SL(2,C) Chern-Simons Theory and Quantum Gravity with a Cosmological Constant

Hal Haggard
Bard College
In collaboration with Muxin Han, Wojciech Kamiński, Aldo Riello

April 13th, 2015

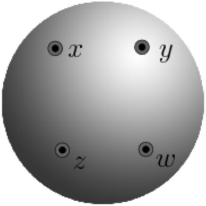
APS April Meeting

hep-th/1412.7546

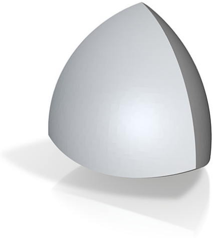
Successful inclusion of a positive cosmological constant in quantum gravity.

Interesting exchange of technical tools between loop gravity and string theory—although not yet of any underlying physics.

Study of flat connections on a Riemann surface

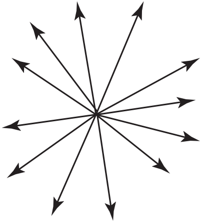


↕

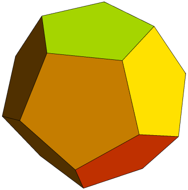


Discrete geometries in spaces of constant curvature

In 1897 H. Minkowski proved



$\Leftrightarrow$

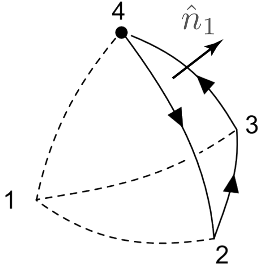


$\vec{A}_1 + \cdots + \vec{A}_N = 0,$ with $\vec{A}_i = A_i \hat{n}_i,$ $A_i =$ area face i
$\hat{n} = \perp$ to face i

Interpreting these area vectors as angular momenta $[\in \mathfrak{su}(2)]$ turns convex polyhedra (discrete geometries) into dynamical systems

We have recently proven

$$O_4 O_3 O_2 O_1 = \mathbb{1} \qquad \Longleftrightarrow$$



Here

$$O = \exp\left(\frac{a}{R^2}\,\hat{n}\cdot\vec{J}\right), \quad O \in SO(3)$$

The closure relation is the automatic homotopy constraint

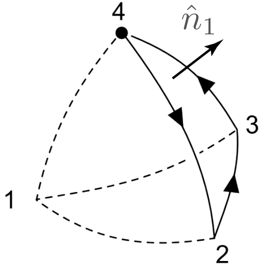
Two immediate checks:

■ For $R \to \infty$

$$O_4 O_3 O_2 O_1 = \mathbb{1} + R^{-2}(a_1 \hat{n}_1 + a_2 \hat{n}_2 + a_3 \hat{n}_3 + a_4 \hat{n}_4) \cdot \vec{J} + \cdots = \mathbb{1}$$

and we recover the flat space Minkowski theorem.

♦ Normals make sense because the faces are flatly embedded (completely geodesic).



The full generalization leads to a new kind of Gram matrix:

$$\text{Gram} = \begin{pmatrix} 1 & \hat{n}_1 \cdot \hat{n}_2 & \hat{n}_1 \cdot \hat{n}_3 & \hat{n}_1 \cdot \hat{n}_4 \\ * & 1 & \hat{n}_2 \cdot \hat{n}_3 & \hat{n}_2 \cdot \mathbf{O}_1 \hat{n}_4 \\ * & * & 1 & \hat{n}_3 \cdot \hat{n}_4 \\ \text{sym} & * & * & 1 \end{pmatrix}$$

The partition function for wilson lines supported on graph Γ is

$$Z(\psi_\Gamma) := \int \mathcal{D}B\mathcal{D}\mathcal{A}\, \mathrm{e}^{\frac{\mathrm{i}}{2\ell_P^2}\int B\wedge\mathcal{F}[\mathcal{A}] - \frac{\Lambda}{6}B\wedge B}\, (f_\gamma\psi_\Gamma)(G[\mathcal{A}])$$

$$= \int \mathcal{D}\mathcal{A}\, \mathrm{e}^{\frac{3\mathrm{i}}{4\Lambda\ell_P^2}\int \mathcal{F}[\mathcal{A}]\wedge\mathcal{F}[\mathcal{A}]}\, (f_\gamma\psi_\Gamma)(G[\mathcal{A}])$$

$$= \int \mathcal{D}\mathcal{A}\, \mathrm{e}^{\frac{3\pi\mathrm{i}}{\Lambda\ell_P^2}\mathsf{CS}[\mathcal{A}]}\, (f_\gamma\psi_\Gamma)(G[\mathcal{A}])$$

where the Chern-Simons functional is

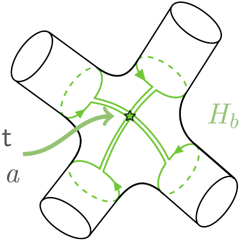
$$\mathsf{CS}[\mathcal{A}] := \frac{1}{4\pi}\oint_{S^3} \mathrm{d}\mathcal{A}\wedge\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}$$

For boundary connection functionals, ΛBF in the bulk is equivalent to CS on the boundary

There are two types of holonomies in $S^3 \setminus \Gamma$:

- transverse $H_b(a)$
- longitudinal G_{ba}

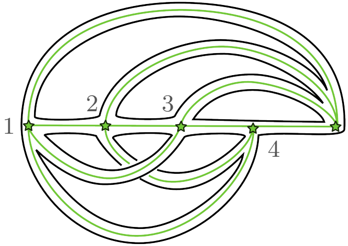
where $a, b, \ldots$ label the graph vertices



vertex a

We need to specify the exact paths
i.e. make a choice of framing for Γ

longitudinal paths
run on the
'top' of the tubes



'Top view' of the
tubular neighborhood

WKB approx. of $Z^{(\alpha)}(S^3 \setminus \Gamma_5|u) \rightsquigarrow$ discrete 4D Einstein gravity

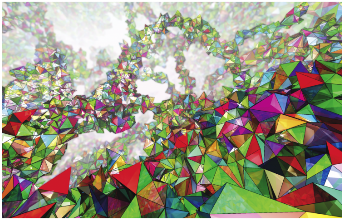
That is, in the semiclassical, $\hbar \to 0$ limit

$$Z^{(\alpha)}(S^3 \setminus \Gamma_5|u) \sim \exp\left[\frac{i}{\hbar} S^{\Lambda}_{\mathsf{Regge}} + o(\log \hbar)\right]$$

where

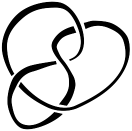
$$S^{\Lambda}_{\mathsf{Regge}} = \sum_{a<b} \mathbf{a}_{ab} \Theta^{\Lambda}_{ab} - \Lambda\, V^{\Lambda}_4$$

■ A great promise of the exchange with string theory is a new approach to the continuum limit of discrete geometries.



♦ Conjecture: the curved Minkowski theorem holds in general $\rightsquigarrow$ study of flat connections on Riemann surfaces closely related to study of discrete, curved polyhedra.

♣ Provide an enriched context for understanding the role of quantum groups in cosmological spacetimes.



Λ