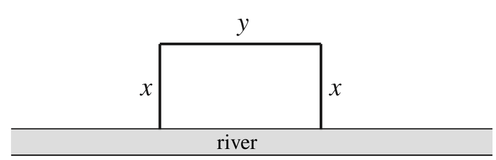
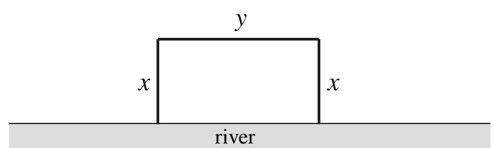
MATH 141 Calculus I
Optimization Problems

Type 1: Closed Bounded Interval

Step	Example
	A farmer has 100 ft. of fence, and wants to make a rectangular pen using this fence, where one side of the rectangle is along a river and does not need a fence. Find the dimensions of the rectangle that has the largest possible area.
Step 1: Make a sketch Be sure to label all the variables.	
Step 2: Find the function that is being maximized or minimized	$A = xy$.
Step 3: Find the constraint	$2x + y = 100$.
Step 4: Solve for one of the variables in the constraint	$y = 100 - 2x$.
Step 5: Rewrite the functions being maximized or minimized in terms of a single variable Simplify the function as much as possible.	$A = x(100 - 2x) = 100x - 2x^2$.
Step 6: Find the possible values of the variable It is crucial to distinguish between a closed bounded interval or not.	The number x is in the interval $[0, 50]$.

Step 7: Find the derivative of the function	$\frac{dA}{dx} = 100 - 4x.$
Step 8: Find the critical points The critical points are where $\frac{dA}{dx} = 0$ or $\frac{dA}{dx}$ does not exist (though A exists).	The derivative in this problem always exists. Solving $100 - 4x = 0$ yields $x = 25$, and that is the only critical point.
Step 9: Find the endpoints	The endpoints are $x = 0$ and $x = 50$.
Step 10: Find the values of A at the endpoints and critical points	$A(0) = 100 \cdot 0 - 2 \cdot 0^2 = 0$, $A(50) = 100 \cdot 50 - 2 \cdot 50^2 = 0$, $A(25) = 100 \cdot 25 - 2 \cdot 25^2 = 1250$.
Step 11: Find the global maximum or global minimum, as needed The global maximum will be the value of x among the endpoints and critical points where $A(x)$ is largest, and the global minimum will be the value of x among the endpoints and critical points where $A(x)$ is smallest.	We want the global maximum in this problem, and it is $x = 25$.
Step 12: Answer the question as asked	This problem asked for the dimensions of the rectangle, which are $x = 25$ and $y = 100 - 2 \cdot 25 = 50$.

Type 2: Single Critical Point

Step	Example
	A farmer wants to make a rectangular pen with area 1250 square ft., where one side of the rectangle is along a river and does not need a fence. Find the dimensions of the rectangle that uses the least amount of fence.
Step 1: Make a sketch Be sure to label all the variables.	
Step 2: Find the function that is being maximized or minimized	$L = 2x + y.$
Step 3: Find the constraint	$xy = 1250.$
Step 4: Solve for one of the variables in the constraint	$y = \frac{1250}{x}.$
Step 5: Rewrite the functions being maximized or minimized in terms of a single variable Simplify the function as much as possible.	$L = 2x + \frac{1250}{x} = 2x + 1250x^{-1}.$
Step 6: Find the possible values of the variable It is crucial to distinguish between a closed bounded interval or not.	The number x is in the interval $(0, \infty)$.
Step 7: Find the derivative of the function	$\frac{dL}{dx} = 2 - 1250x^{-2} = 2 - \frac{1250}{x^2}.$

Step 8: Find the critical points

The critical points are where $\frac{dL}{dx} = 0$ or $\frac{dL}{dx}$ does not exist (though L exists).

The derivative in this problem does not exist at $x = 0$, but that is not a possible value of x.
The equation $2 - \dfrac{1250}{x^2} = 0$ yields $2x^2 = 1250$, which yields $x^2 = 625$, which yields $x = 25$ and $x = -25$, but only $x = 25$ is a possible value of x.

Step 9: Test if the single critical point is a local maximum or local minimum

Use either the First Derivative Test or the Second Derivative Test.

Using the Second Derivative Test, we see that $\dfrac{d^2L}{dx^2} = 2500x^{-3} = \dfrac{2500}{x^3}$, and that is always positive, which means that the critical point $x = 25$ is a local minimum.

Step 10: In the case of a single critical point, conclude that a local maximum is a global maximum, and a local minimum is a global minimum

We deduce that $x = 25$ is a global minimum.

Step 11: Answer the question as asked

This problem asked for the dimensions of the rectangle, which are $x = 25$ and $y = \dfrac{1250}{25} = 50$.
