

**MATH 141B    Calculus 1    Spring 2019**  
**Study Sheet for Midterm Exam**

**The Exam is in class, Wednesday, 13 March 2019**

**GUIDELINES THAT WILL BE ON THE EXAM**

**EXAM GUIDELINES**

In order for this exam to be an honest and accurate reflection of your understanding of the material, you are asked to adhere to the following guidelines:

- The exam is closed book.
- The study sheet is not allowed during the exam.
- Books, notes and online resources are not allowed during the exam.
- Electronic devices (calculators, cell phones, tablets, laptops, etc.) are not allowed during the exam.
- For the duration of the exam, you may not discuss the exam, or related material, with anyone other than the course instructor.
- Giving help to others taking this exam is as much a violation of these guidelines as receiving help.
- Late exams will be allowed only if you discuss it with the course instructor before hand, or if an emergency occurs.
- **Violation of these guidelines will result, at minimum, in a score of zero on this exam.**
- **There will be no opportunity to retake this exam.**

Further comments:

- Write your solutions carefully and clearly.
- Show all your work. You will receive partial credit for work you show, but you will not receive credit for what you do not write down. In particular, correct answers with no work will not receive credit.

## TOPICS

1. Tangent and velocity problems
2. Limits
3. Continuous functions
4. The derivative at a point
5. The derivative as a function
6. Basic rules for taking derivatives
7. Product Rule, Quotient Rule, Chain Rule
8. Derivatives of power functions
9. Derivatives of exponential functions
10. Derivatives of logarithmic functions
11. Derivatives of trigonometric functions
12. Derivatives of inverse trigonometric functions
13. Implicit differentiation

## TIPS FOR STUDYING FOR THE EXAM

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- × **Bad** Forgetting about the homework and the previous quizzes.
- ✓ **Good** Making sure you know how to do all the problems on the homework and previous quizzes; seeking help from the instructor and the tutors about the problems you do not know how to do.

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- × **Bad** Doing all the practice problems from some of the sections, and not having enough time to do practice problems from the rest of the sections.
- ✓ **Good** Doing a few practice problems of each type from every sections.

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- × **Bad** Studying only by reading the book.
- ✓ **Good** Doing a lot of practice problems, and reading the book as needed.

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- × **Bad** Studying only by yourself.
- ✓ **Good** Trying some practice problems by yourself (or with friends), and then seeking help from the instructor and the tutors about the problems you do not know how to do.

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- × **Bad** Doing practice problems while looking everything up in the book.
- ✓ **Good** Doing some of the practice problems the way you would do them on the quiz or exam, which is with closed book and no calculator.

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- × **Bad** Staying up late (or all night) the night before the exam.
- ✓ **Good** Studying hard up through the day before the exam, but getting a good night's sleep the night before the exam.
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### Ethan's Office Hours

- **Monday:** 5:00-6:00
- **Tuesday:** 2:00-3:30 & 5:00-6:00
- **Thursday:** 4:30-6:00
- **Or by appointment**

### Class Tutor & Mathematics Study Room

- **Class Tutor:** Wednesday, 7:30-8:30, Mathematics Common Room (third floor of Albee)
- **Mathematics Study Room:** Sunday-Wednesday, 7:00-10:00, Hegeman 308

**PRACTICE PROBLEMS FROM  
STEWART, CALCULUS CONCEPTS AND CONTEXTS, 4TH ED.**

**Section 2.1:** 1, 3, 5, 7

**Section 2.2:** 1, 3, 5, 7, 13, 15, 17, 19, 21, 23

**Section 2.3:** 1, 9, 11, 13, 15, 17, 19, 21, 23

**Section 2.4:** 9, 15, 17, 33, 35

**Section 2.6:** 5, 7, 11, 13, 15, 17, 19, 21, 23, 27, 29, 31, 33, 35, 37, 41, 45, 47, 49, 51

**Section 2.7:** 1, 3, 5, 7, 9, 11, 13, 19, 21, 23, 25, 27, 29

**Section 3.1:** 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 41, 51, 53, 61, 65, 67, 69

**Section 3.2:** 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 39, 41, 43, 45, 49

**Section 3.3:** 1, 3, 5, 7, 9, 11, 13, 19, 21, 39

**Section 3.4:** 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 49, 51, 53, 57, 59, 61, 63, 67, 69,  
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**Section 3.5:** 3, 5, 7, 9, 11, 13, 15, 21, 23, 25, 27

**Section 3.6:** 17, 19, 21, 23, 25, 27, 29

**Section 3.7:** 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 25, 27

## SOME IMPORTANT CONCEPTS AND FORMULAS

### 1. Slope of Secant Line

Let  $f(x)$  be a function, and let  $a$  and  $b$  be real numbers. The slope of the secant line from  $x = a$  to  $x = b$  is

$$\frac{f(b) - f(a)}{b - a},$$

which is the same as

$$\frac{f(a + h) - f(a)}{h},$$

where  $h = b - a$ .

### 2. Average Velocity

Let  $s(t)$  be the position of an object at time  $t$ . The average velocity from time  $t = a$  to time  $t = b$  is

$$\frac{s(b) - s(a)}{b - a},$$

which is the same as

$$\frac{s(a + h) - s(a)}{h},$$

where  $h = b - a$ .

### 3. Limits

Let  $f(x)$  be a function, and let  $c$  and  $L$  be real numbers.

1. The number  $L$  is the **limit** of  $f(x)$  as  $x$  goes to  $c$ , written

$$\lim_{x \rightarrow c} f(x) = L,$$

if the value of  $f(x)$  gets closer and closer to a number  $L$  as the value of  $x$  gets closer and closer to  $c$ . If  $\lim_{x \rightarrow c} f(x) = L$ , the function  $f(x)$  **converges** to  $L$  as  $x$  goes to  $c$ . If  $f(x)$  converges to some real number as  $x$  goes to  $c$ , the limit  $\lim_{x \rightarrow c} f(x)$  **exists**; otherwise the limit **does not exist**.

2. The above definition, and in particular the use of the phrase “gets closer and closer,” is informal. A rigorous definition of limits will be seen in a Real Analysis course.
3. If  $f(x)$  has a limit as  $x$  goes to  $c$ , the limit is unique.

#### 4. Limits: One Sided

Let  $f(x)$  be a function, and let  $c$  and  $L$  be real numbers.

1. The number  $L$  is the **left-hand limit** of  $f(x)$  as  $x$  goes to  $c$  (also known as the **limit** of  $f(x)$  as  $x$  goes to  $c$  **from the left**), written

$$\lim_{x \rightarrow c^-} f(x) = L,$$

if the value of  $f(x)$  gets closer and closer to a number  $L$  as the value of  $x$  is less than  $c$  and gets closer and closer to  $c$ . If  $\lim_{x \rightarrow c^-} f(x) = L$ , the function  $f(x)$  **converges** to  $L$  as  $x$  goes to  $c$  **from the left**. If  $f(x)$  converges to some real number as  $x$  goes to  $c$  from the left, the limit  $\lim_{x \rightarrow c^-} f(x)$  **exists**; otherwise the limit **does not exist**.

2. The number  $L$  is the **right-hand limit** of  $f(x)$  as  $x$  goes to  $c$  (also known as the **limit** of  $f(x)$  as  $x$  goes to  $c$  **from the right**), written

$$\lim_{x \rightarrow c^+} f(x) = L,$$

if the value of  $f(x)$  gets closer and closer to a number  $L$  as the value of  $x$  is greater than  $c$  and gets closer and closer to  $c$ . If  $\lim_{x \rightarrow c^+} f(x) = L$ , the function  $f(x)$  **converges** to  $L$  as  $x$  goes to  $c$  **from the right**. If  $f(x)$  converges to some real number as  $x$  goes to  $c$  from the right, the limit  $\lim_{x \rightarrow c^+} f(x)$  **exists**; otherwise the limit **does not exist**.

3. The above definition, and in particular the use of the phrase “gets closer and closer,” is informal. A rigorous definition of limits will be seen in a Real Analysis course.
4. If  $f(x)$  has a limit as  $x$  goes to  $c$  from the left or from the right, the limit is unique.
5.  $\lim_{x \rightarrow c} f(x) = L$  if and only if  $\lim_{x \rightarrow c^-} f(x) = L$  and  $\lim_{x \rightarrow c^+} f(x) = L$ .

#### 5. Continuous Functions

Let  $f(x)$  be a function.

1. Let  $c$  be a real number. The function  $f(x)$  is **continuous** at  $c$  if the following three properties hold:
  - (1)  $f(c)$  exists;
  - (2)  $\lim_{x \rightarrow c} f(x)$  exists;
  - (3)  $\lim_{x \rightarrow c} f(x) = f(c)$ .
1. The function  $f(x)$  is **continuous** if it is continuous at all  $c$  in the domain of  $f(x)$ .

**6. Continuity of Some Standard Functions**

The following functions are continuous at all points of their domains.

1. Polynomials.
2. Rational functions and algebraic functions.
3. Trigonometric functions.
4. Inverse trigonometric functions.
5. Exponential functions.
6. Logarithmic functions.

**7. Continuous Functions: Properties**

Let  $f(x)$  and  $g(x)$  be functions, and let  $k$  be a real number. Suppose that  $f(x)$  and  $g(x)$  are continuous.

1.  $f(x) + g(x)$  is continuous.
2.  $f(x) - g(x)$  is continuous.
3.  $kf(x)$  is continuous.
4.  $f(x)g(x)$  is continuous.
5.  $\frac{f(x)}{g(x)}$  is continuous at real numbers  $x$  for which  $g(x) \neq 0$ .

**8. Limits: Properties**

Let  $f(x)$ ,  $g(x)$  and  $h(x)$  be functions, and let  $c$  and  $k$  be real numbers. Suppose that  $\lim_{x \rightarrow c} f(x)$  and  $\lim_{x \rightarrow c} g(x)$  exist.

1.  $\lim_{x \rightarrow c} [f(x) + g(x)]$  exists and  $\lim_{x \rightarrow c} [f(x) + g(x)] = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x)$ .
2.  $\lim_{x \rightarrow c} [f(x) - g(x)]$  exists and  $\lim_{x \rightarrow c} [f(x) - g(x)] = \lim_{x \rightarrow c} f(x) - \lim_{x \rightarrow c} g(x)$ .
3.  $\lim_{x \rightarrow c} kf(x)$  exists and  $\lim_{x \rightarrow c} kf(x) = k \lim_{x \rightarrow c} f(x)$ .
4.  $\lim_{x \rightarrow c} f(x)g(x)$  exists and  $\lim_{x \rightarrow c} f(x)g(x) = [\lim_{x \rightarrow c} f(x)] \cdot [\lim_{x \rightarrow c} g(x)]$ .
5. If  $\lim_{x \rightarrow c} g(x) \neq 0$ , then  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$  exists and  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}$ .
6. If  $f(x) \leq g(x)$  for all  $x$ , then  $\lim_{x \rightarrow c} f(x) \leq \lim_{x \rightarrow c} g(x)$ .
7. (Squeeze Theorem) If  $f(x) \leq h(x) \leq g(x)$  for all  $x$ , and if  $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x)$ , then  $\lim_{x \rightarrow c} h(x)$  exists and  $\lim_{x \rightarrow c} h(x) = \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x)$ .

**9. Slope of Tangent Line**

Let  $f(x)$  be a function, and let  $a$  be a real number. The slope of the tangent line at  $x = a$  is

$$\lim_{b \rightarrow a} \frac{f(b) - f(a)}{b - a},$$

which is the same as

$$\lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}.$$

**10. Instantaneous Velocity**

Let  $s(t)$  be the position of an object at time  $t$ . The velocity at  $t = a$  is

$$v(a) = \lim_{b \rightarrow a} \frac{s(b) - s(a)}{b - a},$$

which is the same as

$$v(a) = \lim_{h \rightarrow 0} \frac{s(a + h) - s(a)}{h}.$$

**11. The Derivative at a Point**

Let  $f(x)$  be a function defined on an open interval. The **derivative** of  $f(x)$  at  $x = a$ , denoted  $f'(a)$ , is the number

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h},$$

provided the limit exists. If the limit does not exist, then we say that  $f'(a)$  does not exist.

**12. The Derivative of a Function**

Let  $f(x)$  be a function defined on an open interval.

1. The **derivative** of  $f(x)$ , denoted  $f'(x)$ , is the function defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h},$$

for those values of  $x$  for which the limit exists.

2. The function  $f(x)$  is **differentiable** if  $f'(x)$  is defined for all values of  $x$ .

**13. Notation for the Derivative**

Let  $y = f(x)$  be a function. The derivative of  $f(x)$  can be written in any of the following ways:

$$f'(x), \quad y', \quad \dot{f}(x), \quad \dot{y}, \quad Df(x), \quad \frac{dy}{dx}, \quad \frac{df}{dx}, \quad \frac{d}{dx}f(x).$$



**14. Higher Derivatives**

Let  $y = f(x)$  be a function.

**Second Derivative** The **second derivative** of  $f(x)$  is the derivative of  $f'(x)$ . The second derivative of  $f(x)$  can be written in any of the following ways:

$$f''(x), \quad y'', \quad \ddot{f}(x), \quad \ddot{y}, \quad D^2 f(x), \quad \frac{d^2 y}{dx^2}, \quad \frac{d^2 f}{dx^2}, \quad \frac{d^2}{dx^2} f(x).$$

**Third Derivative** The **third derivative** of  $f(x)$  is the derivative of  $f''(x)$ . The third derivative of  $f(x)$  can be written in any of the following ways:

$$f'''(x), \quad y''', \quad \ddot{\ddot{f}}(x), \quad \ddot{\ddot{y}}, \quad D^3 f(x), \quad \frac{d^3 y}{dx^3}, \quad \frac{d^3 f}{dx^3}, \quad \frac{d^3}{dx^3} f(x).$$

**$n$ -th Derivative** The  **$n$ -th derivative** of  $f(x)$  is the result of taking the derivative of  $f(x)$   $n$  times. The  $n$ -th derivative of  $f(x)$  can be written in any of the following ways:

$$f^{(n)}(x), \quad y^{(n)}, \quad D^n f(x), \quad \frac{d^n y}{dx^n}, \quad \frac{d^n f}{dx^n}, \quad \frac{d^n}{dx^n} f(x).$$

**15. Basic Rules for Derivatives**

Let  $f(x)$  and  $g(x)$  be functions, and let  $c$  be a real number. Suppose that  $f(x)$  and  $g(x)$  are differentiable.

1.  $[f(x) + g(x)]' = f'(x) + g'(x)$ ;
2.  $[f(x) - g(x)]' = f'(x) - g'(x)$ ;
3.  $[cf(x)]' = cf'(x)$ .

**16. Power Rule**

Let  $r$  be a real number. Then  $(x^r)' = rx^{r-1}$ .

**17. Derivative of the Exponential Function**

$$(e^x)' = e^x.$$

**18. Product Rule**

Let  $f(x)$  and  $g(x)$  be functions. Suppose that  $f(x)$  and  $g(x)$  are differentiable.

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x).$$

**19. Quotient Rule**

Let  $f(x)$  and  $g(x)$  be functions. Suppose that  $f(x)$  and  $g(x)$  are differentiable.

$$\left[ \frac{f(x)}{g(x)} \right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

**20. Derivatives of the Trigonometric Functions**

1.  $(\sin x)' = \cos x$ ;
2.  $(\cos x)' = -\sin x$ .
3.  $(\tan x)' = \sec^2 x$ ;
4.  $(\sec x)' = \sec x \tan x$ ;
5.  $(\csc x)' = -\csc x \cot x$ ;
6.  $(\cot x)' = -\csc^2 x$ .

**21. Chain Rule**

Let  $f(x)$  and  $g(x)$  be functions. Suppose that  $f(x)$  and  $g(x)$  are differentiable.

$$[f(g(x))]' = f'(g(x))g'(x).$$

**22. Chain Rule — Leibniz Notation**

Let  $y = f(u)$  and  $u = g(x)$  be functions. Suppose that  $f(u)$  and  $g(x)$  are differentiable.

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}.$$

**23. Inverse Functions**

Let  $f(x)$  and  $g(x)$  be functions.

1. The functions  $f(x)$  and  $g(x)$  are **inverse functions** if  $g(f(x)) = x$  for all  $x$ , and  $f(g(x)) = x$  for all  $x$ .
2. To say that  $f(x)$  and  $g(x)$  are inverse functions is equivalent to saying that  $y = g(x)$  if and only if  $x = f(y)$ .
3. Not every function has an inverse function. For a function to have an inverse function, it must satisfy the horizontal line test: every horizontal line intersects the graph of the function in at most one point.
4. If a function  $f(x)$  has an inverse, the inverse is unique, and is usually denoted  $f^{-1}(x)$ .

**24. Derivatives of Exponential Functions**

Let  $a$  be a positive real number. Then  $(a^x)' = a^x \ln a$ .

**25. Derivatives of Logarithmic Functions**

1. Let  $a$  be a positive real number. Then  $(\log_a x)' = \frac{1}{x \ln a}$ .

2.  $(\ln x)' = \frac{1}{x}$ .

3.  $(\ln |x|)' = \frac{1}{x}$ .

**26. Inverse Sine Function**

- The function  $f(x) = \sin x$  with domain all real numbers does not have an inverse function. This function does have an inverse function when restricted to  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ ; this inverse function is called **arcsine**, and it is denoted  $\arcsin x$ . The function  $\arcsin x$  has domain  $[-1, 1]$ .
- The function  $\arcsin x$  is also written  $\sin^{-1} x$ , though it is important to recognize that  $\sin^{-1} x$  does not mean  $\frac{1}{\sin x}$ .
- The function  $\arcsin x$  satisfies the equalities  $\arcsin(\sin x) = x$  for all  $x$  in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ , and  $\sin(\arcsin x) = x$  for all  $x$  in  $[-1, 1]$ .
- The function  $\arcsin x$  satisfies the equalities  $\arcsin x = y$  if and only if  $\sin y = x$ .

**27. Inverse Cosine Function**

- The function  $f(x) = \cos x$  with domain all real numbers does not have an inverse function. This function does have an inverse function when restricted to  $[0, \pi]$ ; this inverse function is called **arccosine**, and it is denoted  $\arccos x$ . The function  $\arccos x$  has domain  $[-1, 1]$ .
- The function  $\arccos x$  is also written  $\cos^{-1} x$ , though it is important to recognize that  $\cos^{-1} x$  does not mean  $\frac{1}{\cos x}$ .
- The function  $\arccos x$  satisfies the equalities  $\arccos(\cos x) = x$  for all  $x$  in  $[0, 2\pi]$ , and  $\cos(\arccos x) = x$  for all  $x$  in  $[-1, 1]$ .
- The function  $\arccos x$  satisfies the equalities  $\arccos x = y$  if and only if  $\cos y = x$ .

**28. Inverse Tangent Function**

1. The function  $f(x) = \tan x$  with domain all real numbers does not have an inverse function. This function does have an inverse function when restricted to  $(-\frac{\pi}{2}, \frac{\pi}{2})$ ; this inverse function is called **arctangent**, and it is denoted  $\arctan x$ . The function  $\arctan x$  has domain  $(-\infty, \infty)$ .
2. The function  $\arctan x$  is also written  $\tan^{-1} x$ , though it is important to recognize that  $\tan^{-1} x$  does not mean  $\frac{1}{\tan x}$ .
3. The function  $\arctan x$  satisfies the equalities  $\arctan(\tan x) = x$  for all  $x$  in  $(-\frac{\pi}{2}, \frac{\pi}{2})$ , and  $\tan(\arctan x) = x$  for all  $x$  in  $(-\infty, \infty)$ .
4. The function  $\arctan x$  satisfies the equalities  $\arctan x = y$  if and only if  $\tan y = x$ .

**29. Inverse Secant Function**

1. The function  $f(x) = \sec x$  with domain all real numbers does not have an inverse function. This function does have an inverse function when restricted to  $[0, \frac{\pi}{2})$  and  $(\frac{\pi}{2}, \pi]$ ; this inverse function is called **arcsecant**, and it is denoted  $\operatorname{arcsec} x$ . The function  $\operatorname{arcsec} x$  has domain  $(-\infty, -1]$  and  $[1, \infty)$ .
2. The function  $\operatorname{arcsec} x$  is also written  $\sec^{-1} x$ , though it is important to recognize that  $\sec^{-1} x$  does not mean  $\frac{1}{\sec x}$ .
3. The function  $\operatorname{arcsec} x$  satisfies the equalities  $\operatorname{arcsec}(\sec x) = x$  for all  $x$  in  $[0, \frac{\pi}{2})$  and  $(\frac{\pi}{2}, \pi]$ , and  $\sec(\operatorname{arcsec} x) = x$  for all  $x$  in  $(-\infty, -1]$  and  $[1, \infty)$ .
4. The function  $\operatorname{arcsec} x$  satisfies the equalities  $\operatorname{arcsec} x = y$  if and only if  $\sec y = x$ .

**30. Inverse Cosecant Function**

1. The function  $f(x) = \csc x$  with domain all real numbers does not have an inverse function. This function does have an inverse function when restricted to  $[-\frac{\pi}{2}, 0)$  and  $(0, \frac{\pi}{2}]$ ; this inverse function is called **arccosecant**, and it is denoted  $\operatorname{arccsc} x$ . The function  $\operatorname{arccsc} x$  has domain  $(-\infty, -1]$  and  $[1, \infty)$ .
2. The function  $\operatorname{arccsc} x$  is also written  $\csc^{-1} x$ , though it is important to recognize that  $\csc^{-1} x$  does not mean  $\frac{1}{\csc x}$ .
3. The function  $\operatorname{arccsc} x$  satisfies the equalities  $\operatorname{arccsc}(\csc x) = x$  for all  $x$  in  $[-\frac{\pi}{2}, 0)$  and  $(0, \frac{\pi}{2}]$ , and  $\csc(\operatorname{arccsc} x) = x$  for all  $x$  in  $(-\infty, -1]$  and  $[1, \infty)$ .
4. The function  $\operatorname{arccsc} x$  satisfies the equalities  $\operatorname{arccsc} x = y$  if and only if  $\csc y = x$ .

**31. Inverse Cotangent Function**

1. The function  $f(x) = \cot x$  with domain all real numbers does not have an inverse function. This function does have an inverse function when restricted to  $(0, \pi)$ ; this inverse function is called **arccotangent**, and it is denoted  $\operatorname{arccot} x$ . The function  $\operatorname{arccot} x$  has domain  $(-\infty, \infty)$ .
2. The function  $\operatorname{arccot} x$  is also written  $\cot^{-1} x$ , though it is important to recognize that  $\cot^{-1} x$  does not mean  $\frac{1}{\cot x}$ .
3. The function  $\operatorname{arccot} x$  satisfies the equalities  $\operatorname{arccot}(\cot x) = x$  for all  $x$  in  $(0, \pi)$ , and  $\cot(\operatorname{arccot} x) = x$  for all  $x$  in  $(-\infty, \infty)$ .
4. The function  $\operatorname{arccot} x$  satisfies the equalities  $\operatorname{arccot} x = y$  if and only if  $\cot y = x$ .

**32. Derivatives of Inverse Trigonometric Functions**

1.  $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$ ;
2.  $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$ ;
3.  $(\arctan x)' = \frac{1}{1+x^2}$ ;
4.  $(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$ .
5.  $(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$ ;
6.  $(\operatorname{arccsc} x)' = -\frac{1}{|x|\sqrt{x^2-1}}$ ;

**33. Implicit Differentiation**

Let  $F(x, y)$  and  $G(x, y)$  be functions (with  $G(x, y)$  possibly a constant). The equation

$$F(x, y) = G(x, y)$$

can be thought of as implicitly defining  $y$  as a function of  $x$ . To find the derivative  $\frac{dy}{dx}$ , take the derivative with respect to  $x$  (denoted  $\frac{d}{dx}$ ) of both sides of the equation, and, when encountering any appearance of  $y$  in the equation, think of  $y$  as a function of  $x$ , and use the Chain Rule.

### Basic Rules for Derivatives

1.  $[f(x) + g(x)]' = f'(x) + g'(x)$

2.  $[f(x) - g(x)]' = f'(x) - g'(x)$

3.  $[cf(x)]' = cf'(x)$

4.  $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$

5.  $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

6.  $[f(g(x))]' = f'(g(x))g'(x)$

### Basic Derivatives

1.  $(c)' = 0$

2.  $(x)' = 1$

3.  $(x^r)' = rx^{r-1}$ , for any real number  $r$

4.  $(e^x)' = e^x$

5.  $(a^x)' = a^x \ln a$

6.  $(\ln x)' = \frac{1}{x}$

7.  $(\ln |x|)' = \frac{1}{x}$

8.  $(\log_a x)' = \frac{1}{x \ln a}$

9.  $(\sin x)' = \cos x$

10.  $(\cos x)' = -\sin x$

11.  $(\tan x)' = \sec^2 x$

12.  $(\sec x)' = \sec x \tan x$

13.  $(\csc x)' = -\csc x \cot x$

14.  $(\cot x)' = -\csc^2 x$

15.  $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

16.  $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

17.  $(\arctan x)' = \frac{1}{1+x^2}$

18.  $(\text{arccot } x)' = -\frac{1}{1+x^2}$

19.  $(\text{arcsec } x)' = \frac{1}{|x|\sqrt{x^2-1}}$

20.  $(\text{arccsc } x)' = -\frac{1}{|x|\sqrt{x^2-1}}$