ERRATA FOR
“A First Course in Geometric Topology and Differential Geometry”
Ethan D. Bloch
Birkhäuser, 1997

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Below is an updated list of errata. The fault for all the errors in the book is my own, and I offer my
sincere apologies for any inconvenience caused by the errors in the book.

This list was compiled with the generous assistance of: Bill Bloch, Jonathan Dent, John Engbers, Kyle
Glashower, Tsz Ho Ip, Gerard Venema, Yumi Watanabe, Peter Maria Wirtz, and Bard students Matthew
Brophy, Vasilica Crecea, Tim Goldberg, Jiaming Mao, Kenneth Ober.

If you find any additional errors in the book, or any errors in this list of errors, I would very much
appreciate it if you would let me know by email or regular mail at the following address:

Ethan D. Bloch
Bard College
Annandale-on-Hudson, NY 12504
bloch@bard.edu

<table>
<thead>
<tr>
<th>Page</th>
<th>Line/Item</th>
<th>Text</th>
<th>Comment/Should be</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Exercise 1.2.1 (1)</td>
<td>“a single point in (\mathbb{R}^n)”</td>
<td>Should be “a single point in (\mathbb{R}^2)”</td>
</tr>
<tr>
<td>12</td>
<td>Exercise 1.2.6</td>
<td>“containing (p)”</td>
<td>Should be “such that (p \in V \subset U)”</td>
</tr>
<tr>
<td>13</td>
<td>Exercise 1.2.17</td>
<td>“be a closed set”</td>
<td>Should be “be a non-empty closed set”</td>
</tr>
<tr>
<td>18</td>
<td>l. 13</td>
<td>(f_i = f \circ \pi_i)”</td>
<td>Should be (f_i = \pi_i \circ f)”</td>
</tr>
<tr>
<td>24</td>
<td>l. 6</td>
<td>“(f) is surjective”</td>
<td>Should be “(q) is surjective”</td>
</tr>
<tr>
<td>27</td>
<td>l. 7</td>
<td>(i_1, \ldots, i_r \in {1, \ldots, p})</td>
<td>Should be (i_1, \ldots, i_r \in {0, 1, \ldots, p})</td>
</tr>
<tr>
<td>27</td>
<td>Exercise 1.4.9</td>
<td></td>
<td>We need to assume that (X_i) and (Y_i) are both open or both closed in (X_i \cup Y_i) for (i = 1, 2).</td>
</tr>
<tr>
<td>28</td>
<td>l. -3</td>
<td>(B \cap [b_1, b_2])”</td>
<td>Should be (B_1 \cap [b_1, b_2])”</td>
</tr>
<tr>
<td>39</td>
<td>l. -6</td>
<td>({a, y})&quot;</td>
<td>Should be ({a, y})”</td>
</tr>
</tbody>
</table>
53 l. 17 Remove “containing B”

63 l. 3 “q: D → Q” Should be “q: D → X”

75 l. 13 “A2.1” Should be “A2.2”

81 Exercise 2.6.2 “K^2 # P^2, K^2 # K^2 and T^2 # T^2” Should be “K^2 # P^2 and K^2 # K^2”

89 l. -1 “k times” in two places Should be “d times” in both places

91 l. 6 “k times” in two places Should be “d times” in both places

93 l. 2 “h(\vec{x}\vec{y}) = \vec{x}\vec{y}” Should be “h(\vec{x}\vec{y}) = \overrightarrow{h(x)h(y)}”

98 Figure A2.2.7(i) “D_4” Should be “D_n”

98 Figure A2.2.7(i) “\alpha_4” Should be “\alpha_n”

99 l. -7 “1.5.11” Should be “1.6.11”

99 l. -5 “1.5.6” Should be “1.6.7”

107 Exercise A.2.2.1 The number of the Exercise should be “A2.2.1”

107 Exercise A.2.2.2 The number of the Exercise should be “A2.2.2”

111 l. -5 “p to q” Should be “v to w”

115 l. -15 “\{a_0, \ldots, a_i\}” Should be “\{a_0, \ldots, a_k\}”

115 l. -14 “A face of \sigma that is a k-simplex is called a k-face” Should be “A face of \sigma that is an i-simplex is called an i-face”

116 l. 6 “S^{k-1} = \{x \in \mathbb{R}^k \mid \|x\| < 1\}” Should be “S^{k-1} = \{x \in \mathbb{R}^k \mid \|x\| = 1\}”

118 l. 9 “affine linear take” Should be “affine linear map takes”

124 l. 7 “Exercise 3.3.6” Should be “Lemma 3.3.4”

124 l. 14 “Exercise 3.3.6” Should be “Lemma 3.3.4”
Exercise 3.3.8  

“$\eta \cap \text{Int } \sigma \neq \emptyset$” in two places

Should be “$\text{Int } \eta \cap \text{Int } \sigma \neq \emptyset$” in both places

131  l. -14

“that is simplicial complex”

Should be “that is a simplicial complex”

134 l. 10

There should not be a line break in the middle of “$|\text{star}(w, K)| – \{w\}$”

Exercise 3.5.5

“If $P$ is a 2-dimensional cell complex”

Should be “If $P$ is a 2-dimensional cell complex such that $|P|$ is a topological surface,”

141

Exercise 3.5.5

“If $P$ is a 2-dimensional cell complex”

Should be “If $P$ is a 2-dimensional cell complex such that $|P|$ is a topological surface,”

154 l. 4

$\sum_{\eta \ni v}$

Should be $\sum_{\eta \ni v}$

155 l. 6

$\sum_{\eta \ni v}$

Should be $\sum_{\eta \ni v}$

155 l. 7

$\sum_{\eta \ni v}$

Should be $\sum_{\eta \ni v}$

155 Corollary 3.7.3

“Let $K$ be a 2-complex”

Should be “Let $K$ be a simplicial surface”

162 l. -1

“$f$”

Should be “$r$”

168 l. 11

“is subsequent”

Should be “in subsequent”

174 l. -7

Remove one “definition”

174 l. -5

“$h'(s) = 1/q'(h(s))$”

Should be “$h'(s) = 1/q'(h(s))$”

178 Exercise 4.3.1 (ii)

“$t \ln t – t$”

Should be “$t \ln t$”

179 Exercise 4.3.3

“smooth curve”

Should be “regular curve”

179 l. 14

Remove the second “would”

179 l. -12

“chose”

Should be “choose”

180 Exercise 4.3.11

Hint: Use Exercise 4.2.1; let $G$ be as in Exercise 4.2.1, and then note that $(G \circ c)^{-1}$ is smooth, and that $c^{-1} \circ \tilde{c} = (G \circ c)^{-1} \circ G \circ \tilde{c}$, and this latter expression is smooth
\[ c''(t) = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \]  
Should be \[ c''(t) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \]

\[ c'(t) \times c''(t) = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \]  
Should be \[ c'(t) \times c''(t) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \]

\[ g''(t) \]  
Should be \[ g''(t), \]

\[ \text{the claim} \]  
Should be \[ \text{Exercise 4.6.2} \]

\[ \text{the claim} \]  
Should be \[ \text{Exercise 4.6.2} \]

\[ \text{loose} \]  
Should be \[ \text{lose} \]

\[ \text{counterclockwise} \]  
Should be \[ \text{clockwise} \]

\[ \text{is clockwise} \]  
Should be \[ \text{is counterclockwise} \]

Exercise 4.7.1  
This exercise needs the formula computed in Exercise 4.7.3

Exercise 4.7.1 (ii)  
\[ \mathbb{R}^3 \]  
Should be \[ \mathbb{R}^2 \]

Exercise 4.7.3  
Hint for Exercise 4.7.3: Start as on p. 191, and obtain a formula for \( c''(t) \) in terms of \( \tilde{T} \) and \( \tilde{N} \). Then take the inner product with \( (c')_\perp \)

\[ \text{rank 2, and hence } D(x \circ \phi) \text{ has rank 2.} \]  
Should be \[ \text{rank 2. Because } D\phi \text{ is a } 2 \times 2 \text{ matrix, it follows that } D(x \circ \phi) \text{ has rank 2.} \]

Figure 5.2.2  
\[ A_x \]  
Should be \[ A_{xy} \]

Figure 5.2.2  
\[ A_y \]  
Should be \[ A_{yx} \]

1. 2  
\[ A_{x1} \]  
Should be \[ A_{x1y1} \]

1. 3  
\[ A_{y1} \]  
Should be \[ A_{y1x1} \]

1. 5  
\[ A_{x1} \]  
Should be \[ A_{x1y1} \]
Proposition 5.2.5

Remove “and let \( p \in M \) be a point” from the statement of the proposition.

l. 2

“in \( M \)”

Should be “in \( N \)”

l. -3

“smooth surfaces”

Should be “smooth surfaces in \( \mathbb{R}^3 \), where we think of \( U \) as sitting in the \( x\)-\( y \) plane in \( \mathbb{R}^3 \)”

l. -3

“missing the north and south poles”

Should be “missing half a great circle from the north to the south pole”

l. -1

\( F^{-1}(a) \)”

Should be “\( F^{-1}(a) \)”

l. -10

“a curve”

Should be “a smooth curve”

Equation 5.4.1

\( x_2(t, \theta) \)”

Should be “\( x_2\left(\begin{pmatrix} t \\ \theta \end{pmatrix}\right) \)”

l. 4

Remove “such that \( p \in x(U) \)”

l. 8

“\( n \) is a”

Should be “\( n \) as a”

Exercise 5.4.5

Remove this exercise

l. -7

“the basis \( B \)”

Should be “the given basis”

Exercise 5.5.4

\( \frac{1 + t^2}{t^2} \)”

Should be “\( \frac{1 + t^2}{t^2} \)”

Exercise 5.5.6

“\( 1 + s^2 \tau^2(s) \)”

Should be “\( 1 + t^2 \tau^2(s) \)”

Exercise 5.5.7

\[
\begin{pmatrix}
R \cos t \cos \theta \\
R \cos t \sin \theta \\
R \sin t
\end{pmatrix}
\]

Should be \( \begin{pmatrix}
\cos t \cos \theta \\
\cos t \sin \theta \\
\sin t
\end{pmatrix} \)

l. -11

“(\( a, b \))”

Should be “(\( -\epsilon, \epsilon \))”

l. -8

“\( x^{-1} \circ c(t) \)”

Should be “\( x^{-1} \circ c^1(t) \) and \( x^{-1} \circ c^2(t) \) respectively”

Lemma 5.6.6 (iii)

“\( (\nabla_v f)Z(p) + f(p)(\nabla_v Z) \)”

Should be “\( (\nabla_v f)Z(p) + f(p)(\nabla_v Z) \)”

Exercise 5.6.1

\( p = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) \in S^{2n} \)

Should be “\( p = \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) \in S^1 \times \mathbb{R} \)”
Exercise 5.6.1  

"$T_pS^1 \times \mathbb{R}$" in two places  
Should be "$T_p(S^1 \times \mathbb{R})$" in both places

1. -10  

"$f\left(\frac{x}{y}\right)$"  
Should be "$Z\left(\frac{x}{y}\right)$"

1. 12  

"$(x_1(\bar{p}), \bar{\nabla}_x(x_k(\bar{p})x_j))$"  
Should be "$(x_1(\bar{p}), \bar{\nabla}_x(x_k(\bar{p})x_j))$"

Equation 5.7.4  

"$\left( \begin{array}{c} A^1_{ij} \\ A^2_{ij} \end{array} \right)$"  
Should be "$\left( \begin{array}{c} \frac{\partial g_{ij}}{\partial u_i} + \frac{\partial g_{ij}}{\partial u_j} - \frac{\partial g_{ij}}{\partial u_1} \\ \frac{\partial g_{ij}}{\partial u_1} + \frac{\partial g_{ij}}{\partial u_2} - \frac{\partial g_{ij}}{\partial u_2} \end{array} \right)$"

1. -1  

"we turn tangent"  
Should be "we turn to tangent"

1. -6  

"$v = \frac{\sqrt{2}}{2}x_1(\bar{p}) + \frac{\sqrt{2}}{2}x_2(\bar{p})$"  
Should be "$v = \sqrt{2}x_1(\bar{p}) + \sqrt{2}x_2(\bar{p})$"

1. -5  

"$v^1 = v^2 = \sqrt{2}/2$"  
Should be "$v^1 = v^2 = \sqrt{2}$"
The displayed equation is:

\[ \begin{align*}
\nabla_v Z &= \sum_{k=1}^{2} \sum_{j=1}^{2} \left( \frac{\sqrt{2}}{2} \cdot 0 + \sum_{i=1}^{2} \Gamma_{ij}^{k}(\bar{p}) \frac{\sqrt{2}}{2} Z^i(\bar{p}) \right) x_k(\bar{p}) \\
&= \Gamma_{22}(\bar{p}) \frac{\sqrt{2}}{2} Z^2(\bar{p}) x_1(\bar{p}) + \Gamma_{12}(\bar{p}) \frac{\sqrt{2}}{2} Z^1(\bar{p}) x_2(\bar{p}) \\
&\quad + \Gamma_{21}(\bar{p}) \frac{\sqrt{2}}{2} Z^2(\bar{p}) x_2(\bar{p}) \\
&= \frac{1}{2} \frac{\sqrt{2}}{2} \cdot 1 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \sqrt{2}/2 \end{pmatrix} + (-1) \frac{\sqrt{2}}{2} \cdot 0 \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \end{pmatrix} \\
&\quad + (-1) \frac{\sqrt{2}}{2} \cdot 1 \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \end{pmatrix} = \begin{pmatrix} -1/4 \\ 1/2 \end{pmatrix} \end{align*} \]

The displayed equation should be:

\[ \begin{align*}
\nabla_v Z &= \sum_{k=1}^{2} \sum_{j=1}^{2} \left( \frac{\sqrt{2}}{2} \cdot 0 + \sum_{i=1}^{2} \Gamma_{ij}^{k}(\bar{p}) \frac{\sqrt{2}}{2} Z^i(\bar{p}) \right) x_k(\bar{p}) \\
&= \Gamma_{22}(\bar{p}) \frac{\sqrt{2}}{2} Z^2(\bar{p}) x_1(\bar{p}) + \Gamma_{12}(\bar{p}) \frac{\sqrt{2}}{2} Z^1(\bar{p}) x_2(\bar{p}) \\
&\quad + \Gamma_{21}(\bar{p}) \frac{\sqrt{2}}{2} Z^2(\bar{p}) x_2(\bar{p}) \\
&= \frac{1}{2} \frac{\sqrt{2}}{2} \cdot 1 \cdot \begin{pmatrix} -\sqrt{2}/2 \\ 0 \sqrt{2}/2 \end{pmatrix} + (-1) \frac{\sqrt{2}}{2} \cdot 0 \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \end{pmatrix} \\
&\quad + (-1) \frac{\sqrt{2}}{2} \cdot 1 \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \end{pmatrix} = \begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix} \end{align*} \]
260 l. 10 “as $x$” Should be “as $x | V$”

260 l. -5 “$f \circ x$ has rank 2” Should be “$D(f \circ x)$ has rank 2”

261 l. 7 “local isometry” Should be “local isometry, where we think of $\mathbb{R}^2$ as the $x$-$y$ plane in $\mathbb{R}^3$”

261 l. 14 “$x_1 \times x_2$” Should be “$(f \circ x)_1 \times (f \circ x)_2$”

262 Exercise 5.9.3 “$\mathbb{R}^2$” Should be “$\mathbb{R}^2 - \{O_2\}$”

263 Exercise 5.9.6 This exercise uses Exercise 5.9.8

263 Exercise 5.9.8 Add “and that $d(1_M)_p = I$, where $1_M$ is the identity map on $M$ and $I$ is the $2 \times 2$ identity matrix” at the end of the exercise

263 l. -1 “$3 \times 3$” Should be “$3 \times 3$ matrix”

274 l. 2 “plane that contain” Should be “plane that contains”

274 l. 11 “$\Pi_{T_pM} : U \to T_pM$” Should be “$\Pi_{T_pM} : U \to T_pM$”

277 l. 1 “$= -\frac{1}{R} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -\frac{1}{R} v$” Should be “$= \frac{1}{R} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{R} v$”

277 l. 13 “$II$” Should be “$II$”

278 l. 5 “$\kappa(s) + t^2 \kappa(s) \tau^2(s)$” Should be “$\kappa(s) + t^2 \kappa(s) \tau^2(s) - t \tau'(s)$”

283 l. -9 “oriented” Should be “ordered”

284 l. 5-6 “smooth curve such that $c_{\Omega}((0, \epsilon))$ is an open subset of $\Omega \cap M$ and $c_{\Omega}(0) = p$,” Should be “smooth, unit speed curve such that $c_{\Omega}((0, \epsilon))$ is an open subset of $\Omega \cap M$, that $c_{\Omega}(0) = p$ and that $c_{\Omega}'(0) = v_{\Omega}$,”

284 l. -4 “$-\langle \hat{n} \circ c_{\Omega}'(0), c_{\Omega}'(0) \rangle$” Should be “$-\langle (\hat{n} \circ c_{\Omega})'(0), c_{\Omega}'(0) \rangle$”

285 l. 3 “$\Pi_p$” Should be “$\Pi_p$”

285 l. 4 “$II_p$” Should be “$II_p$”
“eigenvalue” Should be “eigenvector”

“I \Pi_p” Should be “II_p”

“2 (k_1 - k_2) \sin \theta \cos \theta” Should be “2 (k_2 - k_1) \sin \theta \cos \theta”

“K(p) < 0” Should be “K(p) = 0”

Exercise 6.2.6 Should be “Exercise 6.3.6”

Exercise 6.2.7* Should be “Exercise 6.3.7*”

“Theorem 6.6.2” Should be “Theorem 6.5.2”

Equation 6.5.6 \[ \frac{\partial \Gamma_21}{\partial u_2} \] Should be \[ \frac{\partial \Gamma_21}{\partial u_2} \]

“The f is” Should be “The map f is”

Example 7.2.1 (1) “We want find” Should be “We want to find”

\[ \begin{pmatrix} 3t^2 \\ 3t^2 \\ 0 \end{pmatrix} \] Should be \[ \begin{pmatrix} 6t \\ 6t \\ 0 \end{pmatrix} \]

“c|j” Should be “c|J”

“Section 7.3” Should be “Section 7.1”

“c: (-\epsilon, \epsilon) \to U” Should be “c: (-\epsilon, \epsilon) \to x(U)”

\[ \int_a^b \] Should be \[ \int_x^y \]

\[ \frac{dDc'(s)}{ds}(s_0) \neq 0 \] Should be \[ \|\frac{dDc'(s)}{ds}(s_0)\| \neq 0 \]

“assume that \ \frac{Dc'(s)}{ds}(s_0) > 0” Should be “hence \ \|\frac{Dc'(s)}{ds}(s_0)\| > 0”

Drop “the other case is similar”

\[ \frac{dDc'(s)}{ds}(s_0) > 0 \] Should be \[ \|\frac{dDc'(s)}{ds}(s_0)\| > 0 \]
324 l. -13 \((s_0 - \eta, s_0 + \eta) c^{-1}(x(U))\)  

Should be \((s_0 - \eta, s_0 + \eta) \subset c^{-1}(x(U))\)

325 Equation 7.3.4 \(\frac{D c'(s)}{ds}^2\)  

Should be \(\|\frac{D c'(s)}{ds}\|^2\)

325 Equation 7.3.5 \(\frac{D c'(s)}{ds}^2\)  

Should be \(\|\frac{D c'(s)}{ds}\|^2\)

326 l. 11 \(\frac{D c'(s)}{ds}^2\)  

Should be \(\|\frac{D c'(s)}{ds}\|^2\)

332 l. 1 \(\exp_p(O_{\delta_p}(O_3, T_q M))\)  

Should be \(\exp_q(O_{\delta_p}(O_3, T_q M))\)

332 l. 3 \(\exp_p(O_{\delta_p}(O_3, T_q M))\)  

Should be \(\exp_q(O_{\delta_p}(O_3, T_q M))\)

335 l. 1 \(J_p(B)\)  

Should be \(J_p(B)\)

344 l. 14 “geodesic”  

Should be “non-constant geodesic”

347 Figure 8.4.2 higher \(D^{-1}_x(xz)\)  

Should be \(D^{-1}_x(xz)\)

347 Figure 8.4.2 lower \(D^{-1}_x(xz)\)  

Should be \(D^{-1}_x(xy)\)

352 l. 14 \(\sum_{\sigma v}\)  

Should be \(\sum_{\sigma \varepsilon v}\)

358 l. -14 “find surfaces”  

Should be “find such surfaces”

366 l. 3 Remove the line break

373 l. 7 “vertical line”  

Should be “horizontal line”

373 l. 8 “vertical line”  

Should be “horizontal line”

385 l. -2 \(\{1, \ldots, k\}\)  

Should be \(\{0, \ldots, k\}\)

400 l. -11 “D2”  

Should be “D^2”

400 l. -10 “D2”  

Should be “D^2”

400 l. -7 “D2”  

Should be “D^2”

405 l. 2 “4.2.5”  

Should be “4.2.4”
<table>
<thead>
<tr>
<th>Page</th>
<th>Line</th>
<th>Original Text</th>
<th>Corrected Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>405</td>
<td>7</td>
<td>“h(d, e)(a, b)”</td>
<td>Should be “h: (d, e) → (a, b)”</td>
</tr>
<tr>
<td>405</td>
<td>8</td>
<td>“h(t) ≠ 0”</td>
<td>Should be “h'(t) ≠ 0”</td>
</tr>
<tr>
<td>406</td>
<td>1</td>
<td>“a curve”</td>
<td>Should be “a smooth curve”</td>
</tr>
<tr>
<td>406</td>
<td>-1</td>
<td>“|x - (−\frac{1}{\ell} w)|”</td>
<td>Should be “|x - (−\frac{1}{\ell} w)|”</td>
</tr>
<tr>
<td>408</td>
<td>2</td>
<td>“A = U”</td>
<td>Should be “A = x(U)”</td>
</tr>
<tr>
<td>408</td>
<td>12</td>
<td>“(dy)−1(v)”</td>
<td>Should be “(dy)−1(v)”</td>
</tr>
<tr>
<td>408</td>
<td>-6</td>
<td>“7.”</td>
<td>Should be “6.27.”</td>
</tr>
<tr>
<td>410</td>
<td>8.3.1</td>
<td>“Exercise 6.5.2”</td>
<td>Should be “Exercise 6.5.1”</td>
</tr>
<tr>
<td>414</td>
<td>11</td>
<td>“asimplicial”</td>
<td>Should be “a simplicial”</td>
</tr>
<tr>
<td>418</td>
<td></td>
<td>“Kline bottle”</td>
<td>Should be “Klein bottle”</td>
</tr>
</tbody>
</table>