MATH 245 Intermediate Calculus Fall 2018
Study Sheet for Exam #1

GUIDELINES THAT WILL BE ON THE EXAM

EXAM GUIDELINES

In order for this exam to be an honest and accurate reflection of your understanding of the material, you are asked to adhere to the following guidelines:

- The exam is closed book.
- The study sheet is not allowed during the exam.
- Books, notes and online resources are not allowed during the exam.
- Electronic devices (calculators, cell phones, tablets, laptops, etc.) are not allowed during the exam.
- For the duration of the exam, you may not discuss the exam, or related material, with anyone other than the course instructor.
- Giving help to others taking this exam is as much a violation of these guidelines as receiving help.
- Late exams will be allowed only if you discuss it with the course instructor before hand, or if an emergency occurs.
- **Violation of these guidelines will result, at minimum, in a score of zero on this exam.**
- **There will be no opportunity to retake this exam.**

Further comments:

- Write your solutions carefully and clearly.
- Show all your work. You will receive partial credit for work you show, but you will not receive credit for what you do not write down. In particular, correct answers with no work will not receive credit.

TOPICS

1. Graphing functions of two variables
2. Level curves
3. Double integrals and iterated integrals over rectangles
4. Double integrals and iterated integrals over more general regions
5. Polar coordinates
6. Double integrals in polar coordinates
7. Triple integrals
8. Cylindrical and spherical coordinates
9. Triple integrals in cylindrical and spherical coordinates

TIPS FOR STUDYING FOR THE EXAM

×	**Bad**	Forgetting about the homework and the previous quizzes.
✓	**Good**	Making sure you know how to do all the problems on the homework and previous quizzes; seeking help seeking help from the instructor and the tutors about the problems you do not know how to do.
×	**Bad**	Doing all the practice problems from some of the sections, and not having enough time to do practice problems from the rest of the sections.
✓	**Good**	Doing a few practice problems of each type from every sections.
×	**Bad**	Studying only by reading the book.
✓	**Good**	Doing a lot of practice problems, and reading the book as needed.
×	**Bad**	Studying only by yourself.
✓	**Good**	Trying some practice problems by yourself (or with friends), and then seeking help from the instructor and the tutors about the problems you do not know how to do.
×	**Bad**	Doing practice problems while looking everything up in the book.
✓	**Good**	Doing some of the practice problems the way you would do them on the quiz or exam, which is with closed book and no calculator.
×	**Bad**	Staying up late (or all night) the night before the exam.
✓	**Good**	Studying hard up through the day before the exam, but getting a good night's sleep the night before the exam.

Ethan's Office Hours

- **Monday:** 4:30-6:00
- **Tuesday:** 5:00-6:00
- **Wednesday:** 2:00-3:30
- **Or by appointment**

Tutor

- Weronica Nguyen
- **Office hours**: Wednesday: 6:00–7:00, Mathematics Common Room (third floor of Albee)
- **Email to Make an Appointment**: tn3599 "at" bard "dot" edu.

PRACTICE PROBLEMS FROM STEWART, CALCULUS CONCEPTS AND CONTEXTS, 4TH ED.

Section 11.1: 19, 21, 23, 25

Section 12.1: 1, 3, 5

Section 12.2: 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 25, 27, 29

Section 12.3: 1, 3, 5, 7, 9, 15, 17, 19, 21, 23, 25, 27, 29, 41, 43, 45, 47, 49, 51

Section 12.4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 27, 29

Section 12.7: 3, 5, 7, 9, 11, 13, 15, 17

Section 12.8: 5, 7, 9, 11, 17, 19, 21, 23

SOME IMPORTANT CONCEPTS AND FORMULAS

1. $\mathbb{R}^2$

1. The set $\mathbb{R}^2$, thought of as points, is the set of all ordered pairs of the form (x, y).
2. Let (x_1, y_1) and (x_2, y_2) be points in $\mathbb{R}^2$. The distance between the two points is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

2. $\mathbb{R}^3$

1. The set $\mathbb{R}^3$, thought of as points, is the set of all ordered triples of the form (x, y, z).
2. Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be points in $\mathbb{R}^3$. The distance between the two points is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

3. Functions of Two Variables: Level Curves

1. Let $f(x, y)$ be a function. A **level curve** of $f(x, y)$ is any curve in $\mathbb{R}^2$ given by an equation of the form $f(x, y) = k$, for some real number k.
2. A **contour plot** of $f(x, y)$ is a graph showing some of the level curves.

4. Functions of Three Variables: Level Surfaces

Let $f(x, y, z)$ be a function. A **level surface** of $f(x, y, z)$ is any surface in $\mathbb{R}^3$ given by an equation of the form $f(x, y, z) = k$, for some real number k.

5. The Double Integral of a Function over a Rectangle

1. Let $f(x, y)$ be a function defined on a rectangle $R = [a, b] \times [c, d]$. The **integral** of $f(x, y)$ over R is

$$\iint\limits_R f(x, y)\, dA = \lim_{\substack{\max \Delta x_i \to 0 \\ \max \Delta y_j \to 0}} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A_{ij},$$

provided the limit exists, and is the same, for all choices of Riemann sums. If this limit exists, the function $f(x, y)$ is **integrable**.

2. Every continuous function is integrable on any rectangle.

6. Iterated Integrals over Rectangles

Let $f(x, y)$ be a function defined on a rectangle $R = [a, b] \times [c, d]$. Suppose that $f(x, y)$ is continuous.

$$\iint_R f(x, y)\, dA = \int_a^b \int_c^d f(x, y)\, dy\, dx = \int_c^d \int_a^b f(x, y)\, dx\, dy.$$

7. Iterated Integrals over General Regions

Let $f(x, y)$ be a function defined on a closed bounded region D of $\mathbb{R}^2$. Suppose that $f(x, y)$ is continuous.

Type I

Suppose that the region D is given by inequalities of the form

$$\begin{gathered} a \le x \le b \\ g_1(x) \le y \le g_2(x). \end{gathered}$$

Then

$$\iint_D f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx.$$

Type II

Suppose that the region D is given by inequalities of the form

$$\begin{gathered} c \le y \le d \\ h_1(y) \le y \le h_2(y). \end{gathered}$$

Then

$$\iint_D f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy.$$

8. Basic Rules for Double Integrals

Let $f(x, y)$ and $g(x, y)$ be functions defined on a closed bounded region D in $\mathbb{R}^2$, and let k be a real number. Suppose that $f(x, y)$ and $g(x, y)$ are integrable.

1. $\displaystyle\iint_D [f(x, y) + g(x, y)]\, dA = \iint_D f(x, y)\, dA + \iint_D g(x, y)\, dA.$

2. $\displaystyle\iint_D [f(x, y) - g(x, y)]\, dA = \iint_D f(x, y)\, dA - \iint_D g(x, y)\, dA.$

3. $\displaystyle\iint_D k f(x, y)\, dA = k \iint_D f(x, y)\, dA.$

4. $\displaystyle\iint_D k\, dA = k \cdot \text{area}(D).$

9. Breaking up the Region for Double Integrals

Let $f(x, y)$ be a function defined on a closed bounded region D of $\mathbb{R}^2$. Suppose $f(x, y)$ is integrable. Suppose that D is the union of two regions D_1 and D_2 that overlap at most on their boundaries.

$$\iint_D f(x, y)\, dA = \iint_{D_1} f(x, y)\, dA + \iint_{D_2} f(x, y)\, dA.$$

10. Basic Inequalities for Double Integrals

Let $f(x, y)$ and $g(x, y)$ be functions defined on a closed bounded region D in $\mathbb{R}^2$. Suppose that $f(x, y)$ and $g(x, y)$ are integrable.

1. If $f(x, y) \geq 0$ on D, then $\displaystyle\iint_D f(x, y)\, dA \geq 0.$

2. If $f(x, y) \leq g(x, y)$ on D, then $\displaystyle\iint_D f(x, y)\, dA \leq \iint_D g(x, y)\, dA.$

3. If $m \leq f(x, y) \leq M$ on D, then $\displaystyle m \cdot \text{area}(D) \leq \iint_D f(x, y)\, dA \leq M \cdot \text{area}(D).$

11. Polar Coordinates

Let (x, y) be the rectangular coordinates of a point in $\mathbb{R}^2$, and let (r, θ) be the polar coordinates of the same point.

1. $x = r\cos\theta$ and $y = r\sin\theta$.

2. $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$.

12. Double Integrals in Polar Coordinates

Let $f(x, y)$ be a function defined on a closed bounded region D of $\mathbb{R}^2$. Suppose that $f(x, y)$ is continuous. Suppose that the region D is given in polar coordinates by inequalities of the form

$$\alpha \leq \theta \leq \beta$$
$$h_1(\theta) \leq r \leq h_2(\theta).$$

Then

$$\iint_D f(x, y)\, dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta.$$

13. The Triple Integral of a Function over a Box

1. Let $B = [a, b]\times[c, d]\times[r, s]$ be a box in $\mathbb{R}^3$, and let $f\colon B \to \mathbb{R}$ be a function. The integral of f over B is

$$\iiint_B f(x, y, z)\, dV = \lim_{\substack{\max \Delta x_i \to 0 \\ \max \Delta y_j \to 0 \\ \max \Delta z_k \to 0}} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} f(x^*_{ijk}, y^*_{ijk}, z^*_{ijk}) \Delta V_{ijk},$$

provided the limit exists, and is the same, for all choices of Riemann sums. If this limit exists, the function f is **integrable**.

2. Every continuous function is integrable on any box.

14. Iterated Triple Integrals

Let E be a region of $\mathbb{R}^3$ defined by inequalities of the form

$$\begin{gathered} a \leq x \leq b \\ g_1(x) \leq y \leq g_2(x) \\ h_1(x,y) \leq z \leq h_2(x,y), \end{gathered}$$

and let $f\colon E \to \mathbb{R}$ be a function. Suppose that f is continuous.

$$\iiint\limits_E f(x,y,z)\,dV = \int_a^b \int_{g_1(x)}^{g_2(x)} \int_{h_1(x,y)}^{h_2(x,y)} f(x,y,z)\,dz\,dy\,dx.$$

Similarly for any other order of integration.

15. Cylindrical Coordinates

Let (x,y,z) be the rectangular coordinates of a point in $\mathbb{R}^3$, and let (r,θ,z) be the polar coordinates of the same point.

1. $x = r\cos\theta$, and $y = r\sin\theta$ and $z = z$.
2. $r = \sqrt{x^2+y^2}$, and $\tan\theta = \frac{y}{x}$ and $z = z$.

16. Spherical Coordinates

Let (x,y,z) be the rectangular coordinates of a point in $\mathbb{R}^3$, and let (ρ,θ,ϕ) be the polar coordinates of the same point.

1. $x = \rho\sin\phi\cos\theta$ and $y = \rho\sin\phi\sin\theta$ and $z = \rho\cos\phi$.
2. $\rho = \sqrt{x^2+y^2+z^2}$ and $\tan\theta = \frac{y}{x}$ and $\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$.

17. Triple Integrals in Cylindrical Coordinates

Let E be a region of $\mathbb{R}^3$ defined by inequalities of the form

$$\begin{gathered} \alpha \leq \theta \leq \beta \\ g_1(\theta) \leq r \leq g_2(\theta) \\ h_1(r,\theta) \leq z \leq h_2(r,\theta), \end{gathered}$$

and let $f\colon E \to \mathbb{R}$ be a function. Suppose that f is continuous.

$$\iiint\limits_E f(x,y,z)\,dV = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} \int_{h_1(r,\theta)}^{h_2(r,\theta)} f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta.$$

18. Triple Integrals in Spherical Coordinates

Let E be a region of $\mathbb{R}^3$ defined by inequalities of the form

$$\alpha \leq \theta \leq \beta$$
$$\gamma \leq \phi \leq \delta$$
$$h_1(r, \theta) \leq z \leq h_2(r, \theta).$$

and let $f : E \longrightarrow \mathbb{R}$ be a function. Suppose that f is continuous.

$$\iiint_E f(x, y, z)\, dV = \int_\gamma^\delta \int_\alpha^\beta \int_{h_1(\theta,\phi)}^{h_2(\theta,\phi)} f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho \cos\phi)\, \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi.$$