
Show all appropriate work.

1. Solve the following differential equations. Explicitly solve for $y(x)$ or $x(t)$ when possible.
 - (a) $x \frac{dy}{dx} = (1 + y)^2$.
 - (b) $\tan x \, dy + 2y \, dx = 0$.
 - (c) $(x^2 - 1)y' + 2xy^2 = 0$, $y(\sqrt{2}) = 1$.
 - (d) $x \frac{dx}{dt} + t = 1$.
2. Find the solution of $3y^2y' + 16x = 2xy^3$ such that $y(x)$ is bounded as $x \rightarrow \infty$.
3. Suppose the population, $N(t)$, of a given species is not always zero and varies at a rate proportional to its current value. If we call the constant of proportionality λ and assume the initial population is $N(0) = N_0 > 0$, then find $N(t)$. Discuss the behavior of the solution as $t \rightarrow \infty$ for different values of λ .