MATH 141B Calculus 1 Spring 2019

Study Sheet for Final Exam

The Exam is in class, Monday, 20 May 2019

GUIDELINES THAT WILL BE ON THE EXAM

EXAM GUIDELINES

In order for this exam to be an honest and accurate reflection of your understanding of the material, you are asked to adhere to the following guidelines:

- The exam is closed book.
- The study sheet is not allowed during the exam.
- Books, notes and online resources are not allowed during the exam.
- Electronic devices (calculators, cell phones, tablets, laptops, etc.) are not allowed during the exam.
- For the duration of the exam, you may not discuss the exam, or related material, with anyone other than the course instructor.
- Giving help to others taking this exam is as much a violation of these guidelines as receiving help.
- Late exams will be allowed only if you discuss it with the course instructor before hand, or if an emergency occurs.
- **Violation of these guidelines will result, at minimum, in a score of zero on this exam.**
- **There will be no opportunity to retake this exam.**

Further comments:

- Write your solutions carefully and clearly.
- Show all your work. You will receive partial credit for work you show, but you will not receive credit for what you do not write down. In particular, correct answers with no work will not receive credit.

TOPICS

1. Critical points, second critical points
2. Increasing, decreasing, local maxima, local minima
3. Concave up, concave down, inflection points
4. Graphing functions
5. Global maxima, global minima
6. Optimization word problems
7. Linearization
8. Antiderivatives
9. Velocity and acceleration word problems
10. Left hand sums, right hand sums, midpoint sums, Riemann sums
11. The definite integral of a function
12. The definite integral and signed area
13. Basic rules for definite integrals
14. Area functions
15. The Fundamental Theorem of Calculus—Version I
16. The Fundamental Theorem of Calculus—Version II
17. The indefinite integral of a function
18. Integration by substitution

TIPS FOR STUDYING FOR THE EXAM

× **Bad** Forgetting about the homework and the previous quizzes.

✓ **Good** Making sure you know how to do all the problems on the homework and previous quizzes; seeking help seeking help from the instructor and the tutors about the problems you do not know how to do.

× **Bad** Doing all the practice problems from some of the sections, and not having enough time to do practice problems from the rest of the sections.

✓ **Good** Doing a few practice problems of each type from every sections.

× **Bad** Studying only by reading the book.

✓ **Good** Doing a lot of practice problems, and reading the book as needed.

× **Bad** Studying only by yourself.

✓ **Good** Trying some practice problems by yourself (or with friends), and then seeking help from the instructor and the tutors about the problems you do not know how to do.

× **Bad** Doing practice problems while looking everything up in the book.

✓ **Good** Doing some of the practice problems the way you would do them on the quiz or exam, which is with closed book and no calculator.

× **Bad** Staying up late (or all night) the night before the exam.

✓ **Good** Studying hard up through the day before the exam, but getting a good night's sleep the night before the exam.

Ethan's Office Hours

- **Monday:** 5:00-6:00
- **Tuesday:** 2:00-3:30 & 5:00-6:00
- **Thursday:** 4:30-6:00
- **Or by appointment**

Class Tutor & Mathematics Study Room

- **Class Tutor**: Scout Etterson, Wednesday, 7:30-8:30, Mathematics Common Room (third floor of Albee)
- **Mathematics Study Room**: Sunday-Wednesday, 7:00-10:00, Hegeman 308

PRACTICE PROBLEMS FROM STEWART, CALCULUS CONCEPTS AND CONTEXTS, 4TH ED.

Section 4.2: 3, 5, 7, 9, 11, 13, 23, 25, 27, 29, 31, 33, 35, 37, 41, 43, 45, 47, 49, 51, 53, 55

Section 4.3: 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 41, 49

Section 4.6: 3, 5, 11, 13, 17, 19, 23, 25

Section 3.9: 5, 7, 15, 17

Section 4.8: 1, 3, 5, 7, 9, 11, 13, 15, 19, 21, 23, 25, 27, 29, 31, 33, 35, 41, 43, 49, 51, 53

Section 5.1: 3, 5, 11, 13, 15

Section 5.2: 1, 3, 5, 7, 9, 11, 31, 33, 35, 37, 39, 41, 43, 47, 49

Section 5.3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 39, 51, 53, 55, 57

Section 5.4: 3, 7, 9, 11, 13, 15, 17, 19, 21

Section 5.5: 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 41, 43, 45, 47, 49, 51, 53, 55, 57

SOME IMPORTANT CONCEPTS AND FORMULAS

1. Increasing and Decreasing

Let $f(x)$ be a function.

1. The function $f(x)$ is **increasing** if $x < y$ implies $f(x) \leq f(y)$ for all x, y.
2. The function $f(x)$ is **decreasing** if $x < y$ implies $f(x) \geq f(y)$ for all x, y.

2. Increasing and Decreasing: via Derivatives

Let $f(x)$ be a function.

1. If $f'(x) > 0$ on an interval, then $f(x)$ is increasing on that interval.
2. If $f'(x) < 0$ on an interval, then $f(x)$ is decreasing on that interval.

3. Concave Up and Concave Down

Let $f(x)$ be a function.

1. The function $f(x)$ is **concave up** if $f'(x)$ is increasing.
2. The function $f(x)$ is **concave down** if $f'(x)$ is decreasing.

4. Concave Up and Concave Down: via Second Derivatives

Let $f(x)$ be a function.

1. If $f''(x) > 0$ on an interval, then $f(x)$ is concave up on that interval.
2. If $f''(x) < 0$ on an interval, then $f(x)$ is concave down on that interval.

5. Local Maxima and Local Minima

Let $f(x)$ be a function defined on an open interval, and let c be in the interval.

1. The number c is a **local maximum** of $f(x)$ if $f(c) \geq f(x)$ for all x near c.
2. The number c is a **local minimum** of $f(x)$ if $f(c) \leq f(x)$ for all x near c.
3. The number c is a **local extremum** of $f(x)$ if it is either a local maximum or a local minimum.

6. Critical Points

Let $f(x)$ be a function, and let c be a real number.

1. The number c is a **critical point** of $f(x)$ if either one of the following two cases holds:

$$f'(c) = 0 \qquad \text{or} \qquad f'(c) \text{ does not exist.}$$

2. **Caution**: Do not forget the case where $f'(c)$ does not exist.

7. Finding Local Maxima and Local Minima

1. Let $f(x)$ be a function defined on an open interval. The only places where $f(x)$ could have a local maximum or a local minimum is at the critical points.
2. **Caution**: Not every critical point is a local maximum or local minimum.

8. Local Maxima and Local Minima: First Derivative Test

Let $f(x)$ be a function defined on an open interval, and let c be in the interval. Suppose that $f(x)$ is differentiable. Suppose that c is a critical point of $f(x)$.

1. If $f'(x) < 0$ when $x < c$, and $f'(x) > 0$ when $x > c$, then c is a local minimum.
2. If $f'(x) > 0$ when $x < c$, and $f'(x) < 0$ when $x > c$, then c is a local maximum.
3. If $f'(x) < 0$ when $x < c$ and when $x > c$, or if $f'(x) > 0$ when $x < c$ and when $x > c$, then c is neither a local minimum or a local maximum.

9. Local Maxima and Local Minima: Second Derivative Test

Let $f(x)$ be a function defined on an open interval, and let c be in the interval. Suppose that $f(x)$ is differentiable, and that $f''(c)$ exists. Suppose that c is a critical point of $f(x)$.

1. If $f''(c) > 0$, then c is a local minimum.
2. If $f''(c) < 0$, then c is a local maximum.
3. **Caution**: If $f''(c) = 0$, you CANNOT conclude whether c is a local maximum or a local minimum from the Second Derivative Test.

10. Inflection Points

Let $f(x)$ be a function defined on an open interval, and let c be in the interval. The number c is an **inflection point** of $f(x)$ if either $f(x)$ is concave up when $x < c$ and $f(x)$ is concave down when $x > c$, or $f(x)$ is concave down when $x < c$ and $f(x)$ is concave up when $x > c$.

11. Second Critical Points

Let $f(x)$ be a function, and let c be a real number.

1. The number c is a **second critical point** of $f(x)$ if either one of the following two cases holds:

$$f''(c) = 0 \qquad \text{or} \qquad f''(c) \text{ does not exist.}$$

2. **Caution**: Do not forget the case where $f''(c)$ does not exist.

12. Finding Inflection Points

1. Let $f(x)$ be a function defined on an open interval. The only places where $f(x)$ could have an inflection point is at the second critical points.

2. **Caution**: Not every second critical point is an inflection point.

13. Finding Inflection Points: Second Derivative Test

Let $f(x)$ be a function defined on an open interval, and let c be in the interval. Suppose that $f(x)$ is differentiable. Suppose that c is a second critical point of $f(x)$.

1. If $f''(x) < 0$ when $x < c$, and $f''(x) > 0$ when $x > c$, then c is an inflection point.

2. If $f''(x) > 0$ when $x < c$, and $f''(x) < 0$ when $x > c$, then c is an inflection point.

3. If $f''(x) < 0$ when $x < c$ and when $x > c$, or if $f''(x) > 0$ when $x < c$ and when $x > c$, then c is not an inflection point.

14. Steps for Graphing Functions

Let $f(x)$ be a function.

1. Find $f'(x)$.

2. Find $f''(x)$.

3. Find the critical points.

4. Make a chart showing where $f'(x)$ is positive or negative.

5. Find where $f(x)$ is increasing and where it is decreasing.

6. Find the local maxima and local minima of $f(x)$.

7. Find the second critical points.

8. Make a chart showing where $f''(x)$ is positive or negative.

9. Find where $f(x)$ is concave up and where it is concave down.

10. Find the inflection points of $f(x)$.

11. Make a chart showing the critical points and second critical points of $f(x)$ and the behavior between those points.

12. Find the value of $f(x)$ at the critical points and second critical points.

13. Plot $f(x)$ at the critical points and second critical points.

14. Sketch the graph of $f(x)$.

15. Global Maxima and Global Minima

Let $f(x)$ be a function defined on an open interval, and let c be in the interval.

1. The number c is a **global maximum** of $f(x)$ if $f(c) \geq f(x)$ for all x.
2. The number c is a **global minimum** of $f(x)$ if $f(c) \leq f(x)$ for all x.
3. The number c is a **global extremum** of $f(x)$ if it is either a global maximum or a global minimum.
4. Instead of the terms "global maximum," etc., some texts say "absolute maximum," etc.

16. Extreme Value Theorem

Let $f(x)$ be a function defined on a closed bounded interval $[a, b]$. Suppose that $f(x)$ is continuous. Then $f(x)$ has a global maximum and a global minimum in $[a, b]$.

17. Finding Global Maxima and Global Minima: Closed Bounded Interval

Let $f(x)$ be a continuous function defined on a closed bounded interval $[a, b]$.

1. The global maxima and global minima of $f(x)$ can occur only at the critical points of $f(x)$ or the endpoints of $[a, b]$.
2. To find the global maxima and global minima of $f(x)$, find the values of $f(x)$ at the critical points and at the endpoints, and find the largest and smallest of these values.

18. Finding Global Maxima and Global Minima: Single Critical Point

Let $f(x)$ be a continuous function defined on an interval. Suppose that $f(x)$ has exactly one critical point.

1. If the critical point is a local maximum, it is a global maximum.
2. If the critical point is a local minimum, it is a global minimum.

19. Steps for Solving Optimization Word Problems

1. If possible, make a sketch.
2. Figure out what the variables are.
3. Figure out the function you are maximizing or minimizing.
4. Figure out if there are any relations between the variables.
5. Rework the function you are maximizing or minimizing so that it has only one variable.
6. Find the possible values of the variable in the function you are maximizing or minimizing; determine whether or not you have a closed bounded interval.
7. Find the critical points.
8. Find the global maximum or minimum using either the closed bounded interval method or the single critical point method.
9. Verify that you have a global maximum or minimum.
10. Make sure you answer the question as asked.
11. Make sure your answer is intuitively plausible.

20. Linearization

Let $f(x)$ be a function. Suppose that $f(x)$ is differentiable. The **linearization** (also called **tangent line approximation**) of $f(x)$ at $x = a$, denoted $L(x)$, is the function

$$L(x) = f(a) + f'(a)(x - a).$$

21. Antiderivatives

Let $f(x)$ be a function defined on an interval.

1. An **antiderivative** of $f(x)$ is a function $F(x)$ such that $F'(x) = f(x)$.
2. Not every function has an antiderivative.
3. Every continuous function has an antiderivative. Some discontinuous functions have antiderivatives.
4. If a function has an antiderivative, it is not unique.
5. Let $F(x)$ be an antiderivative of $f(x)$. Then every antiderivative of $f(x)$ has the form $F(x) + C$, for some constant C.

22. Basic Antiderivatives

$f(x)$	antiderivative of $f(x)$
1	$x + C$
x^r	$\dfrac{x^{r+1}}{r+1} + C$ when $r \neq -1$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + C$
e^x	$e^x + C$
a^x	$\dfrac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\sec x \tan x$	$\sec x + C$
$\csc^2 x$	$-\cot x + C$
$\csc x \cot x$	$-\csc x + C$
$\dfrac{1}{\sqrt{1-x^2}}$	$\arcsin x + C$
$\dfrac{1}{1+x^2}$	$\arctan x + C$
$\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\operatorname{arcsec} x + C$

23. Velocity and Acceleration

Let $s(t)$ be the position of an object at time t.

1. The **velocity** of the object at time t is $v(t) = s'(t)$.
2. The **acceleration** of the object at time t is $a(t) = v'(t) = s''(t)$.

24. Motion with Constant Acceleration

Suppose that an object moves with constant acceleration g. Let $s_0 = s(0)$ be the initial position of the object, and let $v_0 = v(0)$ be the initial velocity of the object.

1. The acceleration of the object at time t is $a(t) = g$.
2. The velocity of the object at time t is $v(t) = gt + v_0$.
3. The position of the object at time t is $s(t) = \dfrac{1}{2}gt^2 + v_0 t + s_0$.
4. Acceleration due to gravity is a negative number.

25. Right Sums, Left Sums and Midpoint Sums

Let $f(x)$ be a function defined on a closed bounded interval $[a, b]$. Let $n \in \mathbb{N}$, and let $x_0, x_1, \ldots, x_n$ be real numbers such that $a = x_0 < x_1 < \cdots < x_n = b$. For each i, let $\Delta x_i = x_i - x_{i-1}$.

Right Sum

$$R_n = f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + f(x_3)\Delta x_3 + \cdots f(x_n)\Delta x_n.$$

Left Sum

$$L_n = f(x_0)\Delta x_1 + f(x_1)\Delta x_2 + f(x_2)\Delta x_3 + \cdots f(x_{n-1})\Delta x_n.$$

Midpoint Sum

$$M_n = f\left(\tfrac{x_0+x_1}{2}\right)\Delta x_1 + f\left(\tfrac{x_1+x_2}{2}\right)\Delta x_2 + f\left(\tfrac{x_2+x_3}{2}\right)\Delta x_3 + \cdots + f\left(\tfrac{x_{n-1}+x_n}{2}\right)\Delta x_n.$$

26. Riemann Sums

Let $f(x)$ be a function defined on a closed bounded interval $[a, b]$. Let $n \in \mathbb{N}$, and let $x_0, x_1, \ldots, x_n$ be real numbers such that $a = x_0 < x_1 < \cdots < x_n = b$. For each i, let $\Delta x_i = x_i - x_{i-1}$, and let x_i^* be a number in the interval $[x_{i-1}, x_i]$. The **Riemann sum** of $f(x)$ with respect to these choices is

$$\sum_{i=1}^{n} f(x_i^*)\Delta x_i = f(x_1^*)\Delta x_1 + f(x_2^*)\Delta x_2 + f(x_3^*)\Delta x_3 + \cdots f(x_n^*)\Delta x_n.$$

27. The Integral of a Function

Let $f(x)$ be a function defined on a closed bounded interval $[a, b]$. The **integral** of $f(x)$ from a to b (also known as the **definite integral** or the **Riemann integral**) is

$$\int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i,$$

provided the limit exists, and is the same, for all choices of Riemann sums. If this limit exists, the function $f(x)$ is **integrable**.

28. The Integral of a Continuous Function

Every continuous function is integrable on any closed bounded interval.

29. Integrals as Signed Area

Let $f(x)$ be a function defined on a closed bounded interval $[a, b]$.

1. The area of a region in $\mathbb{R}^2$ is always a positive number or zero.
2. If $f(x) \geq 0$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx$ is the area between the graph of $y = f(x)$ and the x-axis, between $x = a$ and $x = b$.
3. If $f(x) \leq 0$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx$ is the negative of the area between the graph of $y = f(x)$ and the x-axis, between $x = a$ and $x = b$.
4. In general, the integral $\int_a^b f(x)\,dx$ is the signed area between the graph of $y = f(x)$ and the x-axis, between $x = a$ and $x = b$, which has the parts above the x-axis and the parts below the x-axis canceling out.

30. Basic Rules for Integrals

Let $f(x)$ and $g(x)$ be functions defined on a closed bounded interval $[a, b]$, and let c be a real number. Suppose that $f(x)$ and $g(x)$ are integrable.

1. $\displaystyle\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx.$
2. $\displaystyle\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx.$
3. $\displaystyle\int_a^b [c f(x)]\,dx = c\int_a^b f(x)\,dx.$
4. $\displaystyle\int_a^b c\,dx = c(b - a).$
5. $\displaystyle\int_a^a f(x)\,dx = 0.$
6. $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx.$

31. Breaking up the Interval for Integrals

Let $f(x)$ be a function, and let a, b and c be real numbers (in any order). Suppose that $f(x)$ is integrable on all intervals with a, b and c as endpoints.

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

32. Basic Inequalities for Integrals

Let $f(x)$ and $g(x)$ be functions defined on a closed bounded interval $[a, b]$. Suppose that $f(x)$ and $g(x)$ are integrable.

1. If $f(x) \geq 0$ on $[a, b]$, then $\displaystyle\int_a^b f(x)\,dx \geq 0$.

2. If $f(x) \geq g(x)$ on $[a, b]$, then $\displaystyle\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.

3. If $m \leq f(x) \leq M$ on $[a, b]$, then $m(b-a) \leq \displaystyle\int_a^b f(x)\,dx \leq M(b-a)$.

33. Fundamental Theorem of Calculus—Version I

Let $f(x)$ be a function defined on an open interval, and let a be in the interval. Suppose that $f(x)$ is continuous. If $g(x)$ is defined by

$$g(x) = \int_a^x f(t)\,dt$$

on the interval, then $g'(x) = f(x)$. An equivalent statement is

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

34. Fundamental Theorem of Calculus—Version II

Let $f(x)$ be a function defined on a closed bounded interval $[a, b]$. Suppose that $f(x)$ is continuous. If $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

35. The Indefinite Integral of a Function

1. Let $f(x)$ be a function. The **indefinite integral** of $f(x)$, denoted

   $$\int f(x)\,dx,$$

   is the general antiderivative of $f(x)$, if an antiderivative exists.

2. To say that $F(x) = \int f(x)\,dx$ simply means that $F'(x) = f(x)$.

3. If a function does not have an antiderivative, it does not have an indefinite integral.

4. **Caution**: When computing indefinite integrals, do not forget the $+C$.

36. Basic Rules for Indefinite Integrals

Let $f(x)$ and $g(x)$ be functions, and let c be a real number. Suppose that $f(x)$ and $g(x)$ have antiderivatives.

1. $\displaystyle\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

2. $\displaystyle\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$

3. $\displaystyle\int c f(x)\,dx = c \int f(x)\,dx.$

37. Basic Indefinite Integrals

1. $\displaystyle\int 1\,dx = x + C.$

2. $\displaystyle\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad \text{when } r \neq -1.$

3. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C.$

4. $\displaystyle\int e^x\,dx = e^x + C.$

5. $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C.$

6. $\displaystyle\int \sin x\,dx = -\cos x + C.$

7. $\displaystyle\int \cos x\,dx = \sin x + C.$

8. $\displaystyle\int \sec^2 x\,dx = \tan x + C.$

9. $\displaystyle\int \sec x \tan x\,dx = \sec x + C.$

10. $\displaystyle\int \csc^2 x\,dx = -\cot x + C.$

11. $\displaystyle\int \csc x \cot x\,dx = -\csc x + C.$

12. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C.$

13. $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan x + C.$

14. $\displaystyle\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x + C.$

38. Substitution

Let $f(x)$ and $g(x)$ be functions. Suppose that $g(x)$ is differentiable.
Let $u = g(x)$. Then $du = g'(x)dx$.

Indefinite Integral

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du.$$

Definite Integral

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du.$$

BASIC RULES FOR DERIVATIVES

1. $[f(x) + g(x)]' = f'(x) + g'(x)$
2. $[f(x) - g(x)]' = f'(x) - g'(x)$
3. $[cf(x)]' = cf'(x)$
4. $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$
5. $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
6. $[f(g(x))]' = f'(g(x))g'(x)$

BASIC DERIVATIVES

1. $(c)' = 0$
2. $(x)' = 1$
3. $(x^r)' = rx^{r-1}$, for any real number r
4. $(e^x)' = e^x$
5. $(a^x)' = a^x \ln a$
6. $(\ln x)' = \dfrac{1}{x}$
7. $(\ln |x|)' = \dfrac{1}{x}$
8. $(\log_a x)' = \dfrac{1}{x}\dfrac{1}{\ln a}$
9. $(\sin x)' = \cos x$
10. $(\cos x)' = -\sin x$
11. $(\tan x)' = \sec^2 x$
12. $(\sec x)' = \sec x \tan x$
13. $(\csc x)' = -\csc x \cot x$
14. $(\cot x)' = -\csc^2 x$
15. $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$
16. $(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$
17. $(\arctan x)' = \dfrac{1}{1+x^2}$
18. $(\operatorname{arccot} x)' = -\dfrac{1}{1+x^2}$
19. $(\operatorname{arcsec} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
20. $(\operatorname{arccsc} x)' = -\dfrac{1}{|x|\sqrt{x^2-1}}$

BASIC RULES FOR INDEFINITE INTEGRALS

1. $\displaystyle\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

2. $\displaystyle\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$

3. $\displaystyle\int c\,f(x)\,dx = c\int f(x)\,dx$

BASIC INDEFINITE INTEGRALS

1. $\displaystyle\int 1\,dx = x + C$

2. $\displaystyle\int x^r\,dx = \frac{x^{r+1}}{r+1} + C$ when $r \neq -1$

3. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

4. $\displaystyle\int e^x\,dx = e^x + C$

5. $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C$

6. $\displaystyle\int \sin x\,dx = -\cos x + C$

7. $\displaystyle\int \cos x\,dx = \sin x + C$

8. $\displaystyle\int \sec^2 x\,dx = \tan x + C$

9. $\displaystyle\int \sec x\tan x\,dx = \sec x + C$

10. $\displaystyle\int \csc^2 x\,dx = -\cot x + C$

11. $\displaystyle\int \csc x\cot x\,dx = -\csc x + C.$

12. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$

13. $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan x + C$

14. $\displaystyle\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x + C$