MATH 141 Calculus I
Graphing Functions

Step	**Example**
	$f(x) = x^3 - 3x^2 - 9x + 7.$
Step 1: Find $f'(x)$	$f'(x) = 3x^2 - 6x - 9 = 3(x + 1)(x - 3).$
Step 2: Find $f''(x)$	$f''(x) = 6x - 6 = 6(x - 1).$
Step 3: Find the critical points The critical points are where $f'(x) = 0$ or $f'(x)$ does not exist (though $f(x)$ exists).	$f'(x)$ always exists in this case. $f'(x) = 0$. So $3(x + 1)(x - 3) = 0$. Critical points: $x = -1$ and $x = 3$.
Step 4: Make a chart for $f'(x)$ Label the critical points, and where $f'(x)$ is positive and where it is negative.	
Step 5: Find where $f(x)$ is increasing and where it is decreasing	Increasing: $(-\infty, -1)$ and $(3, \infty)$. Decreasing: $(-1, 3)$.
Step 6: Find the local maxima and local minima of $f(x)$	Local maximum: $x = -1$. Local minimum: $x = 3$.

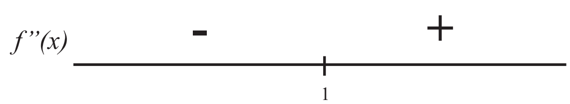
Step 7: Find the second critical points

The second critical points are where $f''(x) = 0$ or $f''(x)$ does not exist (though $f(x)$ exists).

$f''(x)$ always exists in this case.
$f''(x) = 0$. So $6(x-1) = 0$.
Second critical points: $x = 1$.

Step 8: Make a chart for $f''(x)$

Label the second critical points, and where $f''(x)$ is positive and where it is negative.



Step 9: Find where $f(x)$ is concave up and where it is concave down

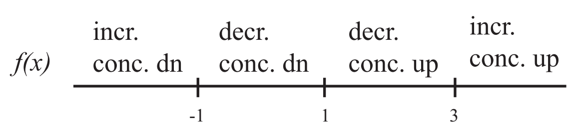
Concave up: $(1, \infty)$.
Concave down: $(-\infty, 1)$.

Step 10: Find the inflection points of $f(x)$

Inflection point: $x = 1$.

Step 11: Make a combined chart for $f(x)$

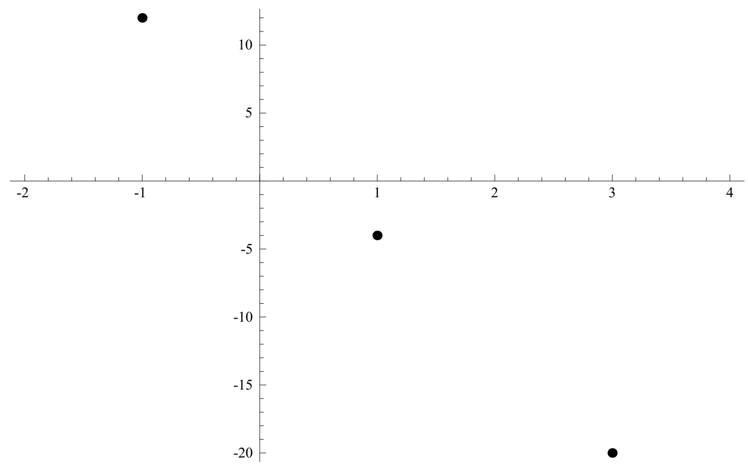
Label the critical points and second critical points, and where $f(x)$ is is increasing concave up, increasing concave down, decreasing concave up, and decreasing concave down.



Step 12: Find the value of $f(x)$ at the critical points and second critical points

$f(-1) = 12$.
$f(1) = -4$.
$f(3) = -20$.

Step 13: Plot $f(x)$ at the critical points and second critical points



Step 14: Sketch the graph of $f(x)$

Use the combined chart to draw each part of the curve between the critical points and second critical points.