Math 332
Abstract Algebra
Spring 2023

What is Abstract Algebra

COURSE CONTENT

- This course is an introduction to modern abstract algebraic systems, specifically groups, rings and fields. The focus of the course is a rigorous treatment of the basic theory of groups (subgroups, quotient groups, homomorphisms, isomorphisms, group actions), and an introduction to rings and fields (ideals, polynomials, factorization).
- This course provides an opportunity for students to develop their skills at formulating and writing rigorous mathematical proofs, and it makes use of the methods and concepts of Proofs and Fundamentals (Math 261).

PREREQUISITES

- The prerequisite is Proofs and Fundamentals (Math 261).
- If you are unsure whether Math 332 is an appropriate course for you, please speak with the instructor.

Basic Information

CLASS

- Tuesday: 1:30–2:50
- Thursday: 1:30–2:50
- Hegeman 308

INSTRUCTOR

- Instructor: Ethan Bloch
- Office: Albee 317
- Phone: (845) 758-7266
- Email: bloch "at" bard "dot" edu
- Website: `http://faculty.bard.edu/bloch/`

Exams

EXAM DATES

- Midterm Exam:
  In Class, *Closed Book*
  Thur., 16 Mar.
- Final Exam:
  Take Home, *Open Book*
  Posted: Thur., 18 May
  Due: Tue., 23 May
- Office hours, no class:
  Thur., 18 May
  Tue., 23 May

Grades

GRADING

- Grades will be determined roughly 50% by the homework assignments and 50% by the exams. Class participation will be taken into account positively, especially in cases of borderline grades.
- Grades will be determined by work completed during the semester, except in cases of medical or personal emergency. There will be no opportunity to do extra credit work after the semester ends.

PASS/FAIL

- This course is graded using letter grades. If you want to take the course Pass/Fail, you must submit a request to do so to the Registrar's Office by the end of the Late Drop Period.

Google Classroom & Email

GOOGLE CLASSROOM

- All the needed information for this class will be available at the Google Classroom site for this class, **on the Classwork page**. You will be invited to join this Google Classroom site at the start of the semester.
- Summary notes will be available at the Google Classroom site.
- All homework assignments will be posted, and submitted, at the Google Classroom site.

EMAIL

- Urgent announcements may be sent out via campus email, so make sure you either check your Bard email regularly or have your Bard email forwarded to the email address of your choice.

Calculators & Electronics

COMPUTERS AND CALCULATORS

- Use of a computer will be needed for typing the homework in LaTeX, which will be required for all homework assignments, as discussed in class.
- Calculators are not needed for this class.

ELECTRONIC DEVICES DURING CLASS

- Electronic devices, including cell phones, tablets and laptop computers, may be used during class only for reasons related to the class, for example as calculators, to take notes or to read the text.
- **Texting, messaging and using social media is not allowed during class.**

Textbook

SUMMARY NOTES

- A set of summary notes with definitions, theorems and exercises is available at the Google Classroom site, **on the Classwork page**, under the heading Materials.

TEXTBOOK

- There is no single required textbook for this course, but the following textbooks are recommended.
- The first two of the following textbooks are the best, but only if they can be found for free or very inexpensively.
- The third textbook is not quite as good as the first two, but is officially free.
- The summary notes are keyed to the corresponding sections of these three texts.
  - Fraleigh, John, A First Course in Abstract Algebra, 7th ed., Addison- Wesley, Reading, MA, 2003.
  - Gallian, Joseph A., Contemporary Abstract Algebra, 7th ed., Brooks/Cole, Belmont, CA, 2010.
  - Judson, Thomas, Abstract Algebra: Theory and Applications (free PDF, at `http://abstract.ups.edu/download/aata-20190710.pdf`).

Office Hours

HELP OUTSIDE OF CLASS

- If you have any problems with the course, or any questions about the material, the homework assignments, the exams or anything else, please see the instructor about it as soon as possible. If you cannot make any of the scheduled office hours, please make an appointment for some other time. To make an appointment, or to discuss anything, talk to the instructor after class, or send him an email message, or just stop by his office.

MASKS DURING OFFICE HOURS

- **Masks are required for office hours** (but are optional for class.)

TIMES—TENTATIVE

- Monday: 4:00 pm – 6:00 pm
- Tuesday: 5:00 pm – 6:00 pm
- Wednesday: 5:00 pm – 6:00 pm
- Thursday: 5:00 pm – 6:00 pm
- Or by appointment

Accommodations

ACCOMMODATIONS

- Students with documented learning and/or other disabilities are entitled to receive reasonable classroom and testing accommodations.
- If you need accommodations, please do the following.
  - Contact the Office of Disability Support Services, who will work with you and will provide documentation to the instructor.
  - Contact the instructor at least one week prior to each exam, quiz or other instance of accommodation, to arrange appropriate scheduling.
  - If you feel comfortable doing so, discuss your accommodations with the instructor at the beginning of the semester.

MISSING CLASS

- If you need to miss a class for any reason (sports team, religious holiday, etc.), it is your responsibility to contact the instructor and find out about the material and assignments you missed.
- Travel plans for spring break and the end of the semester must take into account the dates of the exams.

Covid

CLASS

- Masks are **optional** during class.

OFFICE HOURS

- Masks are **required** for office hours.

Important Dates

- **Wed., 8 Feb.:** End of Drop/Add period
- **Wed., 1 Mar.:** End of Late Drop and Pass/Fail
- **Fri., 17 Mar.:** Fall moderation papers due
- **Mon., 20 Mar.–Fri., 24 Mar.:** Spring break
- **Mon., 1 May–Tue., 2 May:** Advising days (no classes)
- **Wed., 3 May:** Last day to withdraw from a class
- **Wed., 3 May:** Spring senior projects due
- **Thur., 11 May:** Registration for fall classes
- **Tue., 23 May:** Last day of classes

Homework

DUE DATE FOR HOMEWORK

- Homework will be assigned every class, and is due by the start of the following class.
- Late assignments will not be accepted, except in emergency situations.

SUBMITTING HOMEWORK

- Homework assignments will be posted at the class Google Classroom site, **on the Classwork page**, at the location corresponding to the date on which it was assigned.
- The exercise numbers that are posted for homework assignments refer to exercises in the Notes for this class (which are available the the class Google Classroom site); these exercise numbers do NOT refer to the textbook.
- Submit the theory homework via Google Classroom, **on the Classwork page**, at the location corresponding to the date on which it was assigned.

FORMATTING HOMEWORK

- Every homework assignment must be typed in LaTeX, and must use the homework template of the Bard TEXStyle file, which is available at the instructor's TEXwebsite. If you need help with LaTeX in general, or the homework template in particular, ask the instructor.
- Every homework assignment must be uploaded as a SINGLE PDF FILE to the Google Classroom. No other format (for example, a shared file on Overleaf) will be accepted.

DOING HOMEWORK

- You are encouraged to work with other students in solving the homework problems. However, for the sake of better learning, as well as honesty, please adhere to the following guidelines:
  - Write up your solutions yourself.
  - Acknowledge in writing anyone with whom you work and any assistance you receive.
  - Acknowledge in writing any revisions of your work based upon solutions given in class.
- Failure to indicate collaboration, assistance or sources will be construed as plagiarism.

WRITING PROOFS

- Everyone makes honest mathematical mistakes, but there is no reason to get in your own way by writing your proofs with incomplete sentences and other grammatical mistakes, by using undefined symbols for "variables" or by engaging in other forms of sloppy writing. The goal of writing mathematics is two-fold: making sure that a proof is correct in all details, and communicating the proof so that others can understand it. To help achieve those aims, mathematics must be written carefully, and with proper grammar, no differently from any other writing. Properly written proofs entail the following basic points.
  - Write your homework assignments neatly and clearly.
  - Use correct grammar, including full sentences and proper punctuation.
  - Justify each step in a proof, citing the appropriate results from the textbook as needed.
  - Use definitions precisely as stated.
  - Be very careful with quantifiers.
  - Strategize the outline of a proof before working out the details; the outline of a proof is always determined by what is being proved, not by what is known.
  - Distinguish between scratch work and the actual proof; scratch work can be in any order, but the actual proof always starts with what is known and deduces the desired result.
  - Proofs should stand on their own; check your proofs by reading them as if you were not the author.

Learning To Do Rigorous Mathematics

IT IS NOT THE SAME AS CALCULUS

- Proofs-based mathematics courses are very different from computation-based mathematics courses such as Calculus. The ways you studied, did homework and took exams in computation-based courses was appropriate for those courses, but not for proofs-based courses. Approach proofs-based courses with the idea that you will be doing things differently from what you did in computation-based mathematics courses.
- The material in this course is much more abstract, and requires much more precision in both studying and problem solving, than the material you saw in courses such as Calculus. For some students, a proofs-based course such as this one is the first time that they found a mathematics course really challenging, which can be intimidating at first, but is in fact completely normal. Everyone, including the very best mathematicians, reaches a level of mathematics that is a challenge; what varies from person to person is only what that level is. If you made it this far in mathematics and you only now first encounter substantial difficulty in learning the material, you are doing fine.
- In general, the more advanced you get in mathematics (or any subject), the larger the percentage of learning that takes place outside of class, including from the textbook, from other sources, from office hours, from tutors and from your fellow students (not necessarily in that order).
- In Calculus courses, where the material can mostly be learned in class, reading the textbook is not necessarily very important. By contrast, in proofs-based courses reading the textbook carefully, and seeking help with those parts of the textbooks that you find difficult, is crucial.
- In proofs-based courses, reading the textbook is very different from reading fiction, in two ways. First, reading proofs-based mathematics, which cannot be done without pencil and paper in hand, requires active engagement by regularly stopping to work out the details of what is written. Make sure you know why each step in a proof is true before moving on, and if you are unable to figure out one or more steps of a proof, seek help. Second, mathematics is not read in order from beginning to end, but "from the outside in." When you read a proof, start by looking for the overall idea of the proof, and then figure out the strategy that is being used, and only then go through the details one step at a time.
- In Calculus courses, solutions to homework exercises are usually written as a collections of equations, with little or no words explaining the solution. By contrast, rigorous proofs are, fundamentally, convincing arguments, and to make a good argument, words are needed to direct the logical flow of the ideas; to explain what is assumed and what is to be proved; and to state what previous results are used. In particular, rigorous proofs are written using full sentences, and with correct grammar and punctuation, because doing so helps make the arguments more clear and precise.

REVISE AND REVISE

- In Calculus courses, solutions to homework problems are usually written directly, with little revision. By contrast, rigorous proofs should be written the same way a paper in a humanities course is written, by first making an outline (sometimes called "strategizing a proof"); then sketching out a rough draft; then revising the draft repeatedly until the proof works; and, lastly, writing the final draft carefully (and then often typing it in LaTeX).
- Revising a draft of a rigorous proof should be done exactly as revising a draft of a paper in a humanities course, which is to read it as if you are not the author, but rather as if you are someone else in the class, and making sure that each sentence makes sense as written, without recourse to unwritten explanations.

PRACTICE AND GET FEEDBACK

- Learning to write rigorous proofs takes time, and you should not expect to master it instantly.
- A very good way to improve your skill at writing proofs, and to do as well as possible on the homework in this course, is to bring a draft of every homework assignment to office hours before you write up and submit the final draft.

Diversity, Equity, and Inclusion

BARD NOTICE OF NONDISCRIMINATION

- Bard College is committed to ensuring equal access to its educational programs and equal employment without regard to an individual's sex, gender, race, color, national origin, religion, age, disability, gender identity, sexual orientation, predisposing genetic characteristics, marital status, veteran status, military status, domestic violence victim status, ex-offender status, or any other characteristic protected by federal, state, or local law. Students, employees, applicants, and other members of the Bard College community (including, but not limited to, vendors, visitors, and guests) shall not be subject to discrimination or harassment prohibited by law or otherwise treated adversely based upon a protected characteristic. Similarly, the College will not tolerate harassing, violent, intimidating, or discriminatory conduct by its students, employees, or any other member of, or visitor to, the College community. This includes, without limitation, sexual harassment, sexual assault, sexual violence, dating violence, and domestic violence. (From https://www.bard.edu/dei/policies/ .)

REPORTING A BIAS INCIDENT OR HATE CRIME

- Bard College strongly encourages the reporting of all bias incidents and hate crimes that occur on campus, at college-sponsored events, or activities occurring off campus. If you feel that you have been the victim of a bias incident or hate crime, or you believe one has occurred, you are strongly encouraged to report it as quickly as possible. (From https://www.bard.edu/dei/policies/ ; see that site for how to report a bias incident or hate crime at Bard.)

LAND ACKNOWLEDGMENT AND SLAVERY ACKNOWLEDGMENT

- See https://www.bard.edu/dei/resources/ for the Land Acknowledgment for the Bard College Campus in Annandale-on-Hudson and the Slavery Acknowledgment for Bard College.

DIVERSITY, EQUITY, AND INCLUSION IN MATHEMATICS

- The discipline of mathematics – which has its roots in many diverse cultures across the ancient world and is to this day studied and used universally – has had a long history of excluding many people on the basis of race, gender, sexual orientation, religion, class and more. The Mathematics Program at Bard College is committed to joining the broader efforts in the world of mathematics aimed at making our field be not only open but positively welcoming to all who want to study our beautiful and useful subject. Denying access to a good education in mathematics to some categories of people is an unfair obstacle to their intellectual growth and job opportunities, and causes the field of mathematics to miss out on the broad input into our discipline that a diverse population brings.
- Simple statistics from the U.S. show the field of mathematics is not equally accessible to everyone. In 2017, women were 50.7% of the US population; however, only 25% of all U.S. citizens who earned Ph.D.s in Mathematics during the 2017/18 academic year were women. Similarly, Black people make up 13% of the US population but account for only 2.9% of the Ph.D.s earned by U.S. citizens in Mathematics, and 17.6% of the U.S. population is Latinx while accounting for just 3.6% of the Ph.D.s. The underrepresentation that we see in mathematics can only be explained by systemic inequalities such as structural racism and misogyny.
- The mathematics faculty at Bard are committed to sharing our love of mathematics with future generations and helping to ensure the continued growth of the field. We recognize that the members of the Bard community do not all share the same privileges, resources, time and educational background, but we are firm in our knowledge that everyone is capable of succeeding at mathematics, regardless of how they may self-identify or be identified by others, and that everyone deserves to have a positive experience with mathematics, regardless of the discrimination and discouragement they may have previously faced.
- The Mathematics Program at Bard is committed to creating a welcoming, inclusive, and equitable environment with the goal of having our program better represent our broader community. We seek input from our students, both inside and outside the Mathematics Program, as well as from the larger community to help us ensure we include voices and perspectives that have in the past been missing from our discipline.