

ERRATA FOR

“A First Course in Geometric Topology and Differential Geometry”

Ethan D. Bloch
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Below is an updated list of errata. The fault for all the errors in the book is my own, and I offer my sincere apologies for any inconvenience caused by the errors in the book.

This list was compiled with the generous assistance of: Bill Bloch, Jonathan Dent, John Engbers, Kyle Glashower, Tsz Ho Ip, Gerard Venema, Yumi Watanabe, Peter Maria Wirtz, and Bard students Matthew Brophy, Vasilica Crecea, Tim Goldberg, Jiaming Mao, Kenneth Ober.

If you find any additional errors in the book, or any errors in this list of errors, I would very much appreciate it if you would let me know by email or regular mail at the following address:

Ethan D. Bloch
Bard College
Annandale-on-Hudson, NY 12504
bloch@bard.edu

Page	Line/Item	Text	Comment/Should be
11	Exercise 1.2.1 (1)	“a single point in \mathbb{R}^n ”	Should be “a single point in \mathbb{R}^2 ”
12	Exercise 1.2.6	“containing p ”	Should be “such that $p \in V \subset U$ ”
13	Exercise 1.2.17	“be a closed set”	Should be “be a non-empty closed set”
18	l. 13	“ $f_i = f \circ \pi_i$ ”	Should be “ $f_i = \pi_i \circ f$ ”
24	l. 6	“ f is surjective”	Should be “ q is surjective”
27	l. 7	“ $i_1, \dots, i_r \in \{1, \dots, p\}$ ”	Should be “ $i_1, \dots, i_r \in \{0, 1, \dots, p\}$ ”
27	Exercise 1.4.9		We need to assume that X_i and Y_i are both open or both closed in $X_i \cup Y_i$ for $i = 1, 2$.
28	l. -3	“ $B \cap [b_1, b_2]$ ”	Should be “ $B_1 \cap [b_1, b_2]$ ”
39	l. -6	“ $[a, y)$ ”	Should be “ $[a, y]$ ”

53	l. 17		Remove “containing B ”
63	l. 3	“ $q: D \rightarrow Q$ ”	Should be “ $q: D \rightarrow X$ ”
75	l. 13	“A2.1”	Should be “A2.2”
81	Exercise 2.6.2	“ $K^2 \# P^2$, $K^2 \# K^2$ and $T^2 \# T^2$ ”	Should be “ $K^2 \# P^2$ and $K^2 \# K^2$ ”
89	l. -1	“ k times” in two places	Should be “ d times” in both places
91	l. 6	“ k times” in two places	Should be “ d times” in both places
93	l. 2	“ $h(\overrightarrow{xy}) = \overrightarrow{xy}$ ”	Should be “ $h(\overrightarrow{xy}) = \overrightarrow{h(x)h(y)}$ ”
98	Figure A2.2.7(i)	“ D_4 ”	Should be “ D_n ”
98	Figure A2.2.7(i)	“ α_4 ”	Should be “ α_n ”
99	l. -7	“1.5.11”	Should be “1.6.11”
99	l. -5	“1.5.6”	Should be “1.6.7”
107	Exercise A.2.2.1		The number of the Exercise should be “A2.2.1”
107	Exercise A.2.2.2		The number of the Exercise should be “A2.2.2”
111	l. -5	“ p to q ”	Should be “ v to w ”
115	l. -15	“ $\{a_0, \dots, a_i\}$ ”	Should be “ $\{a_0, \dots, a_k\}$ ”
115	l. -14	“A face of σ that is a k -simplex is called a k -face”	Should be “A face of σ that is an i -simplex is called an i -face”
116	l. 6	“ $S^{k-1} = \{x \in \mathbb{R}^k \mid \ x\ < 1\}$ ”	Should be “ $S^{k-1} = \{x \in \mathbb{R}^k \mid \ x\ = 1\}$ ”
118	l. 9	“affine linear take”	Should be “affine linear map takes”
124	l. 7	“Exercise 3.3.6”	Should be “Lemma 3.3.4”
124	l. 14	“Exercise 3.3.6”	Should be “Lemma 3.3.4”

131	Exercise 3.3.8	“ $\eta \cap \text{Int } \sigma \neq \emptyset$ ” in two places	Should be “ $\text{Int } \eta \cap \text{Int } \sigma \neq \emptyset$ ” in both places
131	l. -14	“that is simplicial complex”	Should be “that is a simplicial complex”
134	l. 10		There should not be a line break in the middle of “ $ \text{star}(w, K) - \{w\}$ ”
141	Exercise 3.5.5	“If P is a 2-dimensional cell complex”	Should be “If P is a 2-dimensional cell complex such that $ P $ is a topological surface,”
154	l. 4	“ $\sum_{\eta v}$ ”	Should be “ $\sum_{\eta \ni v}$ ”
155	l. 6	“ $\sum_{\eta v}$ ”	Should be “ $\sum_{\eta \ni v}$ ”
155	l. 7	“ $\sum_{\eta v}$ ”	Should be “ $\sum_{\eta \ni v}$ ”
155	Corollary 3.7.3	“Let K be a 2-complex”	Should be “Let K be a simplicial surface”
162	l. -1	“ f ”	Should be “ r ”
168	l. 11	“is subsequent”	Should be “in subsequent”
174	l. -7		Remove one “definition”
175	l. -2	“ $h'(s) = 1/q'(s)$ ”	Should be “ $h'(s) = 1/q'(h(s))$ ”
178	Exercise 4.3.1 (ii)	“ $t \ln t - t$ ”	Should be “ $t \ln t$ ”
179	Exercise 4.3.3	“smooth curve”	Should be “regular curve”
179	l. 14		Remove the second “would”
179	l. -12	“chose”	Should be “choose”
180	Exercise 4.3.11		Hint: Use Exercise 4.2.1; let G be as in Exercise 4.2.1, and then note that $(G \circ c)^{-1}$ is smooth, and that $c^{-1} \circ \tilde{c} = (G \circ c)^{-1} \circ G \circ \tilde{c}$, and this latter expression is smooth

182	1. -6	$c''(t) = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$	Should be $c''(t) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
182	1. -6	$c'(t) \times c''(t) = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$	Should be $c'(t) \times c''(t) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
191	1. 10	“ $g''(t)$ ”	Should be “ $g'(t)$ ”
194	1. -7	“the claim”	Should be “Exercise 4.6.2”
194	1. -2	“the claim”	Should be “Exercise 4.6.2”
197	1. 4	“loose”	Should be “lose”
197	1. 16	“counterclockwise”	Should be “clockwise”
197	1. 19	“is clockwise”	Should be “is counterclockwise”
200	Exercise 4.7.1		This exercise needs the formula computed in Exercise 4.7.3
200	Exercise 4.7.1 (ii)	“ \mathbb{R}^3 ”	Should be “ \mathbb{R}^2 ”
200	Exercise 4.7.3		Hint for Exercise 4.7.3: Start as on p. 191, and obtain a formula for $c''(t)$ in terms of \tilde{T} and \tilde{N} . Then take the inner product with $(c')^\perp$
206	1. -11	“rank 2, and hence $D(x \circ \phi)$ has rank 2.”	Should be “rank 2. Because $D\phi$ is a 2×2 matrix, it follows that $D(x \circ \phi)$ has rank 2.”
207	Figure 5.2.2	“ A_x ”	Should be “ A_{xy} ”
207	Figure 5.2.2	“ A_y ”	Should be “ A_{yx} ”
208	1. 2	“ A_{x^1} ”	Should be “ $A_{x^1y^1}$ ”
208	1. 3	“ A_{y^1} ”	Should be “ $A_{y^1x^1}$ ”
208	1. 5	“ A_{x^1} ”	Should be “ $A_{x^1y^1}$ ”

208	Proposition 5.2.5		Remove “and let $p \in M$ be a point” from the statement of the proposition
212	1. 2	“in M ”	Should be “in N ”
213	1. -3	“smooth surfaces”	Should be “smooth surfaces in \mathbb{R}^3 , where we think of U as sitting in the x - y plane in \mathbb{R}^3 ”
216	1. -3	“missing the north and south poles”	Should be “missing half a great circle from the north to the south pole”
220	1. -1	“ $F_{-1}(a)$ ”	Should be “ $F^{-1}(a)$ ”
224	1. -10	“a curve”	Should be “a smooth curve”
226	Equation 5.4.1	“ $x_2(t, \theta)$ ”	Should be “ $x_2\left(\begin{pmatrix} t \\ \theta \end{pmatrix}\right)$ ”
227	1. 4		Remove “such that $p \in x(U)$ ”
227	1. 8	“ n is a”	Should be “ n as a”
228	Exercise 5.4.5		Remove this exercise
229	1. -7	“the basis B ”	Should be “the given basis”
233	Exercise 5.5.4	“ $\frac{1+t^2}{t^2}$ ”	Should be “ $\frac{1+t^2}{t^2}$ ”
234	Exercise 5.5.6	“ $1 + s^2\tau^2(s)$ ”	Should be “ $1 + t^2\tau^2(s)$ ”
234	Exercise 5.5.7	“ $\begin{pmatrix} R \cos t \cos \theta \\ R \cos t \sin \theta \\ R \sin t \end{pmatrix}$ ”	Should be “ $\begin{pmatrix} \cos t \cos \theta \\ \cos t \sin \theta \\ \sin t \end{pmatrix}$ ”
235	1. -11	“ (a, b) ”	Should be “ $(-\epsilon, \epsilon)$ ”
235	1. -8	“ $x^{-1} \circ c(t)$ ”	Should be “ $x^{-1} \circ c^1(t)$ and $x^{-1} \circ c^2(t)$ respectively”
239	Lemma 5.6.6 (iii)	“ $(\nabla_v f)Z(p) + f(p)(\nabla_v Z)$ ”	Should be “ $(\tilde{\nabla}_v f)Z(p) + f(p)(\nabla_v Z)$ ”
242	Exercise 5.6.1	“ $p = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \in S^2$ ”	Should be “ $p = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \in S^1 \times \mathbb{R}$ ”

242	Exercise 5.6.1	“ $T_p S^1 \times \mathbb{R}$ ” in two places	Should be “ $T_p (S^1 \times \mathbb{R})$ ” in both places
242	l. -10	“ $f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right)$ ”	Should be “ $Z\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right)$ ”
245	l. 12	“ $\langle x_i(\bar{p}), \bar{\nabla}_{x_k(\bar{p})} x_j \rangle$ ”	Should be “ $\langle x_i(\bar{p}), \bar{\nabla}_{x_k(\bar{p})} x_j \rangle$ ”
246	Equation 5.7.4	“ $\begin{pmatrix} A_{ij}^1 \\ A_{ij}^2 \end{pmatrix}$ ”	Should be “ $\begin{pmatrix} \frac{\partial g_{j1}}{\partial u_i} + \frac{\partial g_{i1}}{\partial u_j} - \frac{\partial g_{ij}}{\partial u_1} \\ \frac{\partial g_{j2}}{\partial u_i} + \frac{\partial g_{i2}}{\partial u_j} - \frac{\partial g_{ij}}{\partial u_2} \end{pmatrix}$ ”
247	l. -1	“we turn tangent”	Should be “we turn to tangent”
249	l. -6	“ $v = (\sqrt{2}/2)x_1(\bar{p}) + (\sqrt{2}/2)x_2(\bar{p})$ ”	Should be “ $v = \sqrt{2}x_1(\bar{p}) + \sqrt{2}x_2(\bar{p})$ ”
249	l. -5	“ $v^1 = v^2 = \sqrt{2}/2$ ”	Should be “ $v^1 = v^2 = \sqrt{2}$ ”

The displayed equation is:

$$\begin{aligned}
\text{“}\nabla_v Z &= \sum_{k=1}^2 \sum_{j=1}^2 \left(\frac{\sqrt{2}}{2} \cdot 0 + \sum_{i=1}^2 \Gamma_{ij}^k(\bar{p}) \frac{\sqrt{2}}{2} Z^i(\bar{p}) \right) x_k(\bar{p}) \\
&= \Gamma_{22}^1(\bar{p}) \frac{\sqrt{2}}{2} Z^2(\bar{p}) x_1(\bar{p}) + \Gamma_{12}^2(\bar{p}) \frac{\sqrt{2}}{2} Z^1(\bar{p}) x_2(\bar{p}) \\
&\quad + \Gamma_{21}^2(\bar{p}) \frac{\sqrt{2}}{2} Z^2(\bar{p}) x_2(\bar{p}) \\
&= \frac{1}{2} \frac{\sqrt{2}}{2} \cdot 1 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} + (-1) \frac{\sqrt{2}}{2} \cdot 0 \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ 0 \end{pmatrix} \\
&\quad + (-1) \frac{\sqrt{2}}{2} \cdot 1 \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/4 \\ 1/2 \\ 1/4 \end{pmatrix}. \quad \diamond''
\end{aligned}$$

The displayed equation should be:

$$\begin{aligned}
\text{“}\nabla_v Z &= \sum_{k=1}^2 \sum_{j=1}^2 \left(\sqrt{2} \cdot 0 + \sum_{i=1}^2 \Gamma_{ij}^k(\bar{p}) \sqrt{2} Z^i(\bar{p}) \right) x_k(\bar{p}) \\
&= \Gamma_{22}^1(\bar{p}) \sqrt{2} Z^2(\bar{p}) x_1(\bar{p}) + \Gamma_{12}^2(\bar{p}) \sqrt{2} Z^1(\bar{p}) x_2(\bar{p}) \\
&\quad + \Gamma_{21}^2(\bar{p}) \sqrt{2} Z^2(\bar{p}) x_2(\bar{p}) \\
&= \frac{1}{2} \sqrt{2} \cdot 1 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} + (-1) \sqrt{2} \cdot 0 \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ 0 \end{pmatrix} \\
&\quad + (-1) \sqrt{2} \cdot 1 \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/2 \\ -1 \\ 1/2 \end{pmatrix}. \quad \diamond''
\end{aligned}$$

255	1. 7	“Exercise 5.5.4”	Should be “Exercise 5.5.5”
258	1. -14	“ $F(W)$ ”	Should be “ $f(W)$ ”
259	1. 7	“ p such that”	Should be “ \bar{p} such that”
259	1. 8	“ $W = f(T) \subset M$ ”	Should be “ $W = x(T) \subset M$ ”

260	1. 10	“as x ”	Should be “as $x V$ ”
260	1. -5	“ $f \circ x$ has rank 2”	Should be “ $D(f \circ x)$ has rank 2”
261	1. 7	“local isometry”	Should be “local isometry, where we think of \mathbb{R}^2 as the x - y plane in \mathbb{R}^3 ”
261	1. 14	“ $x_1 \times x_2$ ”	Should be “ $(f \circ x)_1 \times (f \circ x)_2$ ”
262	Exercise 5.9.3	“ \mathbb{R}^2 ”	Should be “ $\mathbb{R}^2 - \{O_2\}$ ”
263	Exercise 5.9.6		This exercise uses Exercise 5.9.8
263	Exercise 5.9.8		Add “and that $d(1_M)_p = I$, where 1_M is the identity map on M and I is the 2×2 identity matrix” at the end of the exercise
263	1. -1	“ 3×3 ”	Should be “ 3×3 matrix”
274	1. 2	“plane that contain”	Should be “plane that contains”
274	1. 11	“ $\Pi_{T_p M}: U \rightarrow T_p M$ ”	Should be “ $\Pi_{T_p M}: U \rightarrow T_p M$ ”
277	1. 1	“ $= -\frac{1}{R} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -\frac{1}{R}v$ ”	Should be “ $= \frac{1}{R} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{R}v$ ”
277	1. 13	“ $I I$ ”	Should be “ II ”
281	1. 5	“ $\kappa(s) + t^2\kappa(s)\tau^2(s)$ ”	Should be “ $\kappa(s) + t^2\kappa(s)\tau^2(s) - t\tau'(s)$ ”
283	1. -9	“oriented”	Should be “ordered”
284	1. 5-6	“smooth curve such that $c_\Omega((-\epsilon, \epsilon))$ is an open subset of $\Omega \cap M$ and $c_\Omega(0) = p$,”	Should be “smooth, unit speed curve such that $c_\Omega((-\epsilon, \epsilon))$ is an open subset of $\Omega \cap M$, that $c_\Omega(0) = p$ and that $c'_\Omega(0) = v_\Omega$,”
284	1. -4	“ $-\langle \hat{n} \circ c'_\Omega(0), c'_\Omega(0) \rangle$ ”	Should be “ $-\langle (\hat{n} \circ c_\Omega)'(0), c'_\Omega(0) \rangle$ ”
285	1. 3	“ $I I_p$ ”	Should be “ II_p ”
285	1. 4	“ $I I_p$ ”	Should be “ II_p ”

285	1. 5	“eigenvalue”	Should be “eigenvector”
285	1. 6	“ $I I_p$ ”	Should be “ \mathbb{I}_p ”
285	1. -3	“ $2(k_1 - k_2) \sin \theta \cos \theta$ ”	Should be “ $2(k_2 - k_1) \sin \theta \cos \theta$ ”
288	Figure 6.3.2 (iii)	“ $K(p) < 0$ ”	Should be “ $K(p) = 0$ ”
290	Exercise 6.2.6		Should be “Exercise 6.3.6”
290	Exercise 6.2.7*		Should be “Exercise 6.3.7*”
296	1. -10	“Theorem 6.6.2”	Should be “Theorem 6.5.2”
298	Equation 6.5.6	“ $\frac{\partial \Gamma_{21}^r}{\partial u_{12}}$ ”	Should be “ $\frac{\partial \Gamma_{21}^r}{\partial u_2}$ ”
302	1. 4	“ $\begin{pmatrix} t \\ \theta \end{pmatrix}$ ”	Should be “ $\begin{pmatrix} t \\ \theta \end{pmatrix}$ ”
302	1. 7	“The f is”	Should be “The map f is”
311	Example 7.2.1 (1)	“We want find”	Should be “We want to find”
312	1. 13	“ $\begin{pmatrix} 3t^2 \\ 3t^2 \\ 0 \end{pmatrix}$ ”	Should be “ $\begin{pmatrix} 6t \\ 6t \\ 0 \end{pmatrix}$ ”
314	1. 14	“ $c j$ ”	Should be “ $c J$ ”
318	1. 4	“Section 7.3”	Should be “Section 7.1”
320	1. 1	“ $c: (-\epsilon, \epsilon) \rightarrow U$ ”	Should be “ $c: (-\epsilon, \epsilon) \rightarrow x(U)$ ”
324	1. 4	“ \int_a^b ”	Should be “ \int_x^y ”
324	1. 16	“ $\frac{Dc'(s)}{ds}(s_0) \neq 0$ ”	Should be “ $\ \frac{Dc'(s)}{ds}(s_0)\ \neq 0$ ”
324	1. 16	“assume that $\frac{Dc'(s)}{ds}(s_0) > 0$ ”	Should be “hence $\ \frac{Dc'(s)}{ds}(s_0)\ > 0$ ”
324	1. 17		Drop “the other case is similar”
324	1. 17	“ $\frac{Dc'(s)}{ds}(s_0) > 0$ ”	Should be “ $\ \frac{Dc'(s)}{ds}(s_0)\ > 0$ ”

324	l. -13	$“(s_0 - \eta, s_0 + \eta)c^{-1}(x(U))”$	Should be $“(s_0 - \eta, s_0 + \eta) \subset c^{-1}(x(U))”$
325	Equation 7.3.4	$“\left \frac{Dc'(s)}{ds}\right ^2”$	Should be $“\left\ \frac{Dc'(s)}{ds}\right\ ^2”$
325	Equation 7.3.5	$“\left \frac{Dc'(s)}{ds}\right ^2”$	Should be $“\left\ \frac{Dc'(s)}{ds}\right\ ^2”$
326	l. 11	$“\left \frac{Dc'(s)}{ds}\right ^2”$	Should be $“\left\ \frac{Dc'(s)}{ds}\right\ ^2”$
332	l. 1	$“exp_p(O_{\delta_p}(O_3, T_qM))”$	Should be $“exp_q(O_{\delta_p}(O_3, T_qM))”$
332	l. 3	$“exp_p(O_{\delta_p}(O_3, T_qM))”$	Should be $“exp_q(O_{\delta_p}(O_3, T_qM))”$
335	l. 1	$“J_p(B)”$	Should be $“J_{\bar{p}}(B)”$
344	l. 14	“geodesic”	Should be “non-constant geodesic”
347	Figure 8.4.2	higher $“D_x^{-1}(\bar{x}\bar{z})”$	Should be $“D_x^{-1}(\bar{x}\bar{z})”$
347	Figure 8.4.2	lower $“D_x^{-1}(\bar{x}\bar{z})”$	Should be $“D_x^{-1}(\bar{x}\bar{y})”$
352	l. 14	$“\sum_{\sigma v}”$	Should be $“\sum_{\sigma \ni v}”$
358	l. -14	“find surfaces”	Should be “find such surfaces”
366	l. 3		Remove the line break
373	l. 7	“vertical line”	Should be “horizontal line”
373	l. 8	“vertical line”	Should be “horizontal line”
385	l. -2	$“\{1, \dots, k\}”$	Should be $“\{0, \dots, k\}”$
400	l. -11	$“D^2”$	Should be $“D^2”$
400	l. -10	$“D^2”$	Should be $“D^2”$
400	l. -7	$“D^2”$	Should be $“D^2”$
405	l. 2	“4.2.5”	Should be “4.2.4”

405	1. 7	“ $h(d, e)(a, b)$ ”	Should be “ $h: (d, e) \rightarrow (a, b)$ ”
405	1. 8	“ $h(t) \neq 0$ ”	Should be “ $h'(t) \neq 0$ ”
406	1. -4	“a curve”	Should be “a smooth curve”
406	1. -1	“ $\ x - (-\frac{1}{k}w)\ $ ”	Should be “ $\ x - (-\frac{1}{k}w)\ $ ”
408	1. 2	“ $A = U$ ”	Should be “ $A = x(U)$ ”
408	1. 12	“ $(dy_p)^{-1}(v)$ ”	Should be “ $(dy_{\bar{p}})^{-1}(v)$ ”
408	1. -6	“ 7. ”	Should be “ 6.2.7. ”
410	8.3.1	“Exercise 6.5.2”	Should be “Exercise 6.5.1”
414	1. 11	“asimplicial”	Should be “a simplicial”
418		“Kline bottle”	Should be “Klein bottle”