to find the distance from A to each of the vertices B and C.

Section 4.3: In-Class Exercise 2 *[Breadth-First Search]*

All parts of this problem refer to the following graph.



(a) Find the distance from each vertex of the graph to every other vertex of the graph by using Breadth-First Search starting at each vertex of the graph.

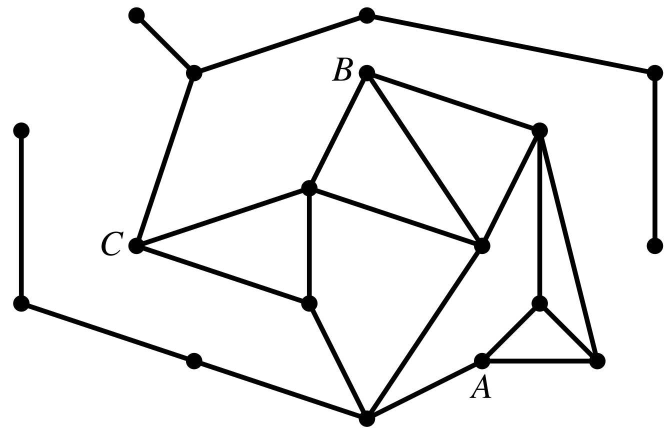
(b) Find the diameter of the graph.

(c) Find the average distance between the vertices of the graph.

Hint: When you find the average distance between the vertices of the graph, make sure you count the distance between each pair of vertices only once.

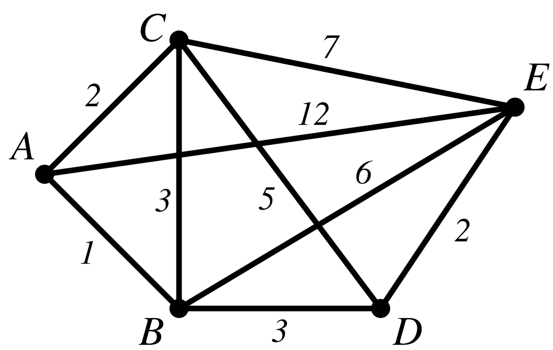
Section 4.3: In-Class Exercise 3 *[Breadth-First Search]*

Use Breadth-First Search starting at vertex A to find a spanning tree for the graph shown below.



Section 4.4: In-Class Exercise 1 *[Dijkstra's Algorithm]*

All parts of this exercise refer to the following weighted graph.



(a) Apply Dijkstra's Algorithm to this graph, where the starting vertex is A.

(b) State the weighted distance in the graph from A to each of the other vertices.

(c) State what the shortest path from A to E is. Give your answer by listing the vertices encountered on this shortest path, starting with vertex A and ending in vertex E.

Section 4.4: In-Class Exercise 2 *[Dijkstra's Algorithm]*

There are seven small towns in Madison County that are connected to each other by gravel roads, as in the following diagram. The grid below shows the distances in miles between the towns. The county wants to pave some of the roads so that people can get from town to town on paved roads, either directly or indirectly. However, Madison County is on a tight budget so the total number of miles paved needs to be minimum.

	A	B	C	D	E	F	G
A	–	10	–	–	–	26	20
B	10	–	25	13	–	15	–
C	–	25	–	18	–	–	–
D	–	13	18	–	19	–	–
E	–	–	–	19	–	14	25
F	26	15	–	–	14	–	18
G	20	–	–	–	25	18	–

(a) Make a graph to represent the above situation.

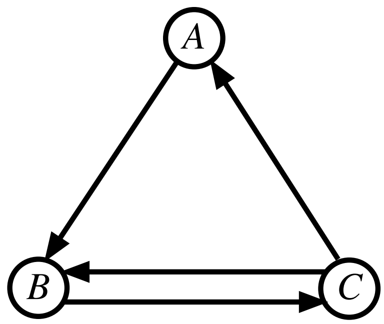
(b) Use Dijkstra's Algorithm to find the shortest distance from town A to each of the other towns.

Smithers, Graph Theory Notes, pp. 77–78, Exercise 2

Section 4.5: In-Class Exercise 1 *[PageRank]*

The goal of this exercise is to do the first few steps for finding the PageRank for the following directed graph.

Give all your answers to three decimal places.



(a) Find the initial scores for each vertex.

(b) Redistribute the scores.

(c) Find the modified scores using $p = 0.85$.

(d) Redistribute the scores again.

(e) Find the modified scores again using $p = 0.85$.

Section 5.1: In-Class Exercise 1 *[Matrices]*

Let

$$M = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} -4 & 3 \\ 0 & -2 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 4 & -3 & 0 \\ 2 & 5 & -1 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} 2 \\ 5 \end{bmatrix}.$$

For each of the following, calculate it if possible, or say why it is not possible.

(a) $4M$.

(b) $M + N$.

(c) $M + P$.

(d) $5M - 3N$.

Section 5.2: In-Class Exercise 1 *[Matrices]*

Let
$$M = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} -4 & 3 \\ 0 & -2 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 4 & -3 & 0 \\ 2 & 5 & -1 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} 2 \\ 5 \end{bmatrix}.$$

For each of the following, calculate it if possible, or say why it is not possible.

(a) MV.

(b) PV.

(c) MN.

(d) NM.

(e) MP.

(f) PM.

Section 5.2: In-Class Exercise 2 *[Matrices]*

Let

$$M = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} -4 & 3 \\ 0 & -2 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 4 & -3 & 0 \\ 2 & 5 & -1 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} 2 \\ 5 \end{bmatrix}.$$

For each of the following, calculate it if possible, or say why it is not possible.

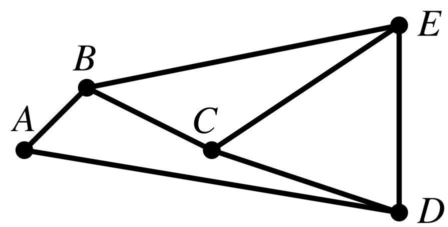
(a) M^2.

(b) P^2.

(c) $M^2 - 2M$.

Section 5.3: In-Class Exercise 1 *[Application of Matrices to Graphs]*

All parts of this exercise refer to the following graph.



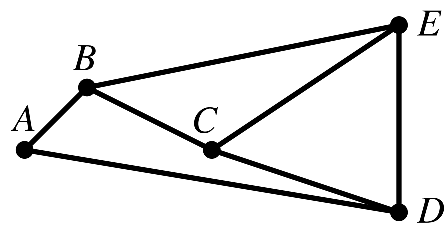
(a) Find the adjacency matrix M.

(b) Compute M^2.

(c) Verify that the diagonal entries of M^2 represent the degrees of the vertices of the graph?

Section 5.3: In-Class Exercise 2 *[Application of Matrices to Graphs]*

Both parts of this exercise refer to the following graph.

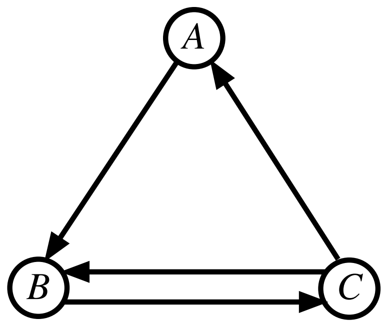


(a) Compute M^3.

(b) Verify that the non-zero diagonal entries of M^3 correspond to the vertices that are in a clique (or cliques)?

Section 5.4: In-Class Exercise 1 *[PageRank]*

The goal of this exercise is to do the first few steps for finding the PageRank for the following directed graph, using matrices. Give all your answers to three decimal places.



(a) Find the initial vector of values v.

(b) Find the transition matrix A.

(c) Let $p = 0.85$. Find the modified transition matrix M.

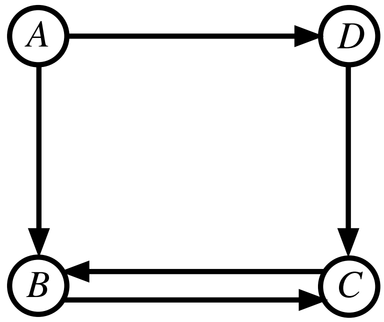
(d) Compute Mv.

(e) Compute M^2v.

(f) Compute M^3v.

Section 5.4: In-Class Exercise 2 *[PageRank]*

The goal of this exercise is to do the first few steps for finding the PageRank for the following directed graph, using matrices. Give all your answers to three decimal places.



(a) Find the initial vector of values v.

(b) Find the transition matrix A.

(c) Let $p = 0.85$. Find the modified transition matrix M.

(d) Compute Mv.

(e) Compute M^2v.

(f) Compute M^3v.

3 Statistics Exercises

Section 7.2: In-Class Exercise 1 *[Frequency Tables and Histograms]*

Both parts of this exercise refer to the following data.

$$8.9, 7.8, 7.8, 6.3, 8.1, 7.5, 6.2, 7.2, 8.3, 7.6, 9.1, 7.4, 8.7, 9.8, 8.4, 7.8, 9.7, 9.3.$$

(a) Construct a frequency table for the above data set, using bins of the form 6.0–6.9, 7.0–7.9, and so on.

(b) Construct a histogram for the above data set, using the same bins used for the frequency table.

Section 7.3: In-Class Exercise 1 *[Statistics Basics]*

All parts of this exercise refer to the numbers

$$5, 11, 2, 4, 2, 6.$$

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the mean of these numbers.

(b) Find the median of these numbers.

(c) Find the mode of these numbers.

Section 7.3: In-Class Exercise 2 *[Statistics Basics]*

All parts of this exercise refer to the numbers

$$-2, 3, 4, -1, 6.$$

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the mean of these numbers.

(b) Find the median of these numbers.

(c) Find the mode of these numbers.

Section 7.3: In-Class Exercise 3 *[Statistics Basics]*

All parts of this exercise refer to the numbers given in the frequency table

value	frequency
1	3
2	2
3	2
5	1
6	2.

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the mean of these numbers.

(b) Find the median of these numbers.

(c) Find the mode of these numbers.

Section 7.4: In-Class Exercise 1 *[Statistics Basics]*

All parts of this exercise refer to the numbers

$$5, 11, 2, 4, 2, 6.$$

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the range of these numbers.

(b) Find the variance of these numbers.

(c) Find the standard deviation of these numbers.

Section 7.4: In-Class Exercise 2 *[Statistics Basics]*

All parts of this exercise refer to the numbers

$$-2, 3, 4, -1, 6.$$

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the range of these numbers.

(b) Find the variance of these numbers.

(c) Find the standard deviation of these numbers.

Section 7.4: In-Class Exercise 3 *[Statistics Basics]*

All parts of this exercise refer to the numbers given in the frequency table

value	frequency
1	3
2	2
3	2
5	1
6	2.

For this exercise, you may use a calculator or computer for addition, subtraction, multiplication, division and square roots, but do not use built-in functions for statistical calculations.

(a) Find the range of these numbers.

(b) Find the variance of these numbers.

(c) Find the standard deviation of these numbers.

Section 8.2: In-Class Exercise 1 *[Probability]*

For all parts of this exercise, two dice are rolled, one after the other.

(a) What is the probability of getting a 2 on the first die and a 3 on the second die?

(b) What is the probability of getting a 2 on both dice?

(c) What is the probability of getting odd numbers on both dice?

(d) What is the probability of getting an odd number on the first die and any number on the second die?

(e) What is the probability of getting a 5 on both dice, or getting a 4 on both dice?

(f) What is the probability of getting an odd number on the first die and a 5 on the second die, or getting a 1 or 3 on the first die and and an odd number on the second die?

Section 8.2: In-Class Exercise 2 *[Probability]*

For all parts of this exercise, a card is drawn from a standard deck of cards (with no jokers), the card is replaced, the deck is shuffled, and a card is drawn again.

(a) What is the probability of getting a king both times a card is drawn?

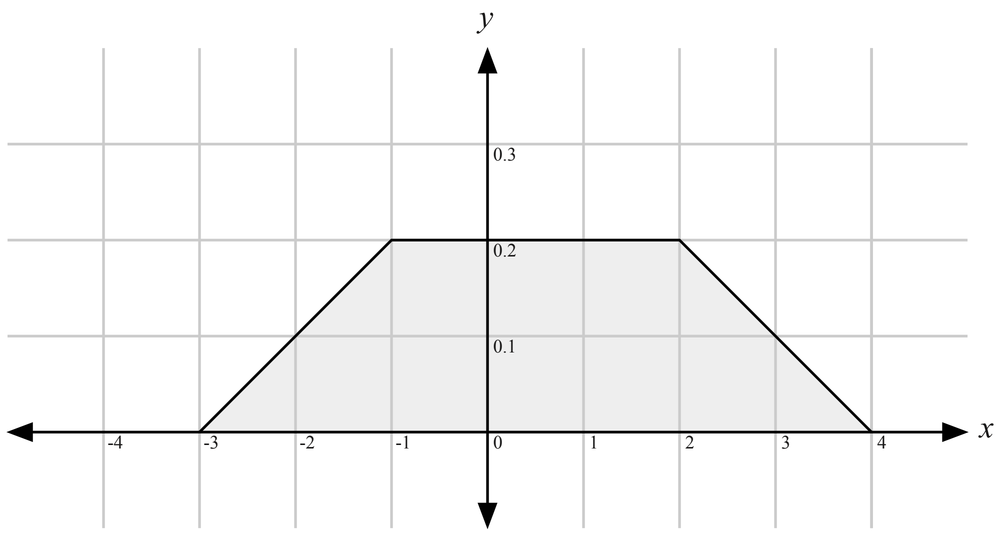
(b) What is the probability of getting a red card the first time a card is drawn and an ace the second time a card is drawn?

(c) What is the probability of getting a 5 both times a card is drawn, or getting a queen both times a card is drawn?

(d) What is the probability of getting a number from 2 to 10 both times a card is drawn, or getting a red card both times a card is drawn?

Section 8.3: In-Class Exercise 1 *[Probability Density Function]*

All parts of this exercise refer to the probability density function seen in the following figure.



(a) Find the probability $P(0 < X < 2)$.

(b) Find the probability $P(X < -1)$.

(c) Find the probability $P(-2 < X < 3)$.

Section 9.2: In-Class Exercise 1 *[Normal Distribution: Z-Score to P-Value]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the probability $P(Z < 0.24)$.

(b) Find the probability $P(-1.82 < Z)$.

(c) Find the probability $P(1.53 < Z < 1.76)$.

Section 9.2: In-Class Exercise 2 *[Normal Distribution: Z-Score to P-Value]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the probability $P(Z < -1.07)$.

(b) Find the probability $P(2.16 < Z)$.

(c) Find the probability $P(-0.71 < Z < 1.63)$.

Section 9.3: In-Class Exercise 1 *[Normal Distribution: P-Value to Z-Score]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the number z^* such that $P(Z < z^*) = 0.0217$.

(b) Find the number z^* such that $P(Z < z^*) = 0.9233$.

(c) Find the number z^* such that $P(z^* < Z) = 0.8708$.

(d) Find the number z^* such that $P(z^* < Z) = 0.1073$.

Section 9.3: In-Class Exercise 2 *[Normal Distribution: P-Value to Z-Score]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the number z^* such that $P(Z < z^*) = 0.9872$.

(b) Find the number z^* such that $P(Z < z^*) = 0.0304$.

(c) Find the number z^* such that $P(z^* < Z) = 0.2577$.

(d) Find the number z^* such that $P(z^* < Z) = 0.9479$.

Section 9.3: In-Class Exercise 3 *[Normal Distribution: P-Value to Z-Score]*

All parts of this exercise refer to the standard normal distribution.

(a) Find the left 92% interval of the standard normal distribution.

(b) Find the middle 92% interval of the standard normal distribution.

(c) Find the right 92% interval of the standard normal distribution.

Section 9.4: In-Class Exercise 1 *[Normal Distribution: Z-Score to P-Value]*

The weight of golf balls made by a particular process is normally distributed with mean 1.361 ounces and standard deviation 0.09 ounce. A regulation golf ball may not weigh more than 1.62 ounces. Find the probability that a golf ball made by this process will meet the weight standard.

Shafer-Zhang, Beginning Statistics, Section 5.3, Exercise 17

Section 9.4: In-Class Exercise 2 *[Normal Distribution: Z-Score to P-Value]*

The systolic blood pressure of adults living in a certain region is normally distributed with mean 112 mm Hg and standard deviation 15 mm Hg. A person is considered "prehypertensive" if her systolic blood pressure is between 120 and 130 mm Hg. Find the probability that the blood pressure of a randomly selected person is prehypertensive

Shafer-Zhang, Beginning Statistics, Section 5.3, Exercise 13

Section 9.4: In-Class Exercise 3 *[Normal Distribution: Z-Score to P-Value]*

Birth weights of full-term babies born in a certain region are normally distributed with mean 7.125 lbs. and standard deviation 1.29 lbs. Find the probability that a randomly selected newborn will weigh less than 5.5 lbs., which is the historic definition of prematurity.

Shafer-Zhang, Beginning Statistics, Section 5.3, Exercise 20

Section 9.5: In-Class Exercise 1 *[Normal Distribution: P-Value to Z-Score]*

Scores on a national exam are normally distributed with mean 382 and standard deviation 26.

(a) Find the score that is the 50^{th} percentile.

(b) Find the score that is the 25^{th} percentile.

(c) Find the height that is the 90^{th} percentile.

Shafer-Zhang, Beginning Statistics, Section 5.4, Exercise 11

Section 9.5: In-Class Exercise 2 *[Normal Distribution: P-Value to Z-Score]*

The tread life of a new tire developed by a tire manufacturer is normally distributed with an estimated mean tread life of $67,350$ miles and standard deviation of $1,120$ miles. The manufacturer will advertise the lifetime of the tire (for example, a "50,000 mile tire") using the largest value for which it is expected that 98% of the tires will last at least that long. Find that advertised value.

Shafer-Zhang, Beginning Statistics, Section 5.4, Exercise 17

Section 9.6: In-Class Exercise 1 *[Central Limit Theorem]*

The speed of vehicles on a particular stretch of roadway are normally distributed with mean 36.6 mph and standard deviation 1.7 mph.

(a) Find the probability that the speed of a randomly selected vehicle is between 35 and 40 mph.

(b) Find the probability that the mean speed of 20 randomly selected vehicles is between 35 and 40 mph.

Shafer-Zhang, Beginning Statistics, Section 6.2, Exercise 17

Section 10.2: In-Class Exercise 1 *[Confidence Intervals]*

A government agency was charged by the legislature with estimating the length of time it takes citizens to fill out various forms. Two hundred randomly selected adults were timed as they filled out a particular form. The times required had mean 12.8 minutes with standard deviation 1.7 minutes. Construct a 90% confidence interval for the mean time taken for all adults to fill out this form.

Shafer-Zhang, Beginning Statistics, Section 7.1, Exercise 7

Section 10.2: In-Class Exercise 2 *[Confidence Intervals]*

A corporation that owns apartment complexes wishes to estimate the average length of time residents remain in the same apartment before moving out. A sample of 150 rental contracts gave a mean length of occupancy of 3.7 years with standard deviation 1.2 years. Construct a 95% confidence interval for the mean length of occupancy of apartments owned by this corporation.

Shafer-Zhang, Beginning Statistics, Section 7.1, Exercise 13

Section 10.3: In-Class Exercise 1 *[Sample Size]*

A software engineer wishes to estimate, to within 3 seconds, the mean time that a new application takes to start up, with 95% confidence. Estimate the minimum size sample required if the standard deviation of start up times for similar software is 12 seconds.

Shafer-Zhang, Beginning Statistics, Section 7.4, Exercise 7

Section 10.3: In-Class Exercise 2 *[Sample Size]*

An economist wishes to estimate, to within 2 minutes, the mean time that employed persons spend commuting each day, with 98% confidence. The range of commuting times is 48 minutes. Estimate the minimum size sample required.

Shafer-Zhang, Beginning Statistics, Section 7.4, Exercise 9

Section 11.1: In-Class Exercise 1 *[Hypothesis Testing]*

State the null hypothesis and alternative hypothesis for each of the following situations.

Specifically, identify the number μ_0, write the null hypothesis as one of $\mu = \mu_0$ or $\mu \leq \mu_0$ or $\mu \geq \mu_0$, and then write the corresponding alternative hypothesis.

(a) The average July temperature in a region historically has been 74.5°F. Perhaps it is higher now.

(b) The average stipend for doctoral students in a particular discipline at a state university is $14,756. The department chair believes that the national average is higher.

(c) The average room rate in hotels in a certain region is $82.53. A travel agent believes that the average in a particular resort area is different.

(d) The average farm size in a predominately rural state was 69.4 acres. The secretary of agriculture of that state asserts that it is less today.

Shafer-Zhang, Beginning Statistics, Section 8.1, Exercise 1

Section 11.2: In-Class Exercise 1 *[Hypothesis Testing]*

In the past the average length of an outgoing telephone call from a business office has been 143 seconds. A manager wishes to check whether that average has decreased after the introduction of policy changes. A sample of 100 telephone calls produced a mean of 133 seconds, with a standard deviation of 35 seconds. Would the manager be justified in saying that the average length of an outgoing call has decreased at the 2% level of significance.

Shafer-Zhang, Beginning Statistics, Section 8.2, Exercise 9

Section 11.2: In-Class Exercise 2 *[Hypothesis Testing]*

An automobile manufacturer recommends oil changes at intervals of $3,000$ miles. To compare actual intervals to the recommendation, the company randomly samples records of 50 oil changes at service facilities and obtains sample mean $3,752$ miles with sample standard deviation 638 miles. Determine whether the data provide sufficient evidence, at the 5% level of significance, that the population mean interval between oil changes exceeds $3,000$ miles.

Shafer-Zhang, Beginning Statistics, Section 8.2, Exercise 13