# Math 313 Discrete and Computational Geometry Fall 2021

# What is Discrete & Computational Geometry

#### **COURSE CONTENT**

- Discrete and computational geometry, which has applications in areas such as pattern recognition, image processing, computer graphics and terrain modeling, is the study of geometric constructs in two- and three-dimensional space that arise from finite sets of points. This class will treat fundamental topics in the field, including convex hull, Delaunay triangulations, Voronoi diagrams, curve reconstruction and polyhedra. The class will combine both theory and algorithms.
- The work for the class will involve both traditional proofs and implementation of algorithms in the Python programming language, which will be discussed in class.
- This course is cross-listed with the Computer Science Program.

#### PREREQUISITES

- The prerequisites for this course are either Proofs and Fundamentals (MATH 261), or Discrete Mathematics (CMSC 145) and some programming experience.
- If you are unsure whether Math 313 is an appropriate course for you, please speak with the instructor.

### **Basic Information**

### CLASS

- Monday: 3:50-5:10
- Wednesday: 3:50–5:10
- Hegeman 204

### **INSTRUCTOR**

- Instructor: Ethan Bloch
- Office: Albee 317
- Phone: (845) 758-7266
- Email: bloch "at" bard "dot" edu
- Website: http://faculty. bard.edu/bloch/

### **Exams**

### EXAM DATES

- Midterm Exam: In Class, Closed Book Wed., Oct. 20
- Final Exam: *Take Home, Open Book* Posted Mon., Dec. 13, Due Wed., Dec. 15

### Grades

### GRADING

- Each of the three exams will count for 30% of the grade, and homework will count for 10% of the grade. Class participation will be taken into account positively, especially in cases of border-line grades.
- Grades will be determined by work completed during the semester, except in cases of medical or personal emergency. There will be no opportunity to do extra credit work after the semester ends.

### PASS/FAIL

• This course is graded using letter grades. If you want to take the course Pass/Fail, you must submit a request to do so to the Registrar's Office by the end of the Late Drop Period.

# **Google Classroom & Email**

### GOOGLE CLASSROOM

- All the needed information for this class will be available at the Google Classroom site for this class. You will be invited to joint this Google Classroom site at the start of the semester.
- Summary notes and programming will be available at the Google Classroom site.
- All homework assignments will be posted at the Google Classroom site. Theory assignments will be submitted via the Google Classroom site, and programming assignments will be submitted via Google Colab.

#### **EMAIL**

• Urgent announcements may be sent out via campus email, so make sure you either check your Bard email regularly or have your Bard email forwarded to the email address of your choice.

# **Computers & Electronics**

### COMPUTERS

- Programming assignments in Python, using Jupyter Notebooks, are an important part of this course. There are two recommended ways to do Jupyter Notebooks for this course: (1) Ananconda, which you can download onto your computer, or (2) Google CoLab, which is done online.
- Theory homework assignments can be done either by hand or in LATEX. If you use LATEX, it must be done using the homework template of the Bard TEX style file, which is available at the instructor's TEXwebsite. If you need help with LATEX, ask the instructor.

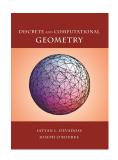
### ELECTRONIC DEVICES DURING CLASS

- Electronic devices, including cell phones, tablets and laptop computers, may be used during class only for reasons related to the class, for example to take notes, to read the text or to do programming.
- Texting, messaging and using social media is not allowed during class.

# Textbook

### TEXTBOOK

- The textbook for the class is
  - Devadoss, Satyan L. and O'Rourke, Joseph, "Discrete and Computational Geometry," Princeton University Press, 2011



### LIST OF ERRORS

- A list of errors for the textbook is found at
  - http://cs.smith.edu/~jorourke/DCG/errata.html

# Office Hours

### HELP OUTSIDE OF CLASS

• If you have any problems with the course, or any questions about the material, the assignments, the quizzes, the exams or anything else, please see the instructor about it as soon as possible. If you cannot make any of the scheduled office hours, please make an appointment for some other time. To make an appointment, or to discuss anything, talk to the instructor after class, or send him an email message, or just stop by his office.

### TIMES

- Tuesday: 5:00-6:30
- Thursday: 1:30-3:00 & 4:00-6:00
- Or by appointment

### Accommodations

#### ACCOMMODATIONS

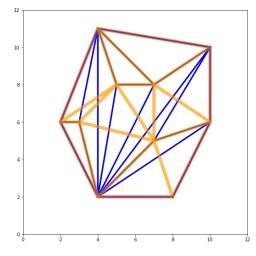
- Students with documented learning and/or other disabilities are entitled to receive reasonable classroom and testing accommodations.
- If you need accommodations, please do the following.
  - Contact the Office of Disability Support Services, who will work with you and will provide documentation to the instructor.
  - Contact the instructor at least one week prior to each exam, quiz or other instance of accommodation, to arrange appropriate scheduling.
  - If you feel comfortable doing so, discuss your accommodations with the instructor at the beginning of the semester.

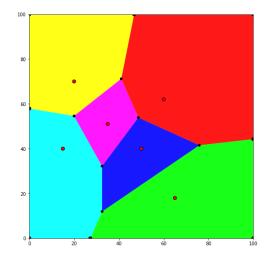
#### **MISSING CLASS**

- If you need to miss a class for any reason (sports team, religious holiday, etc.), it is your responsibility to contact the instructor and find out about the material and assignments you missed.
- Travel plans for fall break, Thanksgiving and the end of the semester must take into account the dates of the exams.

### **Important Dates**

- Wed., Sept. 8: End of Drop/Add period
- Wed., Sept. 29: End of Late Drop and Pass/Fail
- Mon., Oct. 11 Tue., Oct. 12: Fall break
- Fri., Oct. 22: Fall moderation papers due
- Thur., Nov. 25 Fri., Nov. 26: Thanksgiving break
- Fri., Dec. 3: Last day to withdraw from a class
- Mon., Dec. 6: Fall senior projects due
- Wed., Dec. 8: Advising day (no classes)
- Thur., Dec. 9: Registration for spring classes
- Fri., Dec. 17: Last day of classes





### **Homework**

### THEORY AND PROGRAMMING HOMEWORK

- This class has two types of homework assignments: theory assignments and programming assignments.
- Some classes will have both types of homework, and other classes will have only one type. Generally, there will be more theory homework early in the course, and more programming homework later in the course.
- Both types of homework assignments will be posted at the class Google Classroom site, at the location corresponding to the date on which it was assigned.

### DUE DATE FOR HOMEWORK

- Homework will be assigned every class, and is due by the start of the following class.
- Late assignments will not be accepted, except in emergency situations.

### THEORY HOMEWORK – FINDING, SUBMITTING AND FORMATTING

- The exercise numbers that are posted for theory homework refer to exercises in the Theory Notes for this class (which are available the the class Google Classroom site); these exercise numbers do NOT refer to the textbook.
- Submit the theory homework via Google Classroom, at the location corresponding to the date on which it was assigned.
- Theory homework assignments can be done either by hand or in LATEX. If you use LATEX, it must be done using the homework template of the Bard TEX Style file, which is available at the instructor's TeX website.
- Every theory homework assignment must be uploaded to Google Classroom as a SINGLE PDF FILE. No other format will be accepted.
- If you write your theory homework by hand and you are not sure how to scan it and create a single PDF file, please discuss it with the instructor.

### PROGRAMMING HOMEWORK - FINDING, SUBMITTING AND FORMATTING

- The exercise numbers that are posted for programming homework refer to exercises in the Programming Notes for this class (which are available the the class Google Classroom site); these exercise numbers do NOT refer to the textbook.
- Every programming homework exercise must be shared with the instructor via Google Colab, using the email address bloch@bard.edu for sharing.
- Every programming homework exercise must be shared as a separate file, based upon the Homework Template Jupyter notebook (which will be shared with you at the start of the semester). For each programming homework exercise, start it by making a new copy of either the Homework Template or a previous exercise.
- The title for each shared homework assignment must include your name, and also either the date the assignment was posted or the assignment number listed in Google Classroom.

### DOING HOMEWORK

- You are encouraged to work with other students in solving the homework problems. However, for the sake of better learning, as well as honesty, please adhere to the following guidelines:
  - Write up your solutions yourself.
  - Acknowledge in writing anyone with whom you work and any assistance you receive.
  - Acknowledge in writing any revisions of your work based upon solutions given in class.
- Failure to indicate collaboration, assistance or sources will be construed as plagiarism.
- Your solutions should be written clearly and carefully, as described below.

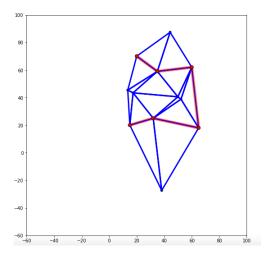
# Writing Proofs and Programs

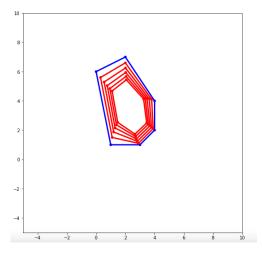
### WRITING PROOFS

- Everyone makes honest mathematical mistakes, but there is no reason to get in your own way by writing your proofs with incomplete sentences and other grammatical mistakes, by using undefined symbols for "variables" or by engaging in other forms of sloppy writing. The goal of writing mathematics is two-fold: making sure that a proof is correct in all details, and communicating the proof so that others can understand it. To help achieve those aims, mathematics must be written carefully, and with proper grammar, no differently from any other writing. Properly written proofs entail the following basic points.
  - Write your homework assignments neatly and clearly.
  - Use correct grammar, including full sentences and proper punctuation.
  - Justify each step in a proof, citing the appropriate results from the class notes and/or the textbook as needed.
  - Use definitions precisely as stated.
  - Be very careful with quantifiers.
  - Strategize the outline of a proof before working out the details; the outline of a proof is always determined by what is being proved, not by what is known.
  - Distinguish between scratch work and the actual proof; scratch work can be in any order, but the actual proof always starts with what is known and deduces the desired result.
  - Proofs should stand on their own; check your proofs by reading them as if they were written by someone else.

### WRITING PROGRAMS

- Similarly to writing mathematics, the goal of writing computer programs is two-fold: making sure that a program is correct in all details, and communicating the details of the program so that others can understand it. Properly written programs entail the following basic points.
  - Adhere to all the conventions of the particular programming language.
  - Break up lengthy programs into smaller pieces via the use of functions, classes, and the like.
  - Cluster related lines of code into "paragraphs," each of which focuses on a single idea or step. Leave a blank lines between paragraphs of code.
  - Use comments generously, for example a comment describing the purpose of each paragraph of code.





# Learning To Do Rigorous Mathematics

### IT IS NOT THE SAME AS CALCULUS

- Proofs-based mathematics courses are very different from computation-based mathematics courses such as Calculus. The ways you studied, did homework and took exams in computation-based courses was appropriate for those courses, but not for proofs-based courses. Approach proofs-based courses with the idea that you will be doing things differently from what you did in computation-based mathematics courses.
- The material in this course is much more abstract, and requires much more precision in both studying and problem solving, than the material you saw in courses such as Calculus. For some students, a proofs-based course such as this one is the first time that they found a mathematics course really challenging, which can be intimidating at first, but is in fact completely normal. Everyone, including the very best mathematicians, reaches a level of mathematics that is a challenge; what varies from person to person is only what that level is. If you made it this far in mathematics and you only now first encounter substantial difficulty in learning the material, you are doing fine.
- In general, the more advanced you get in mathematics (or any subject), the larger the percentage of learning that takes place outside of class, including from the textbook, from other sources, from office hours, from tutors and from your fellow students (not necessarily in that order).
- In Calculus courses, where the material can mostly be learned in class, reading the textbook is not necessarily very important. By contrast, in proofs-based courses reading the textbook carefully, and seeking help with those parts of the textbooks that you find difficult, is crucial.
- In proofs-based courses, reading the textbook is very different from reading fiction, in two ways. First, reading proofs-based mathematics, which cannot be done without pencil and paper in hand, requires active engagement by regularly stopping to work out the details of what is written. Make sure you know why each step in a proof is true before moving on, and if you are unable to figure out one or more steps of a proof, seek help. Second, mathematics is not read in order from beginning to end, but "from the outside in." When you read a proof, start by looking for the overall idea of the proof, and then figure out the strategy that is being used, and only then go through the details one step at a time.
- In Calculus courses, solutions to homework exercises are usually written as a collections of equations, with little or no words explaining the solution. By contrast, rigorous proofs are, fundamentally, convincing arguments, and to make a good argument, words are needed to direct the logical flow of the ideas; to explain what is assumed and what is to be proved; and to state what previous results are used. In particular, rigorous proofs are written using full sentences, and with correct grammar and punctuation, because doing so helps make the arguments more clear and precise.

### **REVISE AND REVISE**

- In Calculus courses, solutions to homework problems are usually written directly, with little revision. By contrast, rigorous proofs should be written the same way a paper in a humanities course is written, by first making an outline (often called "strategizing a proof"); then sketching out a rough draft; then revising the draft repeatedly until the proof works; and, lastly, writing the final draft carefully (and then often typing it in LATEX).
- Revising a draft of a rigorous proof should be done exactly as revising a draft of a paper in a humanities course, which is to read it as if you are not the author, but rather as if you are someone else in the class, and making sure that each sentence makes sense as written, without recourse to unwritten explanations.

### PRACTICE AND GET FEEDBACK

- Learning to write rigorous proofs takes time, and you should not expect to master it instantly.
- A very good way to improve your skill at writing proofs, and to do as well as possible on the homework in this course, is to bring a draft of every homework assignment to office hours before you write up and submit the final draft.

### Diversity, Equity, and Inclusion

### BARD NOTICE OF NONDISCRIMINATION

• Bard College is committed to ensuring equal access to its educational programs and equal employment without regard to an individual's sex, gender, race, color, national origin, religion, age, disability, gender identity, sexual orientation, predisposing genetic characteristics, marital status, veteran status, military status, domestic violence victim status, ex-offender status, or any other characteristic protected by federal, state, or local law. Students, employees, applicants, and other members of the Bard College community (including, but not limited to, vendors, visitors, and guests) shall not be subject to discrimination or harassment prohibited by law or otherwise treated adversely based upon a protected characteristic. Similarly, the College will not tolerate harassing, violent, intimidating, or discriminatory conduct by its students, employees, or any other member of, or visitor to, the College community. This includes, without limitation, sexual harassment, sexual assault, sexual violence, dating violence, and domestic violence. (From https://www.bard.edu/dei/policies/.)

### **REPORTING A BIAS INCIDENT OR HATE CRIME**

• Bard College strongly encourages the reporting of all bias incidents and hate crimes that occur on campus, at college-sponsored events, or activities occurring off campus. If you feel that you have been the victim of a bias incident or hate crime, or you believe one has occurred, you are strongly encouraged to report it as quickly as possible. (From https://www.bard.edu/dei/policies/ ; see that site for how to report a bias incident or hate crime at Bard.)

### DIVERSITY, EQUITY, AND INCLUSION IN MATHEMATICS

- The discipline of mathematics which has its roots in many diverse cultures across the ancient world and is to this day studied and used universally has had a long history of excluding many people on the basis of race, gender, sexual orientation, religion, class and more. The Mathematics Program at Bard College is committed to joining the broader efforts in the world of mathematics aimed at making our field be not only open but positively welcoming to all who want to study our beautiful and useful subject. Denying access to a good education in mathematics to some categories of people is an unfair obstacle to their intellectual growth and job opportunities, and causes the field of mathematics to miss out on the broad input into our discipline that a diverse population brings.
- Simple statistics from the U.S. show the field of mathematics is not equally accessible to everyone. In 2017, women were 50.7% of the US population; however, only 25% of all U.S. citizens who earned Ph.D.s in Mathematics during the 2017/18 academic year were women. Similarly, Black people make up 13% of the US population but account for only 2.9% of the Ph.D.s earned by U.S. citizens in Mathematics, and 17.6% of the U.S. population is Latinx while accounting for just 3.6% of the Ph.D.s. The underrepresentation that we see in mathematics can only be explained by systemic inequalities such as structural racism and misogyny.
- The mathematics faculty at Bard are committed to sharing our love of mathematics with future generations and helping to ensure the continued growth of the field. We recognize that the members of the Bard community do not all share the same privileges, resources, time and educational background, but we are firm in our knowledge that everyone is capable of succeeding at mathematics, regardless of how they may self-identify or be identified by others, and that everyone deserves to have a positive experience with mathematics, regardless of the discrimination and discouragement they may have previously faced.
- The Mathematics Program at Bard is committed to creating a welcoming, inclusive, and equitable environment with the goal of having our program better represent our broader community. We seek input from our students, both inside and outside the Mathematics Program, as well as from the larger community to help us ensure we include voices and perspectives that have in the past been missing from our discipline.