

MATH 261

PROOFS AND FUNDAMENTALS

SPRING 2018

BASIC INFORMATION

Class

- Tue. 3:10-4:30
- Thur. 3:10-4:30
- Hegeman 204

Instructor

- Ethan Bloch
- bloch@bard.edu
- Albee 317
- 758-7266

Office hours

- Mon. 4:00-5:30
- Wed. 2:00-3:30
- Thur. 10:30-12:00
- Or by appointment

Text

- Bloch, Ethan, "Proofs and Fundamentals: A First Course in Abstract Mathematics," 2nd ed., Springer, 2010

Website

- <http://faculty.bard.edu/bloch/math261A.shtml> (includes updated list of assignments)

Errata for Textbook

- http://faculty.bard.edu/bloch/proofs2_errata.pdf

Communications

- Urgent announcements may be sent out via campus email, so make sure you either check your Bard email or have it forwarded to your email address of choice.
-

WHAT IS MATH 261

- This course is an introduction to the methodology of mathematical proof, and to fundamental topics that are used throughout mathematics. Topics for writing proofs include the logic of compound and quantified statements, direct proof, proof by contradiction and mathematical induction. Fundamental mathematical topics include basic set theory, functions, relations and cardinality.
 - The minimal prerequisite is a mathematics course at the level of Calculus II (Math 142); a mathematics course at the level of Elementary Linear Algebra (Math 242) or Intermediate Calculus (Math 245) is preferred.
 - If you are unsure whether Math 261 is an appropriate course for you, please consult with the instructor.
-

COMPUTERS, CALCULATORS AND ELECTRONIC DEVICES

- Calculators and computers are not needed will not be needed during class.
- Use of a computer will be needed for typing the homework in LaTeX, which will be required after spring break, and will be optional before spring break. LaTeX will be discussed in class.
- **Electronic devices, including cell phones, tablets and laptop computers, may not be used during class, other than to read the text.**

WORK FOR THE COURSE

Attendance

- It is expected that students attend all classes.

Homework

- Homework will be assigned at the end of every class. Turn in the homework at the start of the next class. Late assignments will not be accepted, except in genuine emergency situations.
- Prior to spring break, homework may be handed in handwritten. After spring break, all homework must be typed in LaTeX and must use the homework template of the Bard TeX Style file, which is available at the instructor's TeX website. LaTeX will be discussed in class. If you need help with LaTeX, ask the instructor (it can be helpful to bring your laptop to the instructor's office to discuss LaTeX).
- You are encouraged to work with other students in solving the homework problems. However, for the sake of better learning, as well as honesty, please adhere to the following guidelines:
 - › **Write up your solutions yourself.**
 - › **Acknowledge in writing anyone with whom you work and any assistance you receive.**
 - › **Acknowledge in writing any revisions of your work based upon solutions given in class.**
 - › **Do not make use of any solutions found on the web.**
- Failure to indicate collaboration, assistance or sources will be construed as plagiarism.
- The use of homework solutions found on the web or elsewhere will be treated similarly to plagiarism on exams.
- Your solutions should be written clearly and carefully, as described below.

Exams

- Each exam will be in class.

Midterm Exam: Wed., Mar. 15

Final Exam: Mon., May 22

GRADING

- Grades will be determined roughly 50% by the homework assignments and 50% by the exams. Class participation will be taken into account positively, especially in cases of borderline grades.
- Grades will be determined by work completed during the semester, except in cases of medical or personal emergency. There will be no opportunity to do extra credit work after the semester ends.
- This course is graded using letter grades. If you want to take the course Pass/D/Fail, you must submit a request to do so to the Registrar's Office during the Add/Drop period.

OFFICE HOURS

- If you have any problems with the course, or any questions about the material, the assignments, the exams or anything else, please see the instructor about it as soon as possible. If you cannot make any of the scheduled office hours, please make an appointment for some other time. To make an appointment, or to discuss anything, talk to the instructor after class, or send him an email message, or just stop by his office.

ACCOMMODATIONS

- Students with documented learning and/or other disabilities are entitled to receive reasonable classroom and testing accommodations. If you need accommodations, please adhere to the following guidelines:
 - **Discuss your needs with the instructor at the beginning of the semester.**
 - **Provide documentation as appropriate.**
 - **Contact the instructor at least one week prior to each exam or other instance of accommodation.**
 - If you need to miss a class for any reason (sports team, religious holiday, etc.), it is your responsibility to contact the instructor and find out about the material and assignments you missed.
-

RESOURCES

Mathematics Study Room

- The Mathematics Study Room is open Sunday--Thursday, 7pm–10pm, in RKC 111.
 - The Mathematics Study Room is staffed by undergraduate mathematics majors who are available to answer your questions. Not all of the Math Study Room tutors will be prepared to assist you with the material for this course, but some of them will be. You can go to the study room to work on your homework, and then ask for help as needed.
-

IMPORTANT ACADEMIC DATES

Wed., Feb. 7: End of Drop/Add period

Wed., Feb. 28: End of Late Drop; last day to request Pass/Fail

Mon., Mar. 19 — Fri., Mar. 23: Spring break

Mon., Apr. 30 — Tue., May 1: Advising days (no classes)

Tue., May 1: Last day to withdraw from a class

Wed., May. 2: Senior projects due

Tue., May 22: Last day of classes

WRITING PROOFS

- Everyone makes honest mathematical mistakes, but there is no reason to get in your own way by writing your proofs with incomplete sentences and other grammatical mistakes, by using undefined symbols for “variables” or by engaging in other forms of sloppy writing. Mathematics must be written carefully, and with proper grammar, no differently from any other writing.
- This course will offer many opportunities to practice the careful writing of mathematical proofs. Properly written proofs require the writer to observe the following basic points.
 - Write your homework assignments neatly and clearly.
 - Use correct grammar, including full sentences and proper punctuation.
 - Justify each step in a proof, citing the appropriate results from the class notes as needed.
 - Use definitions precisely as stated.
 - Be very careful with quantifiers.
 - Strategize the outline of a proof before working out the details; the outline of a proof is always determined by what is being proved, not by what is known.
 - Distinguish between scratch work and the actual proof; scratch work can be in any order, but the actual proof always starts with what is known and deduces the desired result.
 - Proofs should stand on their own; check your proofs by reading them as if they were written by someone else.

LEARNING TO DO RIGOROUS MATHEMATICS

- Proofs-based mathematics courses are very different from computation-based mathematics courses such as Calculus. The ways you studied, did homework and took exams in computation-based courses was appropriate for those courses, but not for proofs-based courses. Approach proofs-based courses with the idea that you will be doing things differently from what you did in computation-based mathematics courses.
- The material in this course is much more abstract, and requires much more precision in both studying and problem solving, than the material you saw in courses such as Calculus. For some students, a proofs-based course such as this one is the first time that they found a mathematics course really challenging, which can be intimidating at first, but is in fact completely normal. Everyone, including the very best mathematicians, reaches a level of mathematics that he or she finds difficult; what varies from person to person is only what that level is. If you made it this far in mathematics and you only now first encounter substantial difficulty in learning the material, you are doing fine.
- In general, the more advanced you get in mathematics (or any subject), the larger the percentage of learning that takes place outside of class, including from the textbook, from other sources, from office hours, from tutors and from your fellow students (not necessarily in that order).
- In Calculus courses, where the material can mostly be learned in class, reading the textbook is not necessarily very important. By contrast, in proofs-based courses reading the textbook carefully, and seeking help with those parts of the textbooks that you find difficult, is crucial.
- In proofs-based courses, reading the textbook is very different from reading fiction, in two ways. First, reading proofs-based mathematics, which cannot be done without pencil and paper in hand, requires active engagement by regularly stopping to work out the details of what is written. Make sure you know why each step in a proof is true before moving on, and if you are unable to figure out one or more steps of a proof, seek help. Second, mathematics is not read in order from beginning to end, but “from the outside in.” When you read a proof, start by looking for the overall idea of the proof, and then figure out the strategy that is being used, and only then go through the details one step at a time.
- In Calculus courses, solutions to homework exercises are usually written as a collections of equations, with little or no words explaining the solution. By contrast, rigorous proofs are, fundamentally, convincing arguments, and to make a good argument, words are needed to direct the logical flow of the ideas; to explain what is assumed and what is to be proved; and to state what previous results are used. In particular, rigorous proofs are written using full sentences, and with correct grammar and punctuation, because doing so helps make the arguments more clear and precise.
- In Calculus courses, solutions to homework problems are usually written directly, with little revision. By contrast, rigorous proofs should be written the same way a paper in a humanities course is written, by first making an outline (often called “strategizing a proof”); then sketching out a rough draft; then revising the draft repeatedly until the proof works; and, lastly, writing the final draft carefully and typing it in LaTeX.
- Revising a draft of a rigorous proof should be done exactly as revising a draft of a paper in a humanities course, which is to read it as if you are not the author, but rather as if you are someone else in the class, and making sure that each sentence makes sense as written, without recourse to unwritten explanations.
- Learning to write rigorous proofs takes time, and you should not expect to master it instantly.
- **A very good way to improve your skill at writing proofs, and to do as well as possible on the homework in this course, is to bring a draft of every homework assignment to office hours before you write up and submit the final draft.**