

MATH 142: Practice Midterm 1

Show all appropriate work. Variables may represent any real number.

1. (a) Use integration by parts with $dv = \cos(x) dx$ to prove the reduction formula
$$\int \cos^n(x) dx = \frac{1}{n} \cos^{n-1}(x) \sin(x) + \frac{n-1}{n} \int \cos^{n-2}(x) dx.$$

(b) Use your answer to part (a) to find $\int_0^{\frac{\pi}{2}} \cos^5(x) dx$.
2. Let $f(x) = 2 \cos \left[(2x - x^2) \frac{\pi}{2} \right]$ and $g(x) = 2x \cos \left[(2x - x^2) \frac{\pi}{2} \right]$. Given that $f(x) \geq g(x)$ on $[0, 1]$, find the area bound by $f(x)$ and $g(x)$ on the interval from 0 to 1.
3. Evaluate the following integrals:
 - (a) $\int e^{-y} \cos y dy$.
 - (b) $\int \sqrt{x} e^{\sqrt{x}} dx$.
 - (c) $\int \frac{x^4 + x^3 - x^2 - x + 1}{x^3 - x} dx$.