3d gravity and quantum groups - Day 1

Bard Summer School on Quantum Gravity

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I. OUTLINE

- Lecture 1: Introduction.
- Lecture 2: Poisson Lie groups and Lie bialgebras.
- Lecture 3: The Loop Gravity phase space as an Heisenberg double and 3D Loop Gravity with a non-zero cosmological constant.
- Lecture 4: Quantum groups.
- Lecture 5: 3D Loop Quantum Gravity with a non-zero cosmological constant.

II. IN PREPARATION OF LECTURE 2

A. A quick overview of Poisson Lie groups - Few definitions to read before lecture 2!

- A Poisson structure on M is a $\mathbb{R}$ bilinear map $\{\ ,\ \}$: $C^\infty(M) \times C^\infty(M) \to C^\infty(M)$, called the *Poisson bracket*, which satisfies the following conditions

  1. $\{f_1, f_2\} = -\{f_2, f_1\}$
  2. $\{f_1, \{f_2, f_3\}\} + \{f_2, \{f_3, f_1\}\} + \{f_3, \{f_1, f_2\}\} = 0$
  3. $\{f_1 f_2, f_3\} = f_1\{f_2, f_3\} + \{f_1, f_3\}f_2$

  $(M, \{\ ,\ \})$ is a *Poisson manifold.*

- A smooth map $F : N \to M$ between Poisson manifolds is called a *Poisson map* if it preserves the Poisson brackets of M and N, *i.e.*

$$\{f_1, f_2\}_M \circ F = \{f_1 \circ F, f_2 \circ F\}_N.$$

- If M and N are Poisson manifolds, their product $M \times N$ is a Poisson manifold. For $f_1, f_2 \in C^\infty(M \times N)$, $x \in M$, $y \in N$, the *product Poisson structure* is given by

$$\{f_1, f_2\}_{M\times N}(x, y) = \{f_1(\ , y), f_2(\ , y)\}_M(x) + \{f_1(x,\ ), f_2(x,\ )\}_n(y).$$

- A *Poisson-Lie group* is a Lie group G which has a Poisson structure $\{\ ,\ \}$ compatible with the product of G *i.e.* the multiplication map $\mu : G \times G \to G$ ($\mu(g_1 g_2) = g_1 g_2$) is a Poisson map ($G \times G$ is given the product Poisson structure).

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B. $\mathfrak{so}(3)$ generators in the spin 1 representation

The $\mathfrak{so}(3)$ generators in the spin 1 representation can be written as 3×3 anti-symmetric matrices,

$$j_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad j_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad j_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

They satisfy the following commutation relation $[j_i, j_j] = \epsilon_{ij}{}^{k} j_k$.