MATH 104 Data and Decisions Spring 2017

Study Sheet for Exam #1
Voting

- This study sheet will not be allowed during the exam.
- Books, notes and online resources will not be allowed during the exam.
- A calculator will be handed out for use during the exam. **No other electronic devices (calculators, cell phones, tablets, laptops, etc.) will be allowed during the exam.**

Topics

1. Social choice methods
2. Fairness criteria for social choice methods
3. Arrow's Impossibility Theorem
4. Apportionment methods
5. Fairness criteria for apportionment methods
6. Gerrymandering

Practice Problems from Homework Exericses

- Exercise 1.1
- Exercise 1.2
- Exercise 1.3
- Exercise 1.4
- Exercise 1.5
- Exercise 1.6
- Exercise 1.7
- Exercise 1.8
- Exercise 1.9
- Exercise 1.10
- Exercise 1.11
- Exercise 1.12
- Exercise 1.13
- Exercise 1.14 (a)(b)

Practice Problems from In-Class Exericses

- Exercise 1.1
- Exercise 1.2
- Exercise 1.3
- Exercise 1.4
- Exercise 1.5
- Exercise 1.6 (a)(b)

Tips for Studying for the Exams

×	**Bad**	Forgetting about the homework.
✓	**Good**	Making sure you know how to do all the relevant problems from the homework exercises and in-class exercises; seeking help from the instructor and the tutors about the problems you do not know how to do.
×	**Bad**	Studying only by reading the texts.
✓	**Good**	Doing exercises, and reading the texts as needed.
×	**Bad**	Studying only by yourself.
✓	**Good**	Trying some exercises by yourself (or with friends), and then seeking help from the instructor about the exercises you do not know how to do.
×	**Bad**	Doing exercises while looking everything up in the texts or summaries.
✓	**Good**	Doing some of the exercises the way you would do them on the exams, which is with closed book.
×	**Bad**	Staying up late (or all night) the night before the exam.
✓	**Good**	Studying hard up through the day before the exam, but getting a good night's sleep the night before the exam.

Ethan's Office Hours

- **Monday:** 11:00-12:30
- **Tuesday:** 2:00-3:30 & 5:00-6:00
- **Thursday:** 2:30-4:00
- **Or by appointment**

Concepts and Formulas You Need to Know for the Exam

1. Social Choice Method

A **social choice method** (also known as **social choice function**) is a procedure that takes as input the preferences among the candidates expressed by the voters, and gives as output either a single winning candidate, or tied winning candidates, or a statement that there is no winner.

2. Plurality Voting

Suppose an election has more than two candidates. Every voter votes for one candidate. The method of **Plurality Voting** is that the candidate with the most votes wins the election.

3. Single Runoff Voting with Two Ballots

Suppose an election has more than two candidates. Every voter votes for one candidate. The method of **Single Runoff Voting** is that if a candidate has more than 50% of the votes, that candidate wins; if no candidate has more than 50% of the votes, then there is a runoff vote between the two candidates with the highest numbers of votes in the first round.

4. Single Runoff Voting with One Ballot

Suppose an election has more than two candidates. Every voter ranks the candidates without ties. The method of **Single Runoff Voting** is that if a candidate has more than 50% of the first place rankings, that candidate wins; if no candidate has more than 50% of the first place rankings, then all the candidates except the two with the largest number of the first place rankings are dropped from the rankings, and the candidate who has more than 50% of the first place rankings among the remaining two candidates is the winner.

5. Instant Runoff Voting

Suppose an election has more than two candidates. Every voter ranks the candidates without ties. The method of **Instant Runoff Voting** (also called the **Hare System**, among other names) is that if a candidate has more than 50% of the first place rankings, that candidate wins; if no candidate has more than 50% of the first place rankings, then the candidate with the least number of the first place rankings is dropped from the rankings, and the process is repeated, as many times as needed, until a candidate has more than 50% of the first place rankings among the remaining candidates, and that candidate is the winner.

6. Borda Count Voting

Suppose an election has more than two candidates. Suppose that there are n candidates. Every voter ranks the candidates without ties. The method of **Borda Count Voting** is that each candidate receives n point for each first place ranking, and $n-1$ points for each second place ranking, and so forth, concluding with 1 point for each last place ranking, and the candidate with the highest total number of points wins the election.

7. Sequential Pairwise Voting with Fixed Agenda

Suppose an election has more than two candidates. Every voter ranks the candidates without ties. The method of **Sequential Pairwise Voting with Fixed Agenda** is as follows. First, the candidates are listed in some order (called an "agenda"). Next, the first two candidates in the given order are compared, where the winner is the candidate who is ranked higher than the other candidate on a majority of the ballots. Next, the winner among the first two candidates is the compared with the third candidate in the given order, and the process is repeated until the last candidate is compared with the winner of the previous comparisons, and the final winner wins the election.

8. Condorcet Voting

Suppose an election has more than two candidates. Every voter ranks the candidates without ties. The method of **Condorcet Voting** is that every pair of candidates are compared, where the winner is the candidate who is ranked higher than the other candidate on a majority of the ballots, and if a candidate defeats everyone else, that candidate wins the election; if no candidate defeats everyone else, then there is no winner.

9. Copeland Voting

Suppose an election has more than two candidates. Every voter ranks the candidates without ties. The method of **Copeland Voting** is that every pair of candidates are compared, where the winner is the candidate who is ranked higher than the other candidate on a majority of the ballots, and in every pairwise comparison, the candidate who defeats the other receives 1 point, and tied candidates receive $\frac{1}{2}$ point each, and the candidate with the highest total number of points wins the election.

10. Dictatorship

Suppose an election has more than two candidates, and that one voter is chosen as the "dictator." Every voter ranks the candidates without ties. The method of **Dictatorship** is that the candidate with the highest ranking on the dictator's ballot wins the election.

11. Approval Voting

Suppose an election has two or more candidates. Every voter votes for as many candidates as she wants. The method of **Approval Voting** is that each candidate receives 1 point for each vote, and the candidate with the highest total number of points wins the election.

12. Range Voting

Suppose an election has two or more candidates. Every voter gives each candidate some number of points within a preset range of possible scores. The method of **Range Voting** is that the candidate with the highest total number of points wins the election.

13. Standard Quota

Suppose an apportionment has n states, denoted $S_1, \dots, S_n$, and H representatives. Suppose that the states have populations $P_1, \dots, P_n$, respectively. The total population is $P = P_1 + \cdots + P_n$.

1. The **standard quota** for state S_k, denoted Q_k, is $\frac{P_k}{P} H$.
2. The **standard quota rounded down** for state S_k, denoted D_k, is the result of rounding Q_k down to the nearest whole number.
3. The **standard quota remainder** for state S_k, denoted R_k, is $Q_k - D_k$.

14. Hamilton's Method

Suppose an apportionment has n states, denoted $S_1, \dots, S_n$, and H representatives. Suppose that the states have populations $P_1, \dots, P_n$, respectively.

Hamilton's Method allocates the representatives in the following steps.

1. For each state, find its standard quota, standard quota rounded down and standard quota remainder.
2. Allocate representatives according to the standard quotas rounded down.
3. Add up the representatives allocated in the previous step, and find the remaining number of representatives.
4. Allocate the remaining representatives by giving one representative at a time, starting with the state with the largest standard quota remainder, then the state with the second largest standard quota remainder, etc., until all the remaining representatives are allocated.

15. Standard Divisor

Suppose an apportionment has n states, denoted $S_1, \dots, S_n$, and H representatives. Suppose that the states have populations $P_1, \dots, P_n$, respectively. The total population is $P = P_1 + \cdots + P_n$.

1. The **standard divisor** for these states, denoted T, is $\frac{P}{H}$.
2. The standard quota for state S_k equals $\frac{P_k}{T}$.

16. Jefferson's Method

Suppose an apportionment has n states, denoted $S_1, \ldots, S_n$, and H representatives. Suppose that the states have populations $P_1, \ldots, P_n$, respectively.

Jefferson's Method allocates the representatives in the following steps.

1. Find the standard divisor, denoted T.
2. For each state S_k, find its standard quota, which equals $\frac{P_k}{T}$, and its standard quota rounded down, which is the whole number that is the result of doing rounding down to the standard quota.
3. Allocate representatives according to the standard quotas rounded down.
4. Add up the representatives allocated using the standard quotas rounded down. If the number of representatives allocated using the standard quotas rounded down equals H, the allocation is complete.
5. If the number of representatives allocated using the standard quotas rounded down does not equal H, choose a modified divisor, denoted $\hat{T}$, which is different from the standard divisor. For each state S_k, find its modified quota, which equals $\frac{P_k}{\hat{T}}$, and its modified quota rounded down, which is the whole number that is the result of doing rounding down to the modified quota.
6. Allocate representatives according to the modified quotas rounded down.
7. Add up the representatives allocated using the modified quotas rounded down. If the number of representatives allocated using the modified quotas rounded down equals H, the allocation is complete.
8. If the number of representatives allocated using the modified quotas rounded down does not equal H, choose a new modified divisor, and try again, and keep trying until a modified divisor is found so that the number of representatives allocated using the modified quotas rounded down equals H

17. Adams' Method

Suppose an apportionment has n states, denoted $S_1, \ldots, S_n$, and H representatives. Suppose that the states have populations $P_1, \ldots, P_n$, respectively.

Adams' Method allocates the representatives in the following steps.

1. Find the standard divisor, denoted T.
2. For each state S_k, find its standard quota, which equals $\frac{P_k}{T}$, and its standard quota rounded up, which is the whole number that is the result of doing rounding up to the standard quota.
3. Allocate representatives according to the standard quotas rounded up.
4. Add up the representatives allocated using the standard quotas rounded up. If the number of representatives allocated using the standard quotas rounded up equals H, the allocation is complete.
5. If the number of representatives allocated using the standard quotas rounded up does not equal H, choose a modified divisor, denoted $\hat{T}$, which is different from the standard divisor. For each state S_k, find its modified quota, which equals $\frac{P_k}{\hat{T}}$, and its modified quota rounded up, which is the whole number that is the result of doing rounding up to the modified quota.
6. Allocate representatives according to the modified quotas rounded up.
7. Add up the representatives allocated using the modified quotas rounded up. If the number of representatives allocated using the modified quotas rounded up equals H, the allocation is complete.
8. If the number of representatives allocated using the modified quotas rounded up does not equal H, choose a new modified divisor, and try again, and keep trying until a modified divisor is found so that the number of representatives allocated using the modified quotas rounded up equals H

18. Webster's Method

Suppose an apportionment has n states, denoted $S_1, \dots, S_n$, and H representatives. Suppose that the states have populations $P_1, \dots, P_n$, respectively.

Webster's Method allocates the representatives in the following steps.

1. Find the standard divisor, denoted T.
2. For each state S_k, find its standard quota, which equals $\frac{P_k}{T}$, and its standard quota rounded, which is the whole number that is the result of doing standard rounding to the standard quota.
3. Allocate representatives according to the standard quotas rounded.
4. Add up the representatives allocated using the standard quotas rounded. If the number of representatives allocated using the standard quotas rounded equals H, the allocation is complete.
5. If the number of representatives allocated using the standard quotas rounded does not equal H, choose a modified divisor, denoted $\hat{T}$, which is different from the standard divisor. For each state S_k, find its modified quota, which equals $\frac{P_k}{\hat{T}}$, and its modified quota rounded, which is the whole number that is the result of doing standard rounding to the modified quota.
6. Allocate representatives according to the modified quotas rounded.
7. Add up the representatives allocated using the modified quotas rounded. If the number of representatives allocated using the modified quotas rounded equals H, the allocation is complete.
8. If the number of representatives allocated using the modified quotas rounded does not equal H, choose a new modified divisor, and try again, and keep trying until a modified divisor is found so that the number of representatives allocated using the modified quotas rounded equals H

19. Huntington-Hill Method

Suppose an apportionment has n states, denoted $S_1, \dots, S_n$, and H representatives. Suppose that the states have populations $P_1, \dots, P_n$, respectively.

The **Huntington-Hill Method** allocates the representatives in the following steps.

1. Find the standard divisor, denoted T.
2. For each state S_k, find its standard quota, which equals $\frac{P_k}{T}$, and its standard quota geometric mean rounded, which is the whole number that is the result of doing geometric mean rounding to the standard quota.
3. Allocate representatives according to the standard quotas geometric mean rounded.
4. Add up the representatives allocated using the standard quotas geometric mean rounded. If the number of representatives allocated using the standard quotas geometric mean rounded equals H, the allocation is complete.
5. If the number of representatives allocated using the standard quotas geometric mean rounded does not equal H, choose a modified divisor, denoted $\hat{T}$, which is different from the standard divisor. For each state S_k, find its modified quota, which equals $\frac{P_k}{\hat{T}}$, and its modified quota geometric mean rounded, which is the whole number that is the result of doing geometric mean rounding to the modified quota.
6. Allocate representatives according to the modified quotas geometric mean rounded.
7. Add up the representatives allocated using the modified quotas geometric mean rounded. If the number of representatives allocated using the modified quotas geometric mean rounded equals H, the allocation is complete.
8. If the number of representatives allocated using the modified quotas geometric mean rounded does not equal H, choose a new modified divisor, and try again, and keep trying until a modified divisor is found so that the number of representatives allocated using the modified quotas geometric mean rounded equals H

Concepts and Formulas that Will Be Stated on the Exam If Needed

1. Majority Criterion

A social choice method in which the voters rank the candidates without ties satisfies the **Majority Criterion**, abbreviated **MAJ**, if the following holds: if candidate A receives a majority of first place votes, then A is the winner.

2. Always a Winner Criterion

A social choice method in which the voters rank the candidates without ties satisfies the **Always a Winner Criterion**, abbreviated **AAW**, if the following holds: the method always produces a winner or a group of candidates tied for winner.

3. Condorcet Winner

Suppose an election has more than two candidates. Every voter ranks the candidates without ties. A **Condorcet winner** is a candidate who, when compared with every other candidates, is ranked higher than the other candidate on a majority of the ballots.

4. Condorcet Winner Criterion

A social choice method in which the voters rank the candidates without ties satisfies the **Condorcet Winner Criterion**, abbreviated **CWC**, if the following holds: if candidate A is a Condorcet winner, then A is the winner by the social choice method.

5. Monotonicity Criterion

A social choice method in which the voters rank the candidates without ties satisfies the **Monotonicity Criterion**, abbreviated **MON**, if the following holds: if candidate A is the winner (or tied for winner), and if one or more voters changes her ballot by exchanging A with the candidate she previously ranked just above A, then A would still be the winner (or tied for winner).

6. Pareto Criterion

A social choice method in which the voters rank the candidates without ties satisfies the **Pareto Criterion**, abbreviated **PAR**, if the following holds: if all voters rank candidate A higher than candidate B, then candidate B is not the winner (or tied for winner).

7. Independence of Irrelevant Alternatives Criterion

A social choice method in which the voters rank the candidates without ties satisfies the **Independence of Irrelevant Alternatives Criterion**, abbreviated **IIA**, if the following holds: if candidate A is the winner (or tied for winner) and candidate B is not the winner (or tied for winner), and if one or more voters changes her ballot but does not change which of A or B is ranked higher than the other, then B would still not be the winner (or tied for winner).

8. Simple Impossibility Theorem

There is no social welfare method for three or more candidates in which the voters rank the candidates without ties, and that satisfies Always a Winner Criterion, Independence of Irrelevant Alternatives Criterion and Condorcet Winner Criterion.

9. Social Welfare Method

A **social welfare method** (also known as **social welfare function**) is a procedure that takes as input the preferences among the candidates expressed by the voters, and gives as output a ranking of the candidates, perhaps with ties.

10. Pareto Criterion for Social Welfare Methods

A social welfare method in which the voters rank the candidates without ties satisfies the **Pareto Criterion for Social Welfare Methods** (also called **Unanimity Criterion**), abbreviated **PAR-SWM**, if the following holds: if all voters rank candidate A higher than candidate B, then candidate A is ranked higher than candidate B in the social welfare ranking.

11. Arrow's Impossibility Theorem

There is no social choice method for three or more candidates in which the voters rank the candidates without ties, and that satisfies Pareto Criterion for Social Welfare Methods, Independence of Irrelevant Alternatives Criterion and Monotonicity Criterion, other than dictatorship.

12. Intensity of Independence of Irrelevant Alternatives Criterion

A social choice method in which the voters give each candidate some number of points within a preset range of possible scores satisfies the **Intensity of Independence of Irrelevant Alternatives Criterion**, abbreviated **IIIA**, if the following holds: if candidate A is the winner (or tied for winner) and candidate B is not the winner (or tied for winner), and if one or more voters changes her ballot but does not change her intensity of preference for A over B, then B would still not be the winner (or tied for winner).

13. Non-Manipulable Criterion

A social choice method in which the voters rank the candidates without ties satisfies the **Non Manipulable Criterion**, abbreviated **NM**, if the following holds: there is no voter who can change her ranking of the candidates from her sincere ranking to an insincere ranking and by doing so cause there to be a winner who is higher ranked on her sincere ranking than occurred when she voted sincerely.

14. Gibbard-Satterthwaite Theorem

There is no social choice method for three or more candidates in which the voters rank the candidates without ties, and that satisfies Pareto Criterion and Non-Manipulable Criterion, other than dictatorship.

15. Quota Criterion

An apportionment method satisfies the **Quota Criterion**, abbreviated **QUO**, if the following holds: the number of representatives allocated to each state is either its standard quota rounded down or its standard quota rounded up.

16. House Monotonicity Criterion

An apportionment method satisfies the **House Monotone Criterion**, abbreviated **HMON**, if the following holds: if the number of representatives is increased, no state loses representatives.

17. Population Criterion

An apportionment method satisfies the **Population Criterion**, abbreviated **POP**, if the following holds: if the population of state A increases and the population of state B decreases, then it cannot happen that A loses representatives and B gains representatives or stays the same.

18. One Version of Balinski-Young Theorem

An apportionment method that satisfies House Monotone Criterion and Population Criterion will not satisfy Quota Criterion, and an apportionment method that satisfies Quota Criterion will not satisfy House Monotone Criterion and Population Criterion.