

MATH 321: Practice Midterm 1

Show all appropriate work. Variables may represent any real number.

1. Consider a function of time, $y = f(t)$, whose rate of change is proportional to the square of the time multiplied by the function value at that time.
 - (a) Find the general form of the function f .
 - (b) If the constant of proportionality is 3 and $f(2) = 9$, find $f(t)$.
2. Construct a linear first-order differential equation of the form $xy' + f(x)y = g(x)$, such that the homogenous solution is $y_h = \frac{C}{x^3}$ and it has the particular solution $y_p = x^3$.
3. Consider the differential equation $(x^2 + y^2) dx + (x^2 - xy) dy = 0$.
 - (a) Show that the differential equation is not exact.
 - (b) Make the substitution $y = vx$ to obtain a separable differential equation.
 - (c) Solve the differential equation.
4. Determine if the following differential equations are exact. If they are solve them. If not, find an integrating factor to make the equation exact and then solve.
 - (a) $(2y \sin(x) \cos(x) - y + 2y^2 e^{xy^2}) dx = (x - \sin^2(x) - 4xye^{xy^2}) dy$.
 - (b) $y(x + y + 1) dx + (x + 2y) dy = 0$.