The Black Hole Spin Puzzle

Physics Friday

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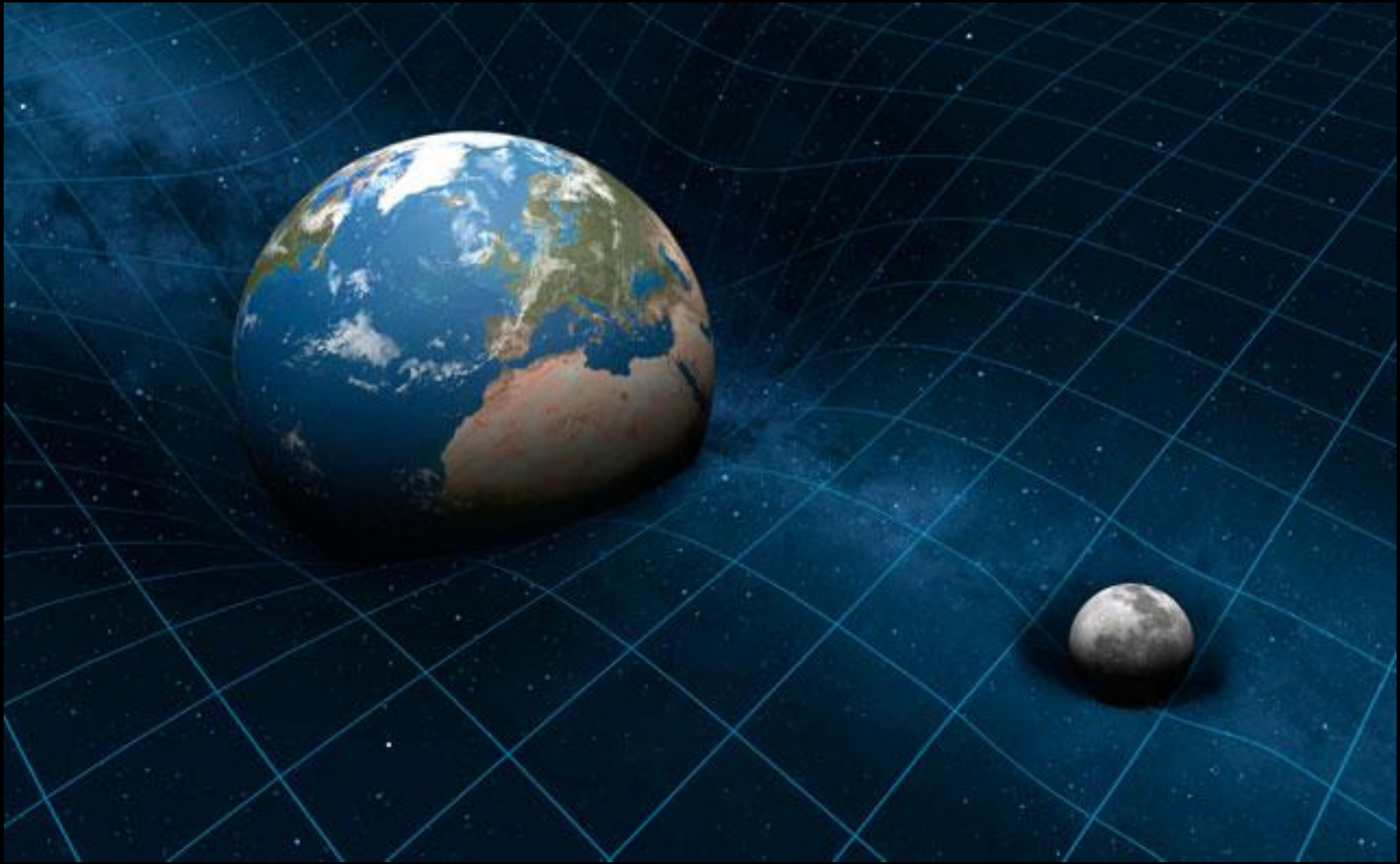
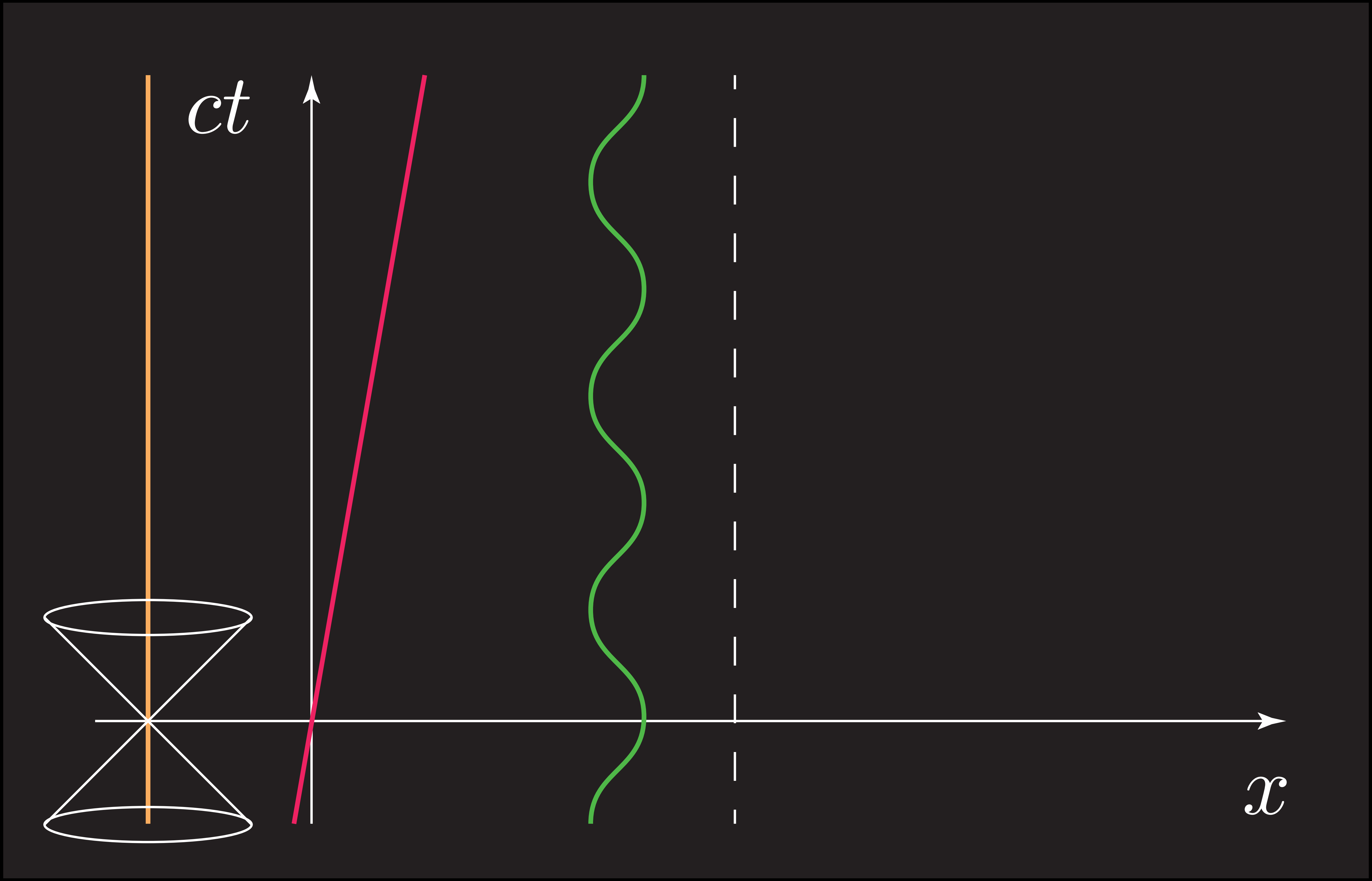
February 28th, 2020





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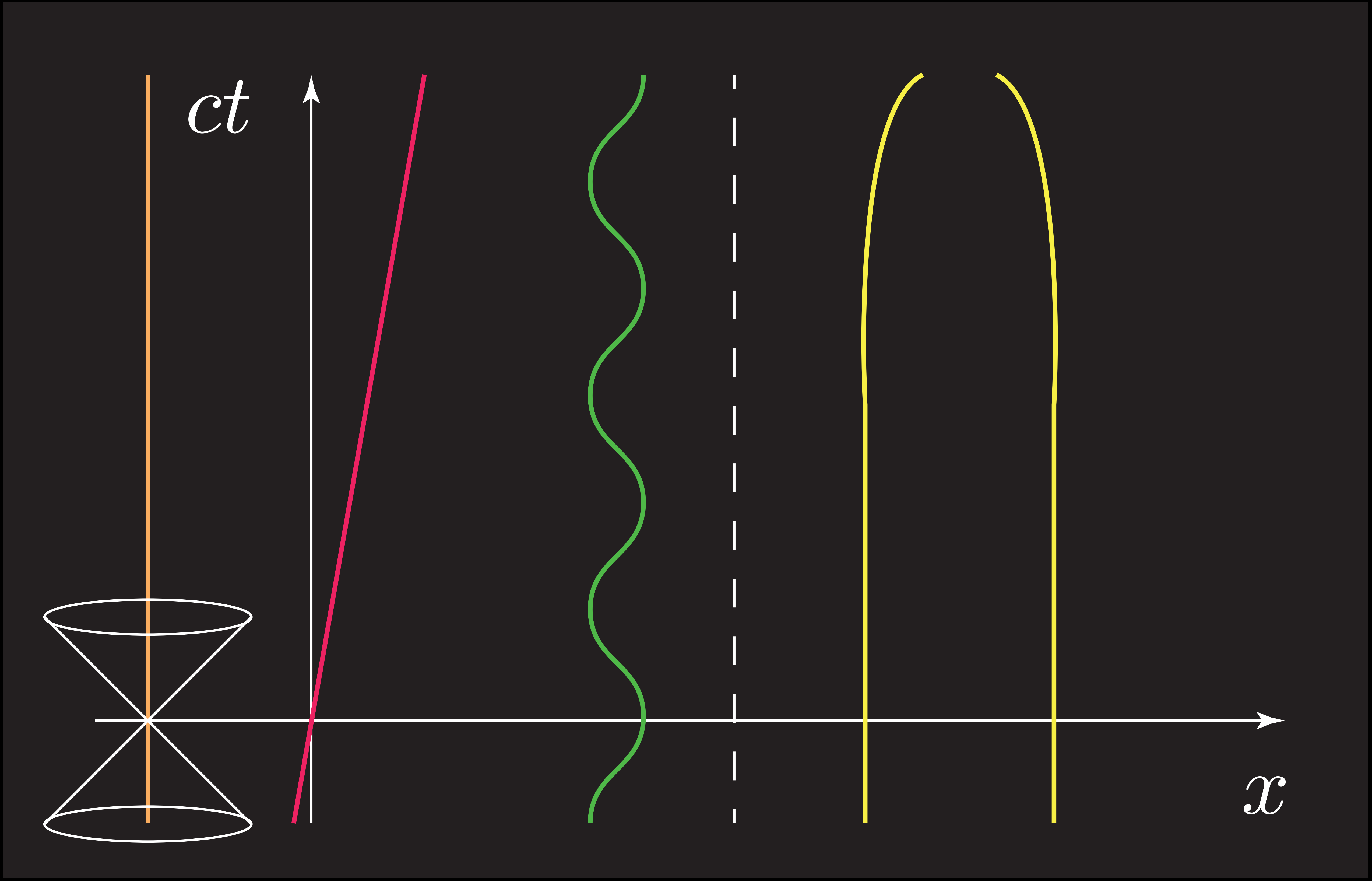

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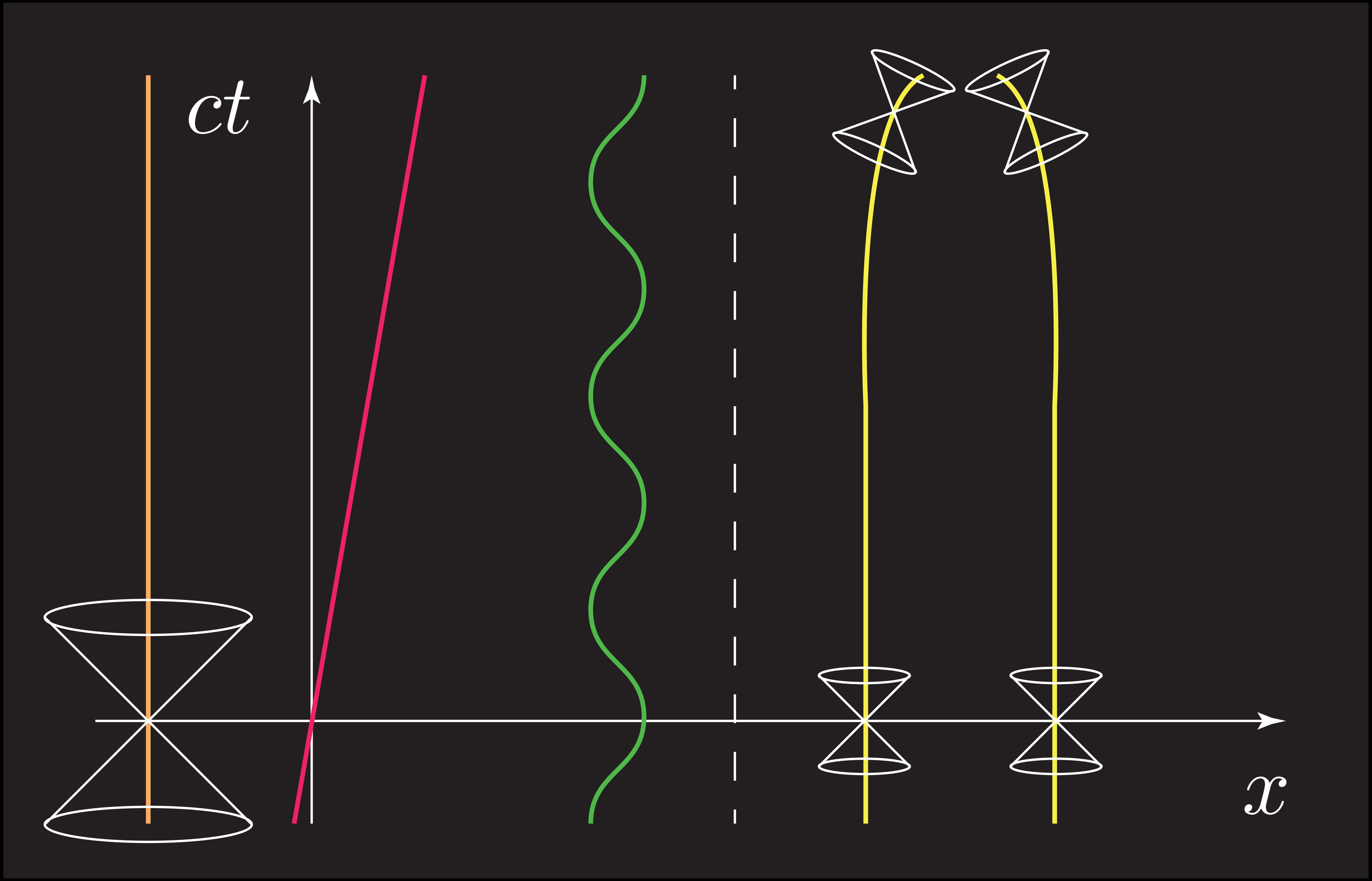
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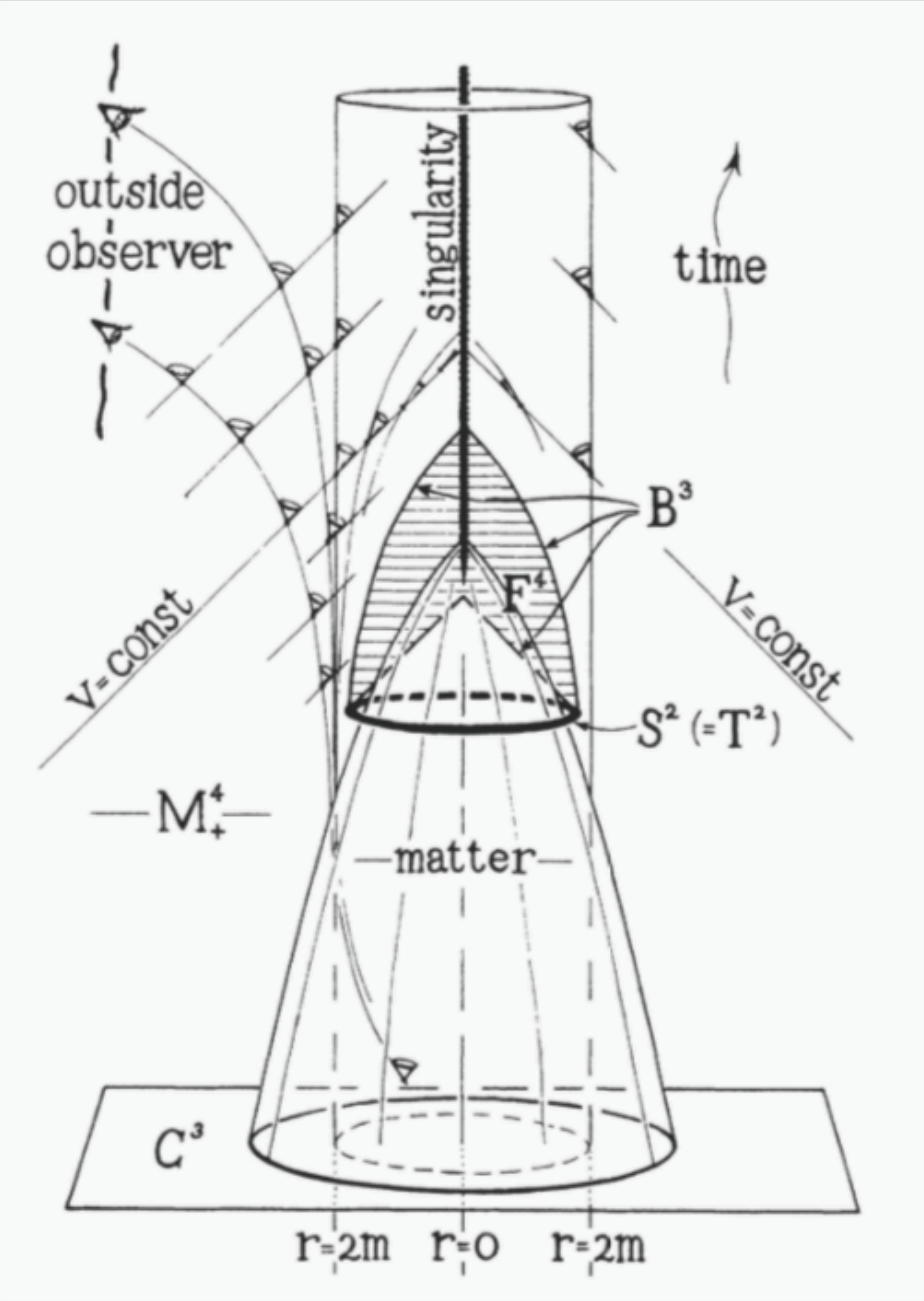
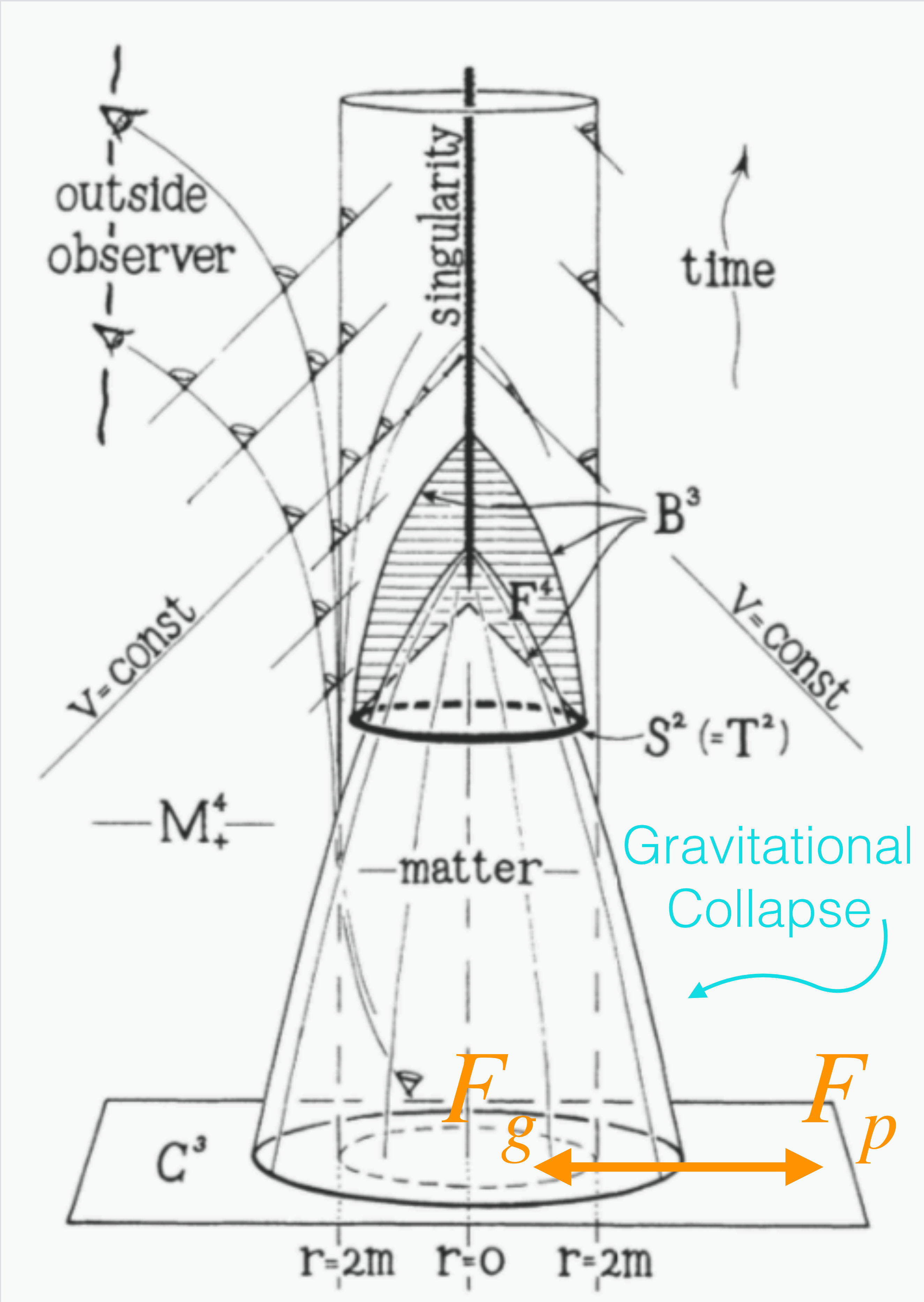


The event horizon is "a perfect unidirectional membrane: causal influences can cross it in only one direction”.
—D. Finkelstein 1958



Why do black holes form?

The non-linearity of gravity leads to all sorts of wonderful instabilities.

Gravity for shell of mass m:

$$F_g = -G\frac{Mm}{r^2}$$

Pressure on this shell:

$$F_p = pA$$

And (!):

$$m = m_o + \frac{pV}{c^2}$$

W

$$F_g = F_p \rightsquigarrow r \lesssim \frac{2GM}{c^2}$$

What are the spins of black holes that form via collapse?

$$0 \leq a = \frac{|\vec{S}|}{GM^2/c} \leq 1$$

Table 1 The masses and spins, measured via continuum-fitting, of ten stellar black holes[a].

System	a_*	$M/M_\odot$	References
Persistent			
Cyg X-1	> 0.95	14.8 ± 1.0	Gou et al. 2011; Orosz et al. 2011a
LMC X-1	$0.92^{+0.05}_{-0.07}$	10.9 ± 1.4	Gou et al. 2009; Orosz et al. 2009
M33 X-7	0.84 ± 0.05	15.65 ± 1.45	Liu et al. 2008; Orosz et al. 2007
Transient			
GRS 1915+105	$> 0.95^b$	10.1 ± 0.6	McClintock et al. 2006; Steeghs et al. 2013
4U 1543–47	0.80 ± 0.10^b	9.4 ± 1.0	Shafee et al. 2006; Orosz 2003
GRO J1655–40	0.70 ± 0.10^b	6.3 ± 0.5	Shafee et al. 2006; Greene et al. 2001
XTE J1550–564	$0.34^{+0.20}_{-0.28}$	9.1 ± 0.6	Steiner et al. 2011; Orosz et al. 2011b
H1743–322	0.2 ± 0.3	$\sim 8^c$	Steiner et al. 2012a
LMC X-3	$< 0.3^d$	7.6 ± 1.6	Davis et al. 2006; Orosz 2003
A0620–00	0.12 ± 0.19	6.6 ± 0.25	Gou et al. 2010; Cantrell et al. 2010

Outline

I. Don't all Black Holes Spin?

II. Stars Collapse, but The Universe Expands

III. Boxing in Black Holes

Binary Black Holes
in
Gravitational Waves

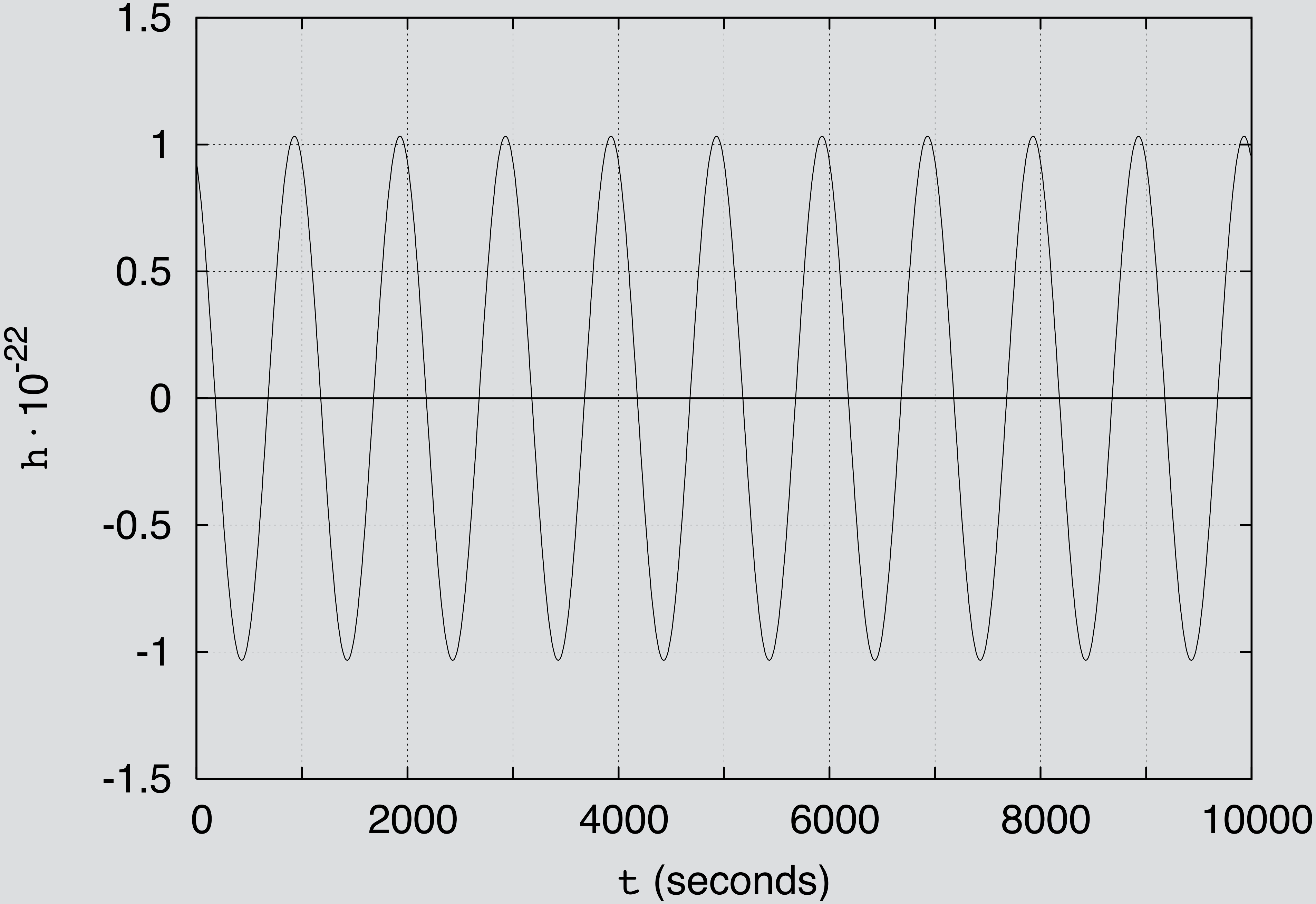
Kepler's 3rd Law

$$P^3 = \frac{4\pi^2 R^2}{G(M_1 + M_2)}$$

When the wave carries energy off,
the black holes get closer.

Smaller $R \rightsquigarrow$ smaller P

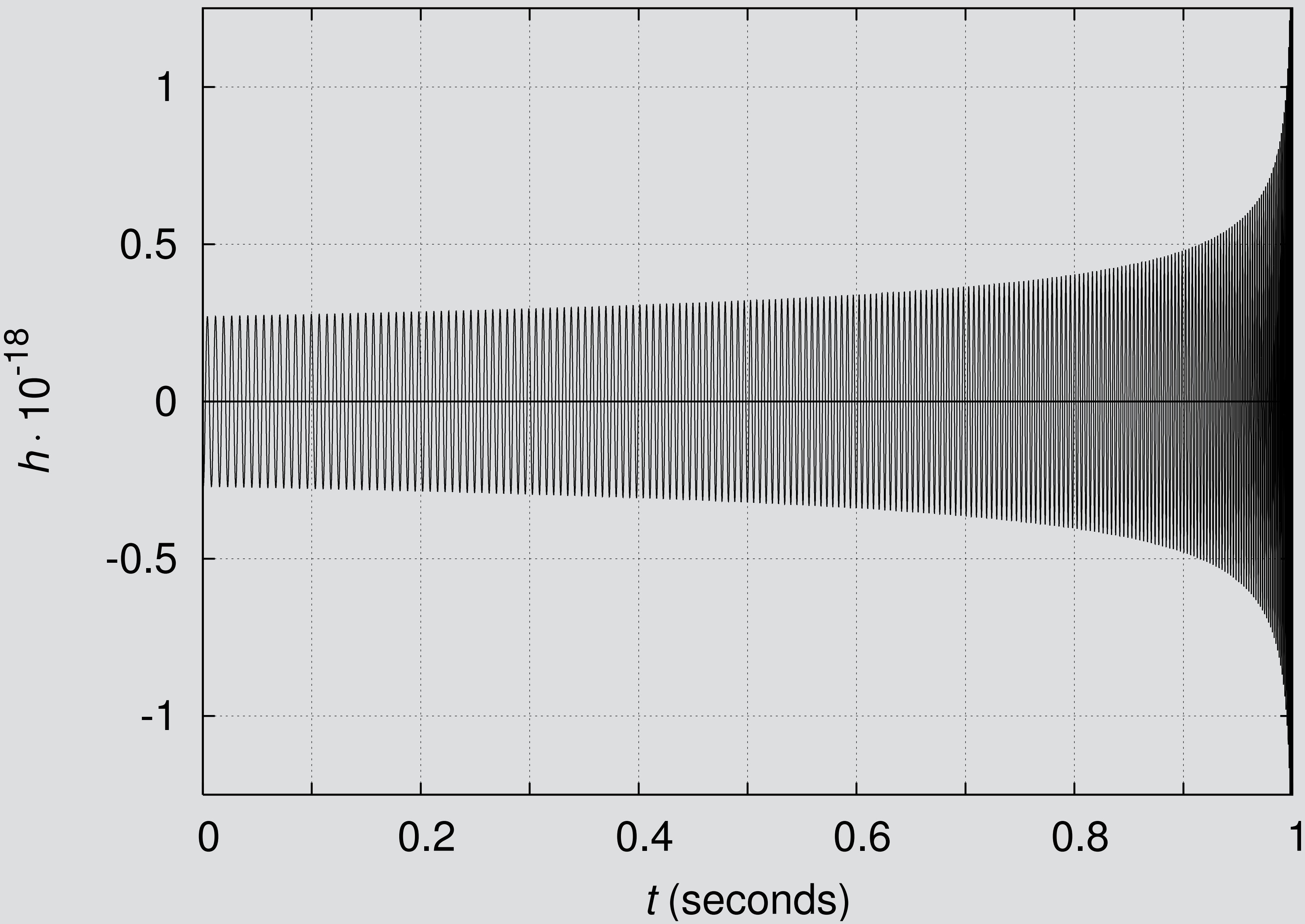
Far from the binary system...



$$h(t) = \mathscr{A}(t)\cos(\Phi(t))$$

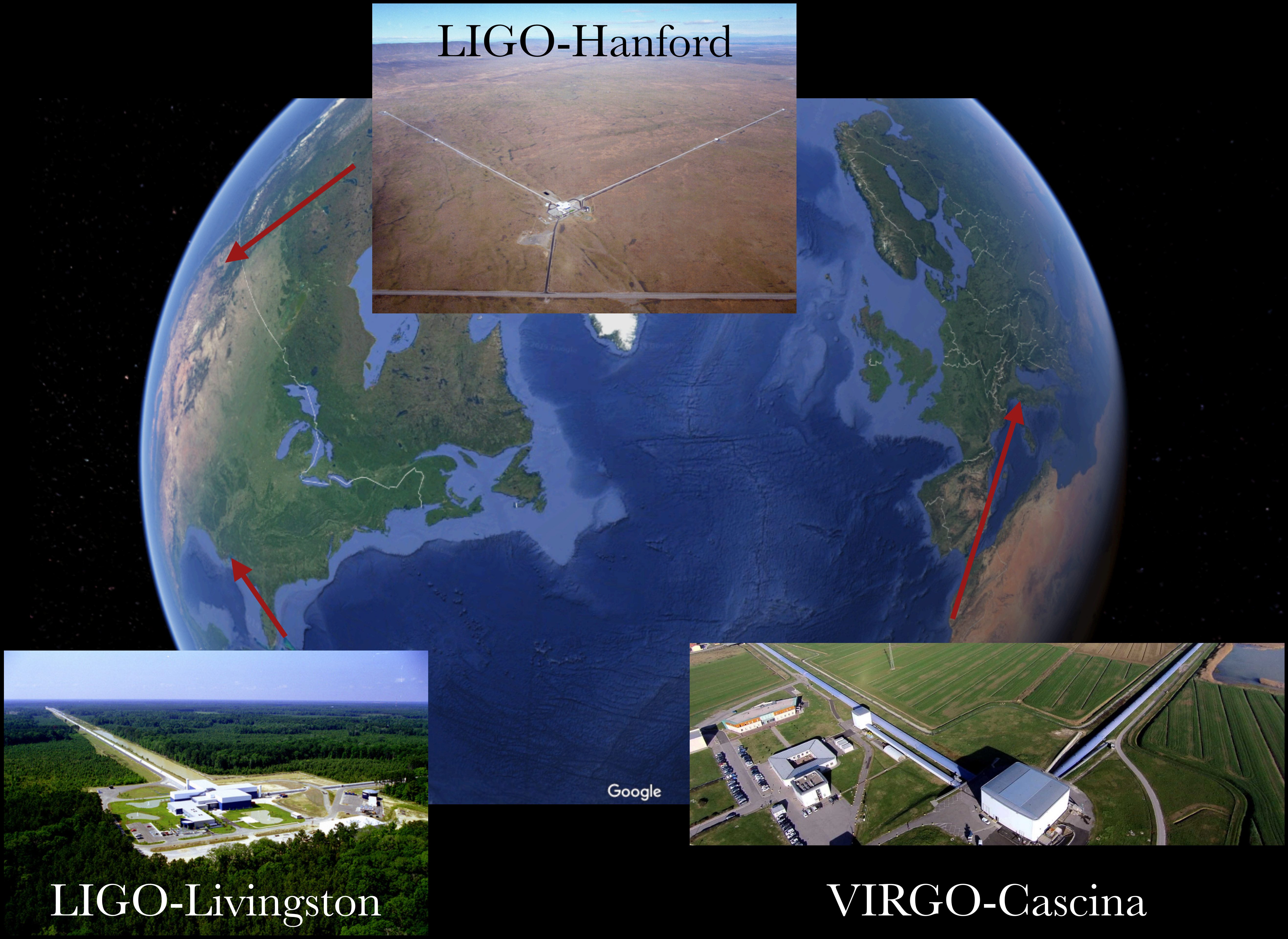
…you get a wave pattern that repeats with period P_{gw}.

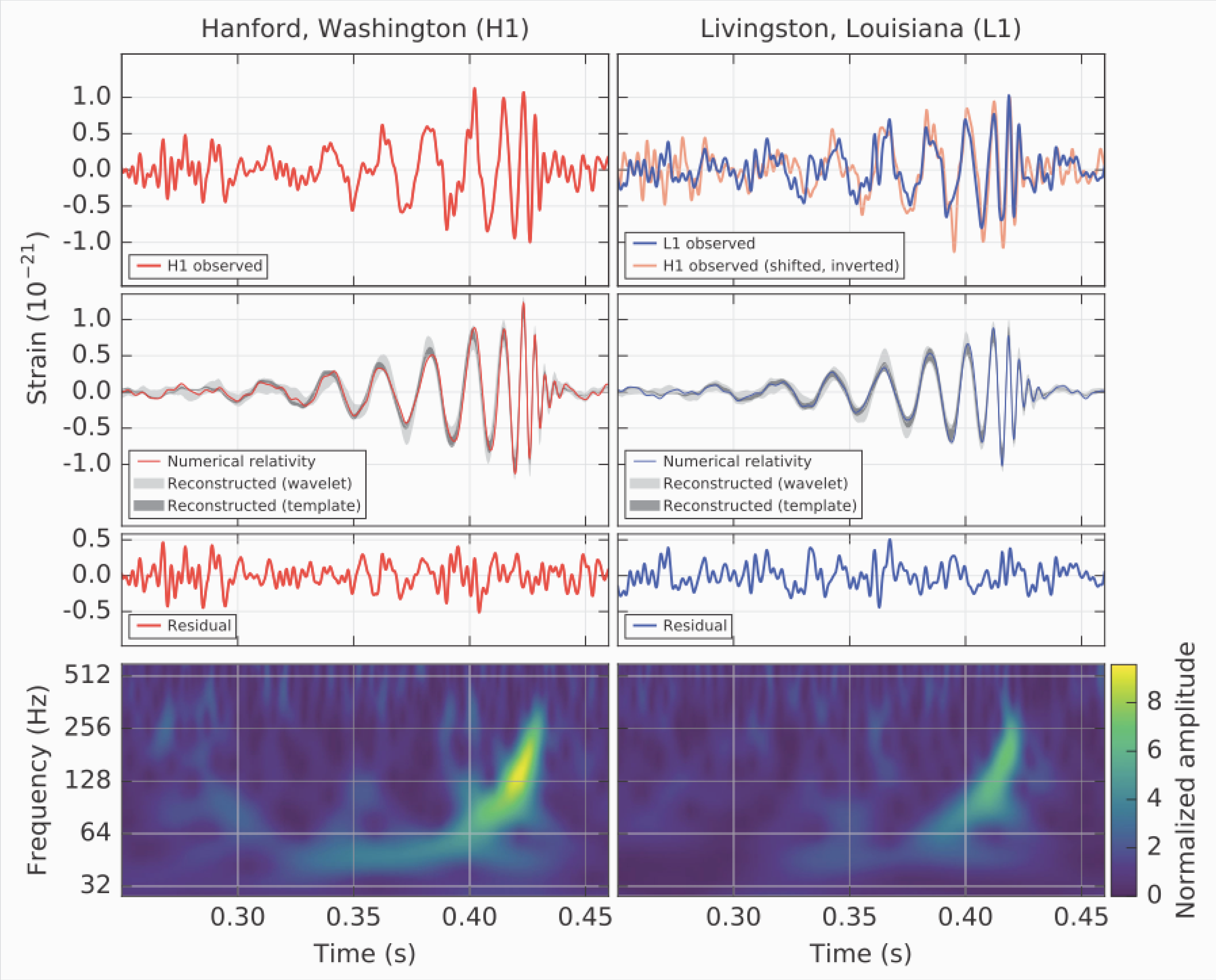
The wave begins to chirp

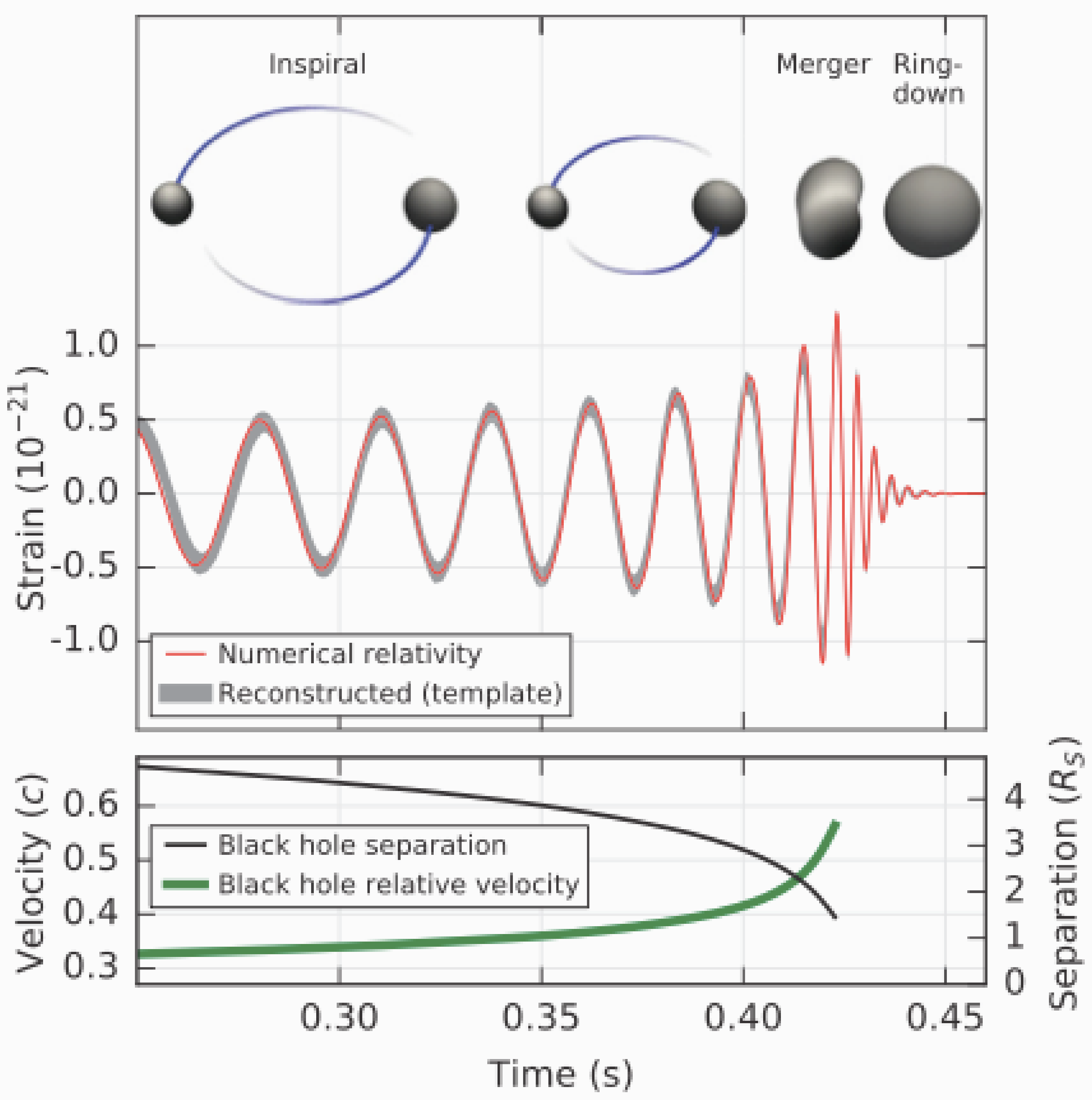


$$h(t) = \mathscr{A}(t)\cos(\Phi(t))$$

$$\mathscr{A}(t) = \frac{2(G\mathscr{M})^{5/3}}{c^4 r}\left(\frac{\pi}{P_{gw}(t)}\right)^{2/3} \quad \text{and} \quad \mathscr{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$


LIGO-Hanford
LIGO-Livingston
VIRGO-Cascina
Google


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (s)
Normalized amplitude
0
2
4
6
8



Inspiral
Merger
Ring-down
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
Numerical relativity
Reconstructed (template)
Velocity (c)
0.6
0.5
0.4
0.3
Black hole separation
Black hole relative velocity
Separation (R_S)
4
3
2
1
0
0.30
0.35
0.40
0.45
Time (s)

Some Fun Scales

Hal's favorite scale: $\ell_{Pl} = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}$ m

$$R_{\text{observable universe}} \sim R_H \sim 10^{26} \text{ m}$$

So, you ($L_{\text{you}} \sim 2$ m) are closer to the size of the observable universe, than to my favorite length scale!

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So, you ($L_{\text{you}} \sim 2$ m) are closer to the size of the observable universe, than to my favorite length scale!

$$t_{Pl} = \frac{\ell_{Pl}}{c} = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-44} \text{ s} \quad \text{and} \quad E_{Pl} = \frac{\hbar}{t_{Pl}} \sim 10^9 \text{ J}$$

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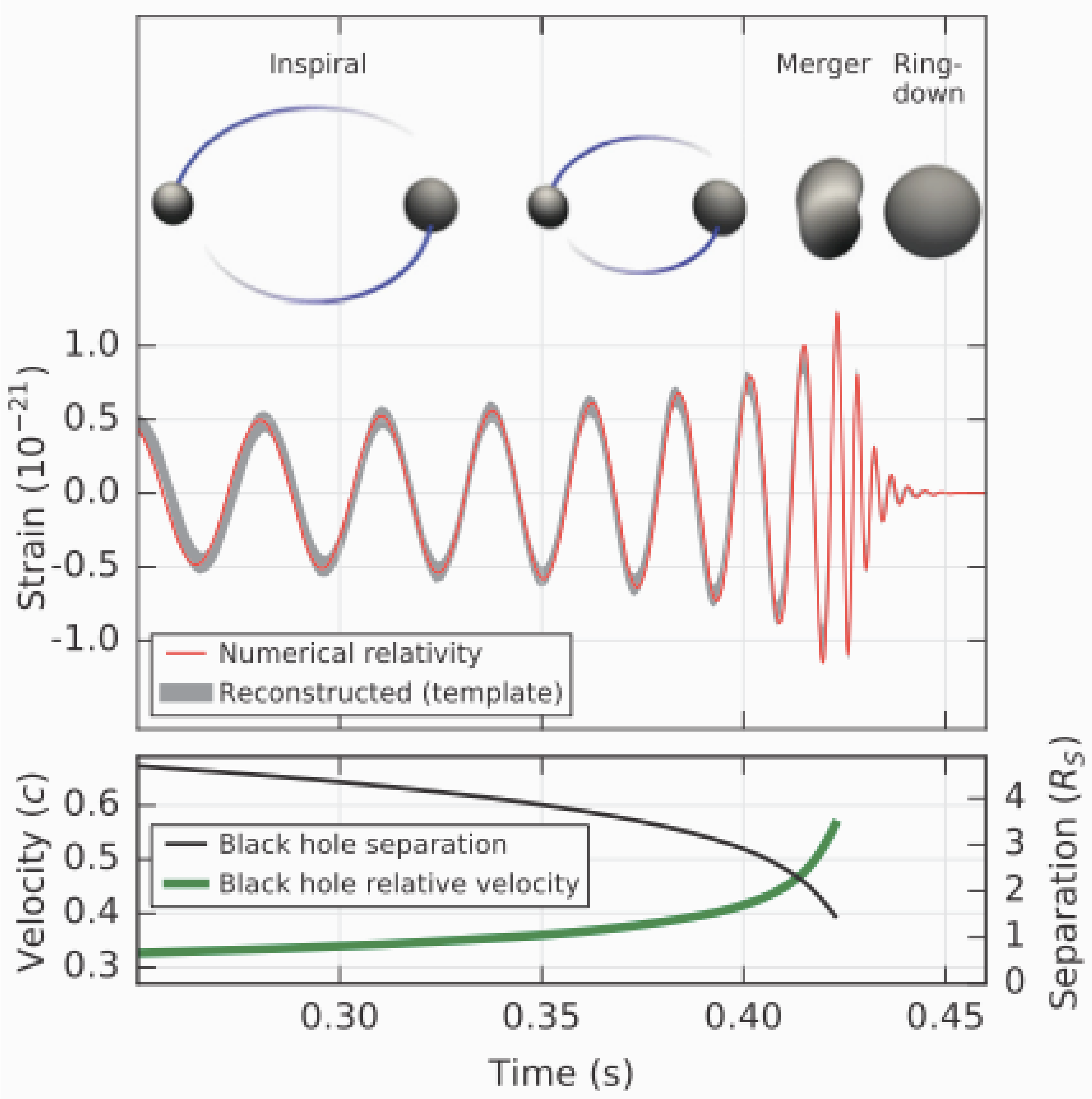
But, interestingly, there are non-quantum Planck scales:

$$P_{Pl} = \frac{E_{Pl}}{t_{Pl}} = \frac{\hbar}{t_{Pl}^2} = \frac{c^5}{G} \sim 10^{52}\ \text{W}\,.$$



Inspiral
Merger
Ring-down
1.0
0.5
0.0
-0.5
-1.0
Strain (10^{-21})
Numerical relativity
Reconstructed (template)
0.6
0.5
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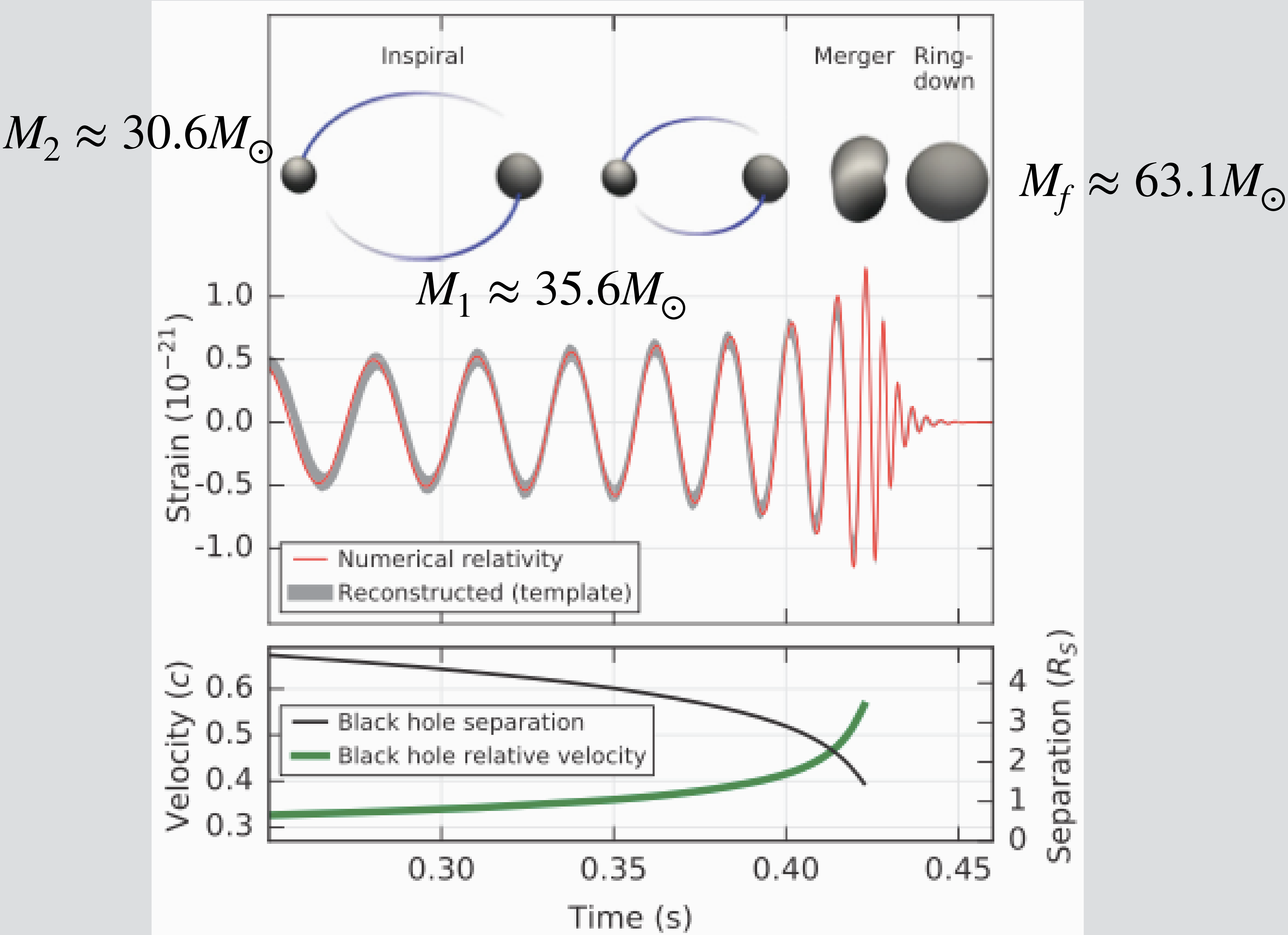
0.3
Velocity (c)
Black hole separation
Black hole relative velocity
4
3
2
1
0
Separation (R_S)
0.30
0.35
0.40
0.45
Time (s)



$M_2 \approx 30.6 M_\odot$
$M_1 \approx 35.6 M_\odot$
$M_f \approx 63.1 M_\odot$
Inspiral
Merger
Ring-down
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
Numerical relativity
Reconstructed (template)
Velocity (c)
0.6
0.5
0.4
0.3
Black hole separation
Black hole relative velocity
Separation (R_S)
4
3
2
1
0
0.30
0.35
0.40
0.45
Time (s)

$$M_1 \approx 35.6M_\odot + M_2 \approx 30.6M_\odot - M_f \approx 63.1M_\odot = 3.1M_\odot$$

The collision let off 3 suns worth of energy in about 0.2 seconds, which is

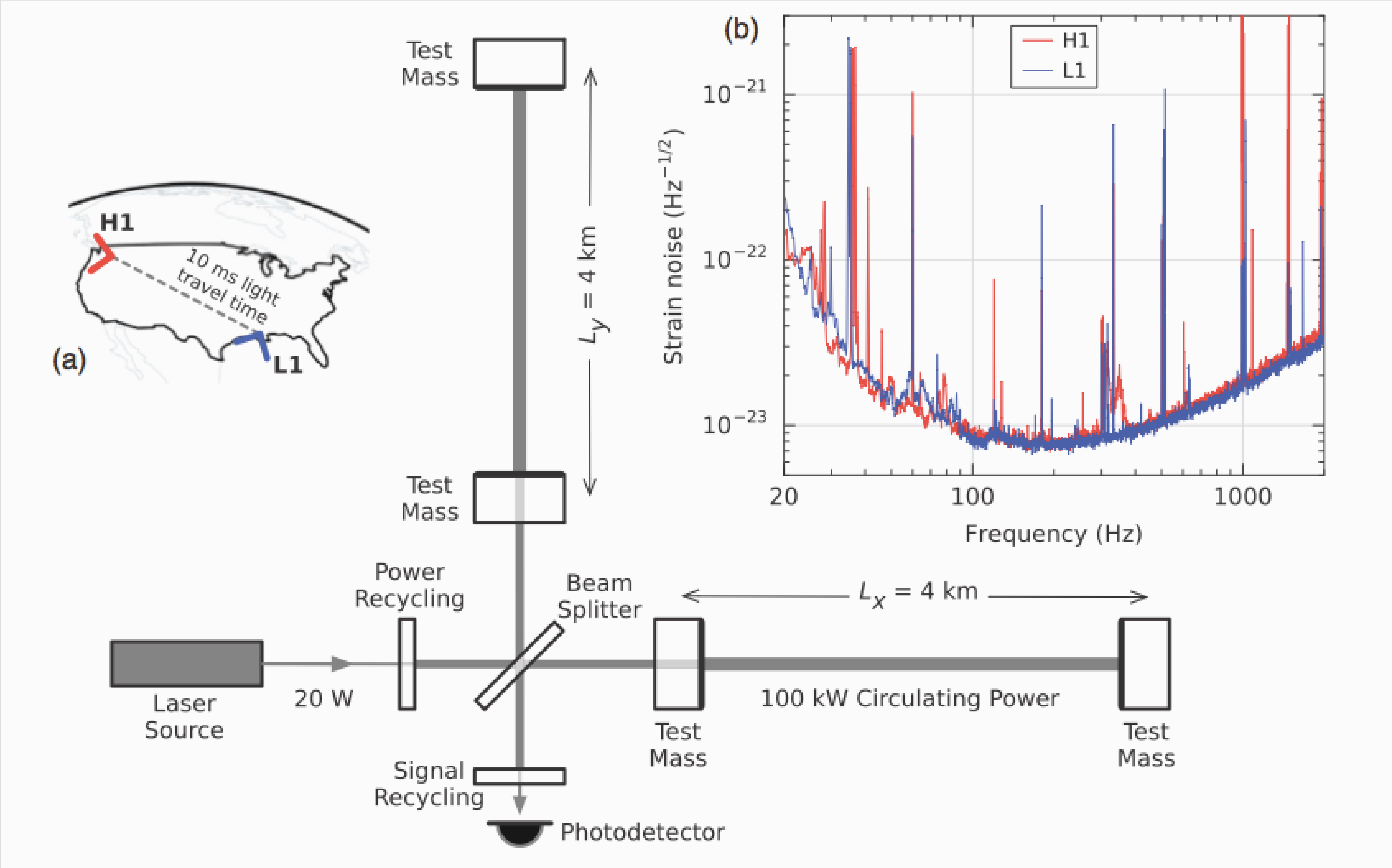
$$3.1M_\odot c^2/(0.2\ \mathrm{s}) \sim 10^{48}\ \mathrm{W}\,.$$

Our guess, $10^{52}\ \mathrm{W}$, was essentially right. If you put in all the details,

$$P = \frac{32}{5}\frac{G^4}{c^5}\frac{M_1^2 M_2^2 (M_1 + M_2)}{r^5},$$

and you get the right correction of 10^{-4}.

How did they do something so spectacular?



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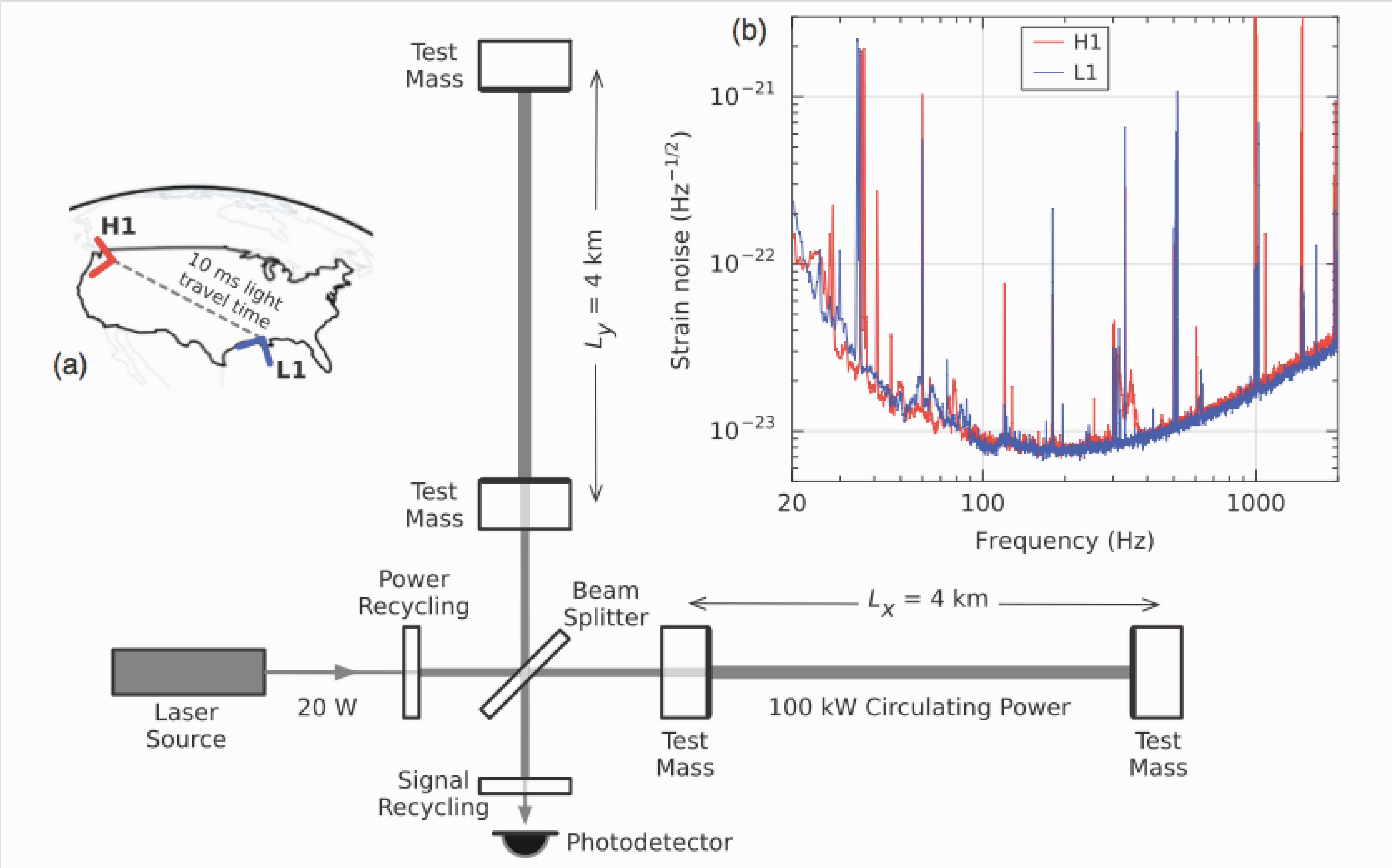
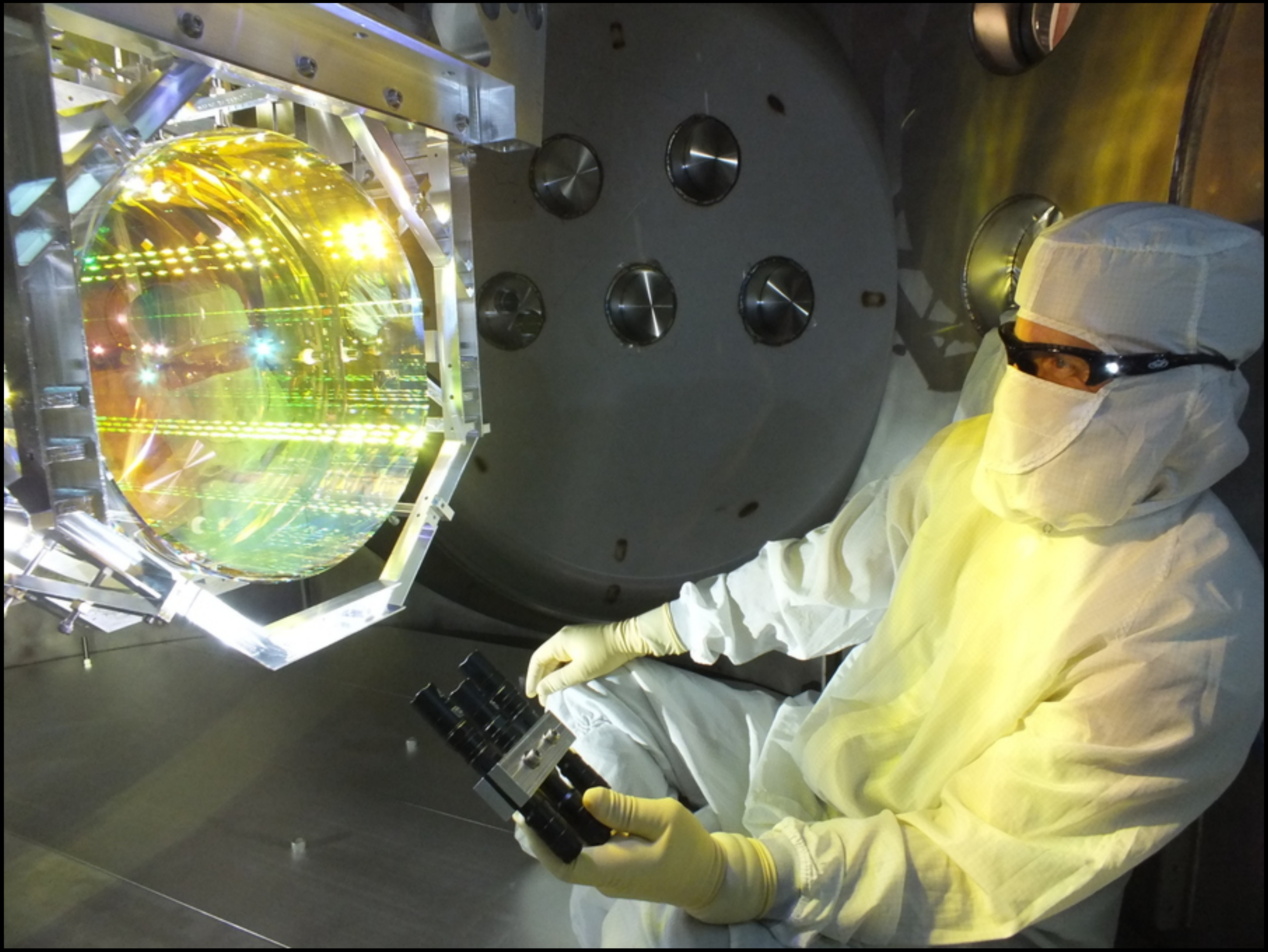
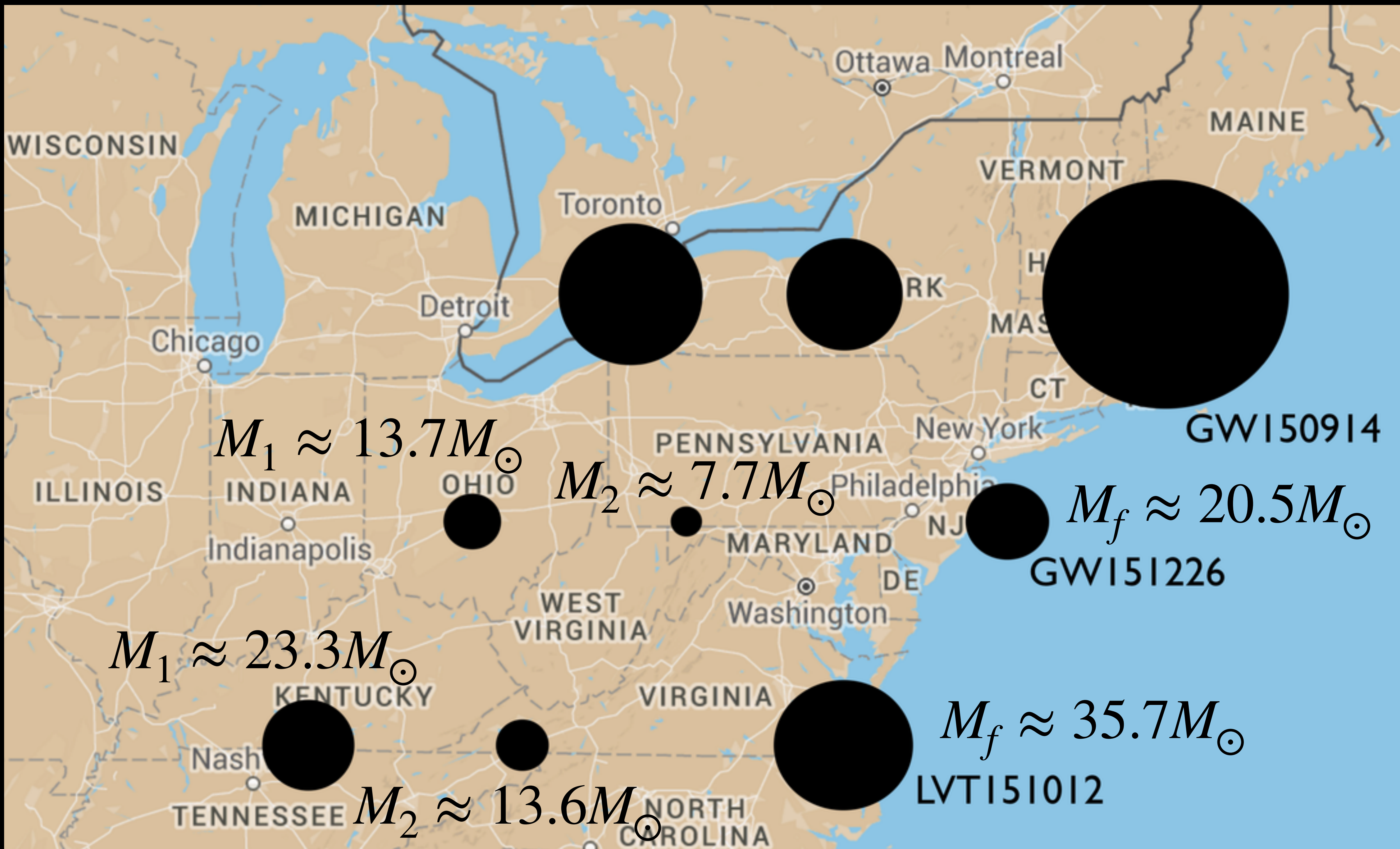


I don't know, ask Antonios!

The mirrors





Ottawa
Montreal
MAINE
WISCONSIN
VERMONT
MICHIGAN
Toronto
Detroit
Chicago
CT
New York
GW150914
$M_1 \approx 13.7 M_\odot$
PENNSYLVANIA
ILLINOIS
INDIANA
OHIO
$M_2 \approx 7.7 M_\odot$
Philadelphia
$M_f \approx 20.5 M_\odot$
NJ
Indianapolis
MARYLAND
GW151226
DE
WEST
VIRGINIA
Washington
$M_1 \approx 23.3 M_\odot$
KENTUCKY
VIRGINIA
$M_f \approx 35.7 M_\odot$
LVT151012
TENNESSEE
$M_2 \approx 13.6 M_\odot$
NORTH
CAROLINA

What do the waves tell us?

Observation of black-hole spins in GW events

$$(M_1, \vec{a}_1) \;+\; (M_2, \vec{a}_2) \;+\; \vec{L} \;\longrightarrow\; (M_f, \vec{a}_f) \;+\; GW$$

Dimensionless spin

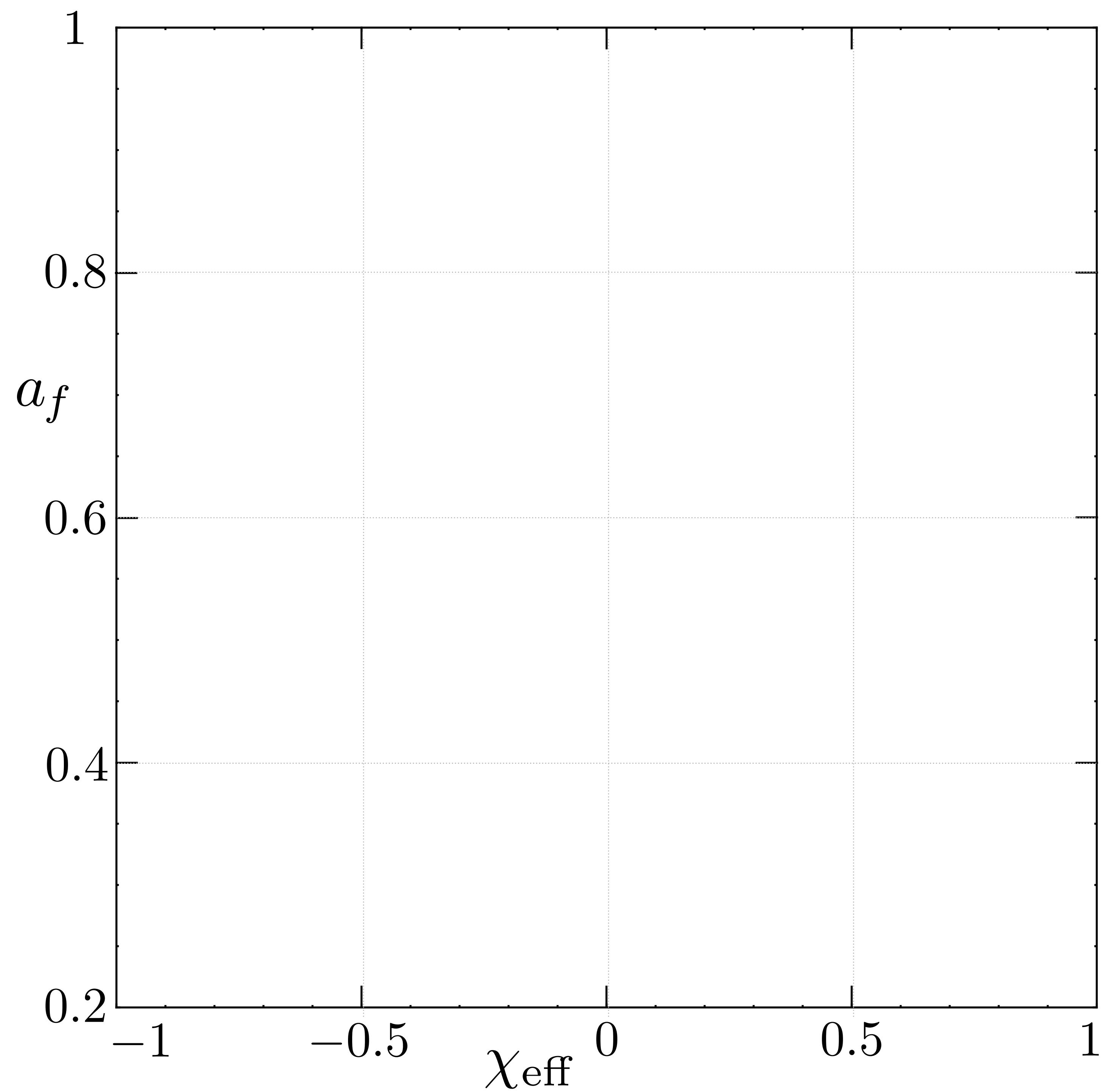
$$a = \frac{|\vec{S}|}{GM^2/c}$$

$a \in [0, 1].$

Effective initial spin

$$\chi_{\rm eff} = \frac{M_1\vec{a}_1 + M_2\vec{a}_2}{M_1 + M_2} \cdot \frac{\vec{L}}{|\vec{L}|},$$

with $\chi_{\rm eff} \in [-1, 1].$



Observation of black-hole spins in GW events

$$(M_1, \vec{a}_1) \; + \; (M_2, \vec{a}_2) \; + \; \vec{L} \; \longrightarrow \; (M_f, \vec{a}_f) \; + \; GW$$

Dimensionless spin

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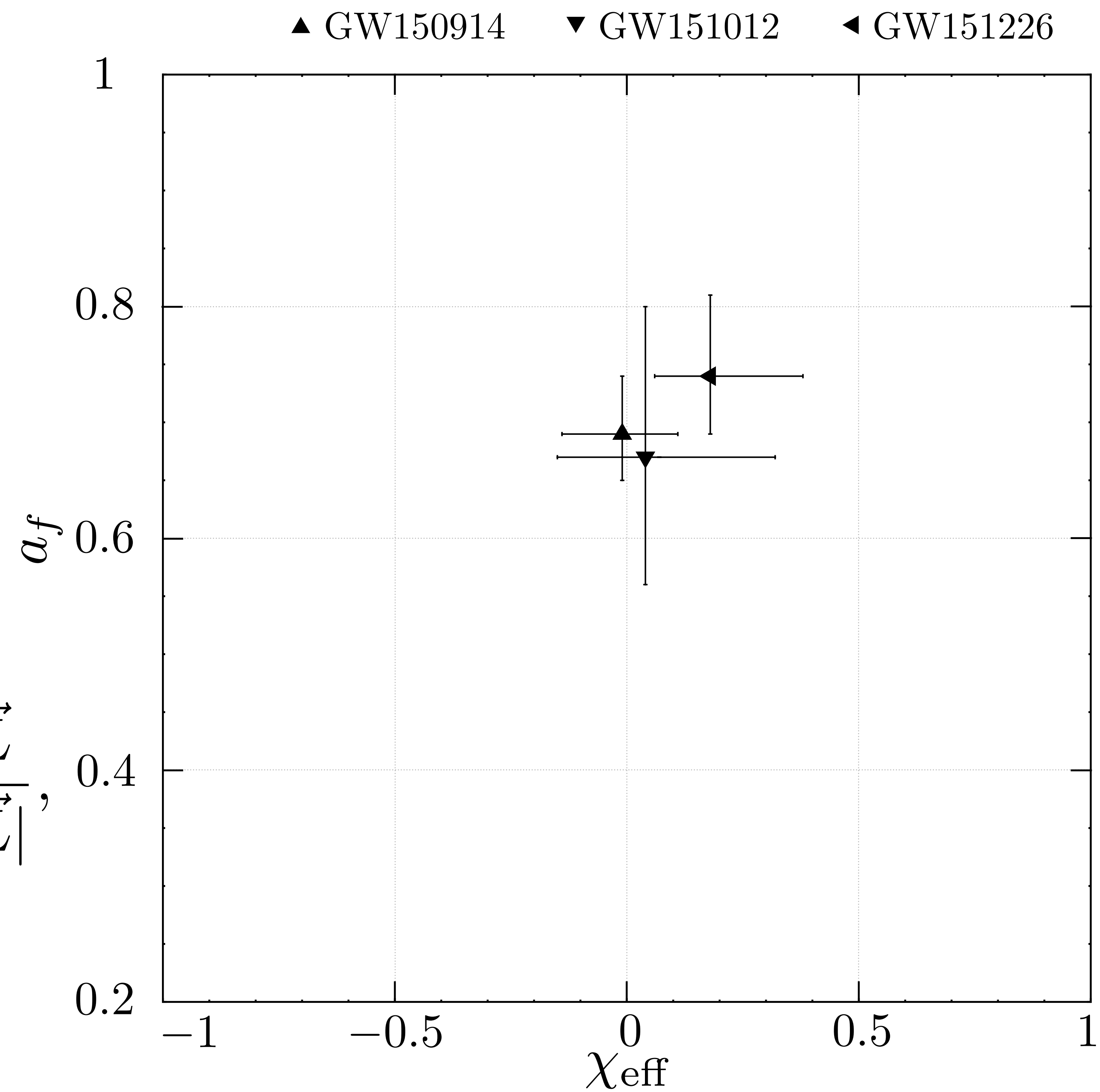

Observation of black-hole spins in GW events

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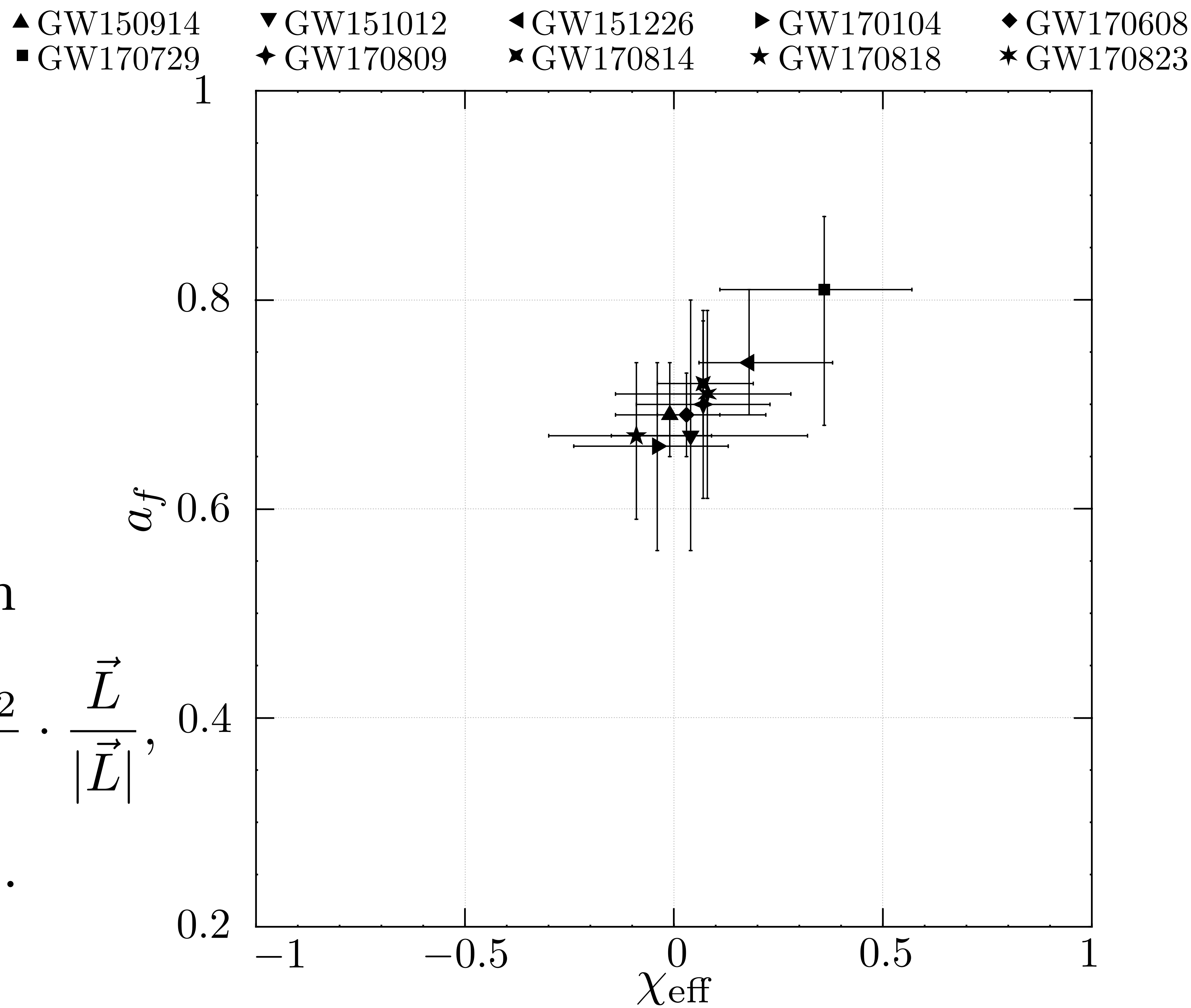
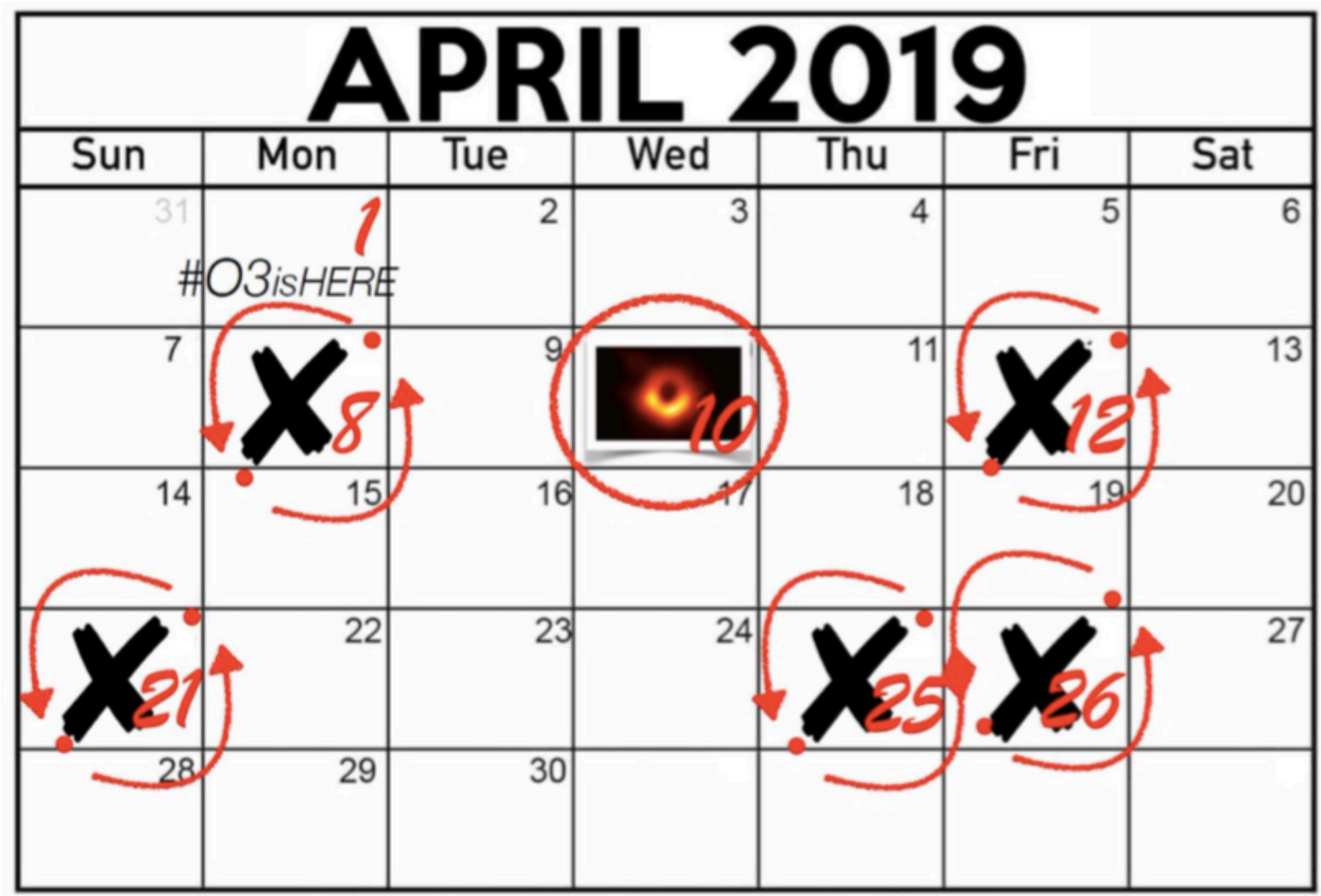


Do all black holes spin?

Is there another way to form black holes?

Are these first 10 results typical?


APRIL 2019
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MAY 2019

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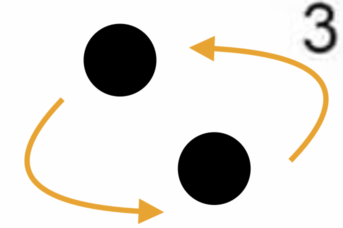
28	29	30	1	2	3 	4
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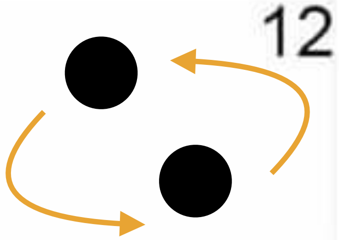
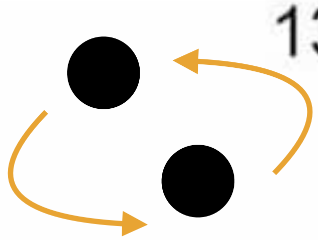
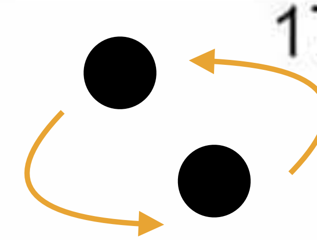
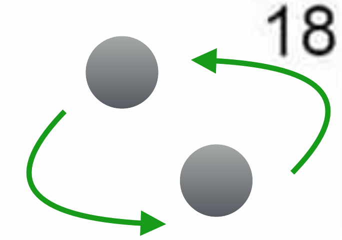
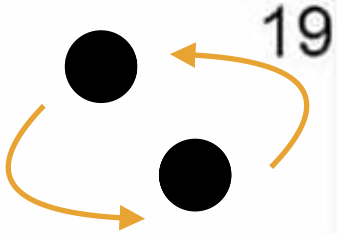
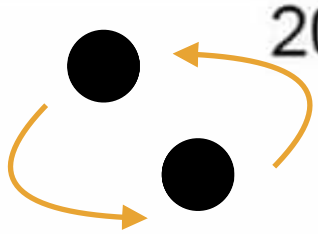
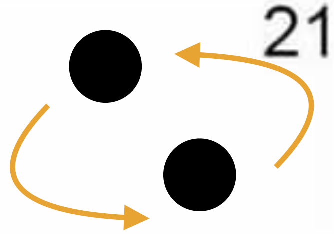
12 	13 	14	15	16	17 	18 
19 	20 	21 	22	23	24	25
26	27	28	29	30	31	

GraceDB

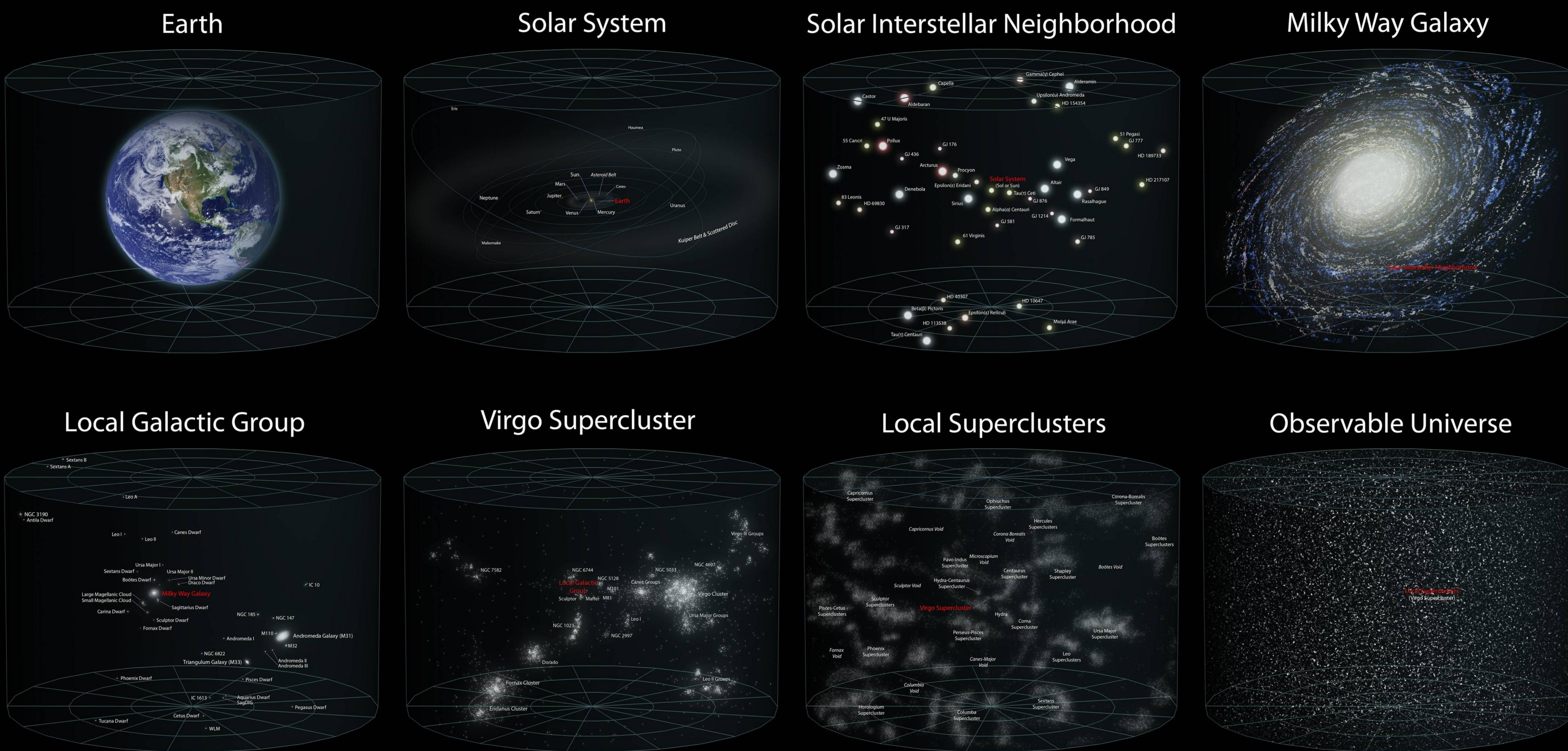
https://gracedb.ligo.org/superevents/public/O3/

Outline

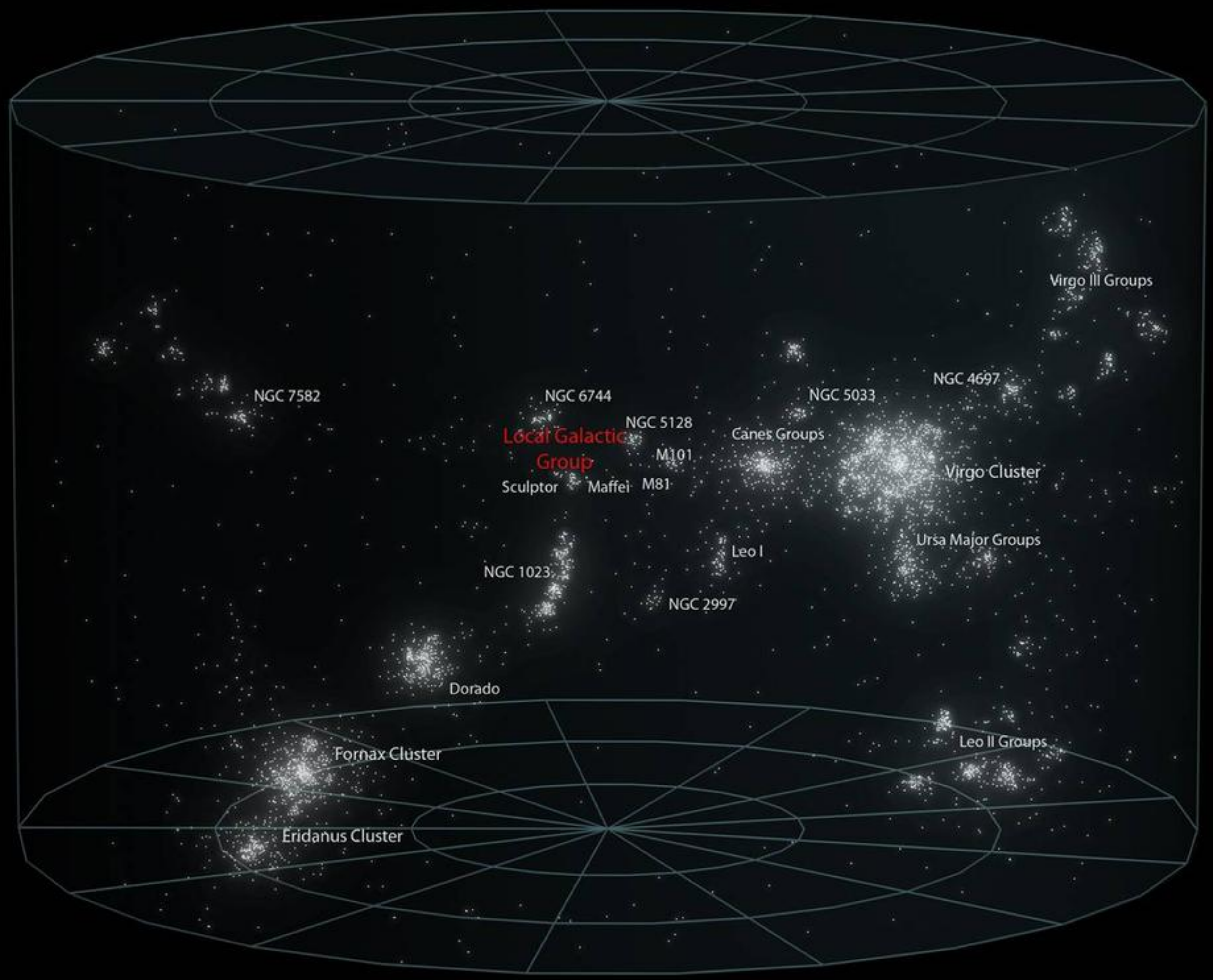
I. Don’t all Black Holes Spin?

II. Stars Collapse, but
The Universe Expands

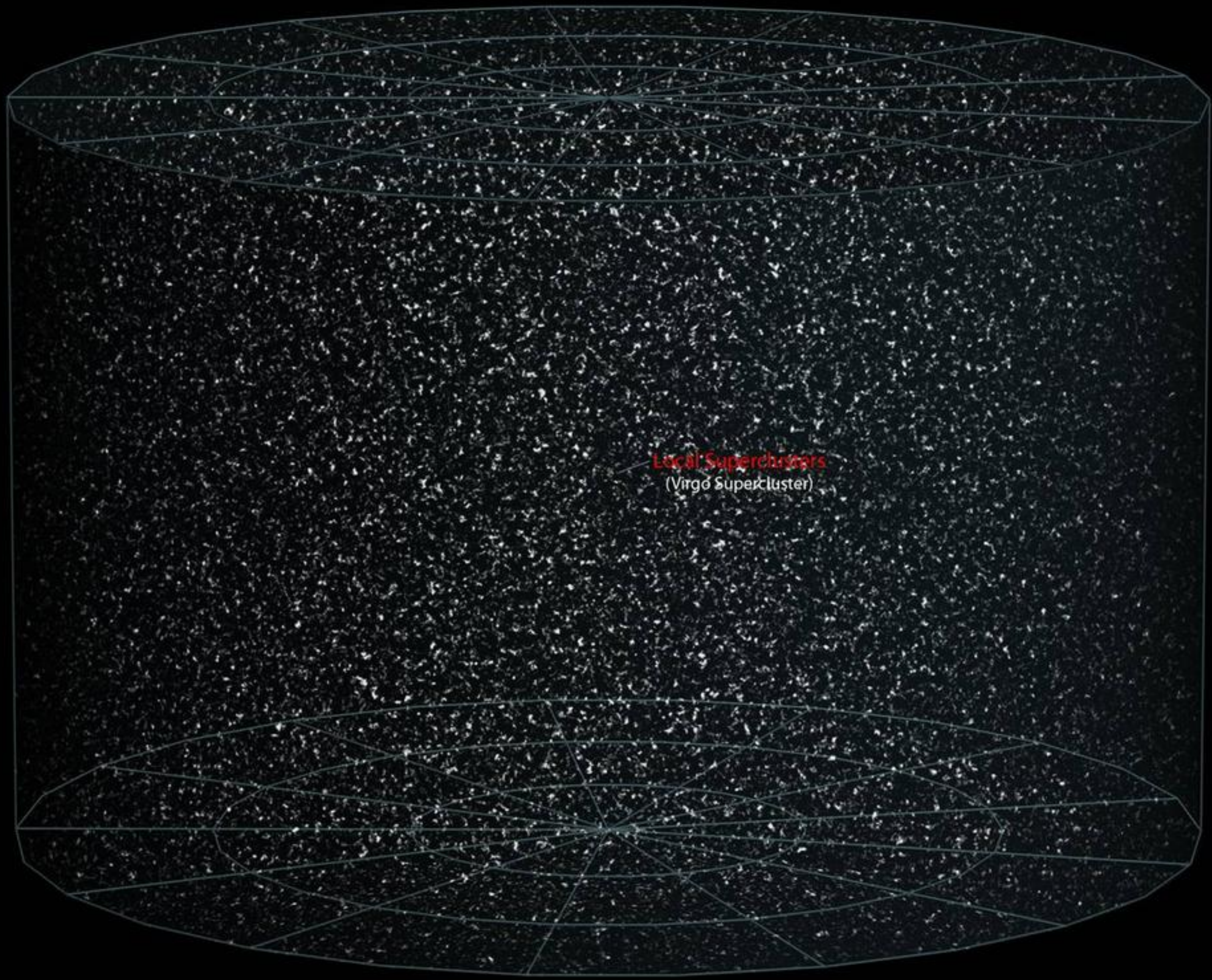
III. Boxing in Black Holes


Earth
Solar System
Solar Interstellar Neighborhood
Milky Way Galaxy
Local Galactic Group
Virgo Supercluster
Local Superclusters
Observable Universe

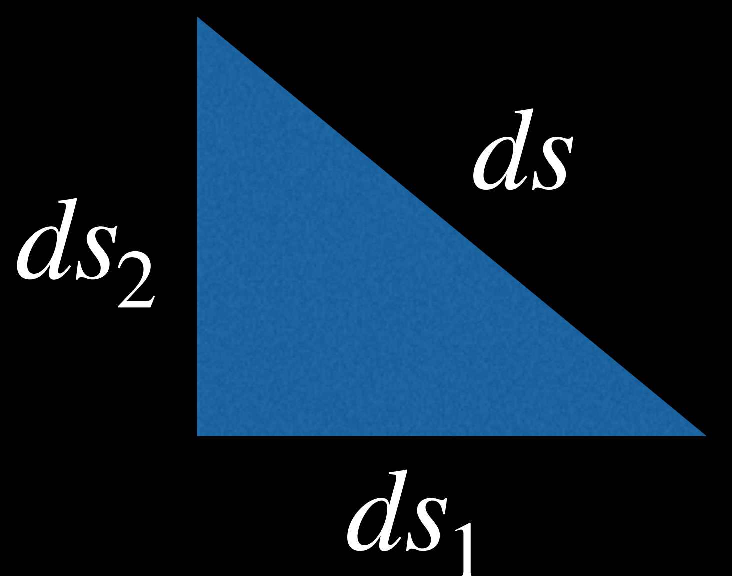
Virgo Supercluster



Observable Universe



The observable universe is nearly homogeneous and isotropic—



Recall Pythagoras

$$ds^2 = ds_1^2 + ds_2^2$$

—and it's dynamical(!):

$$ds^2 = c^2dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

For fun, the metric

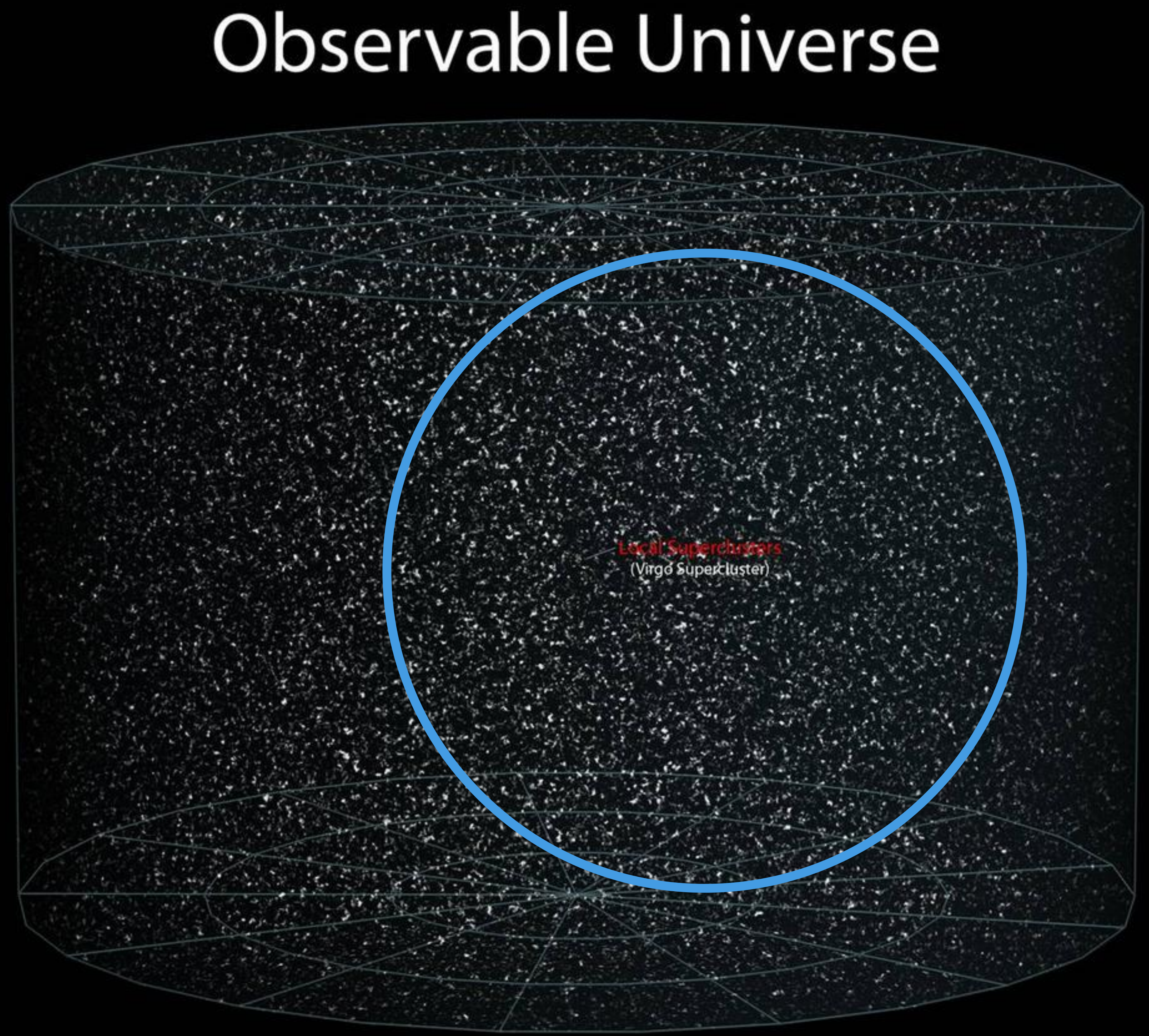
$$ds^2 = c^2dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

evolves according to the Einstein equations for the universe:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3p)$$

$$\dot{\rho} = -3H(\rho + p)$$

Strikingly, an 'apparent' horizon can form



Outline

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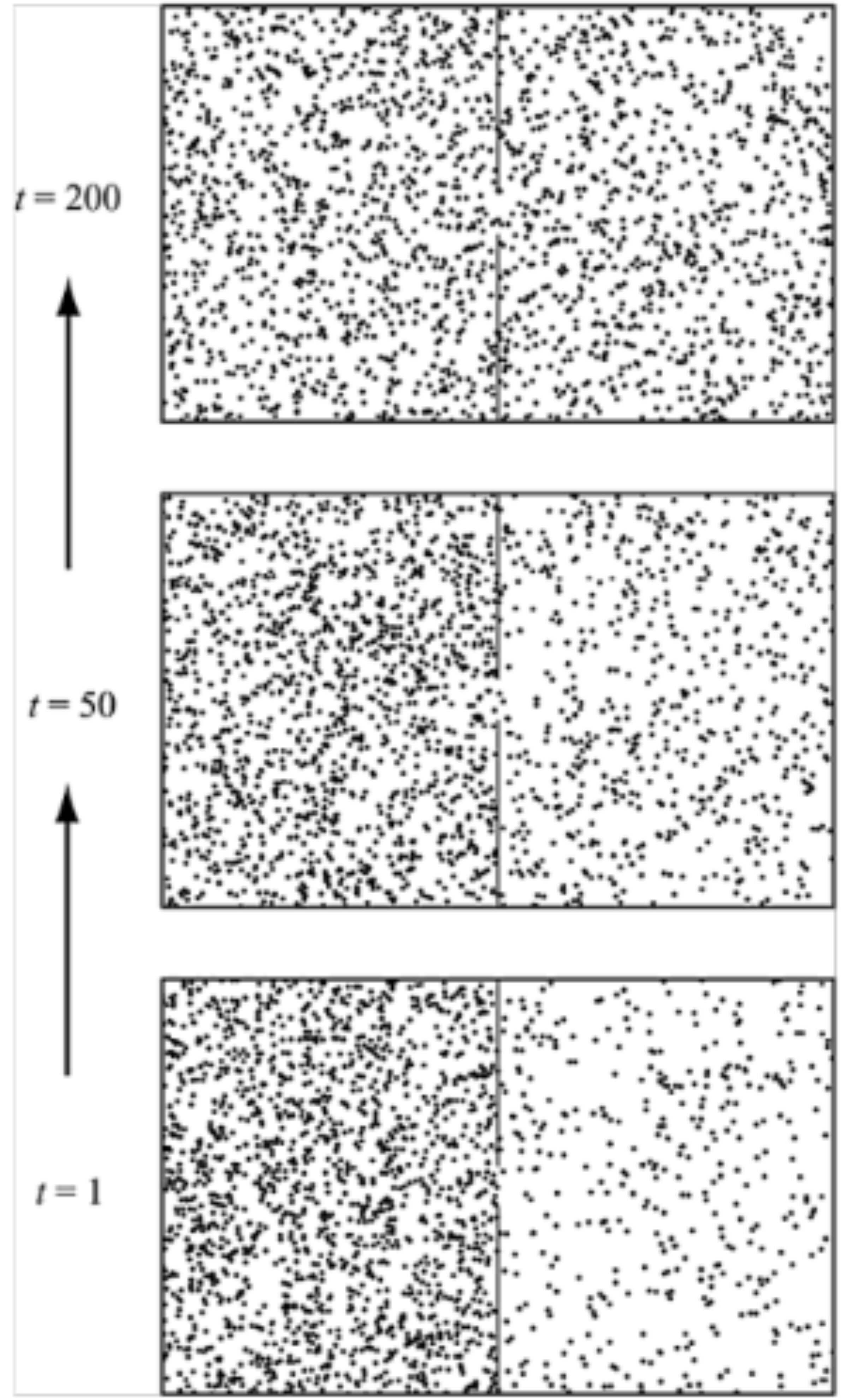
Equal left and right

1400 molecules on the left,
600 on the right

1600 molecules on the left,
400 on the right



In the early 1970's Bekenstein and Hawking argued that black holes have an enormous entropy…

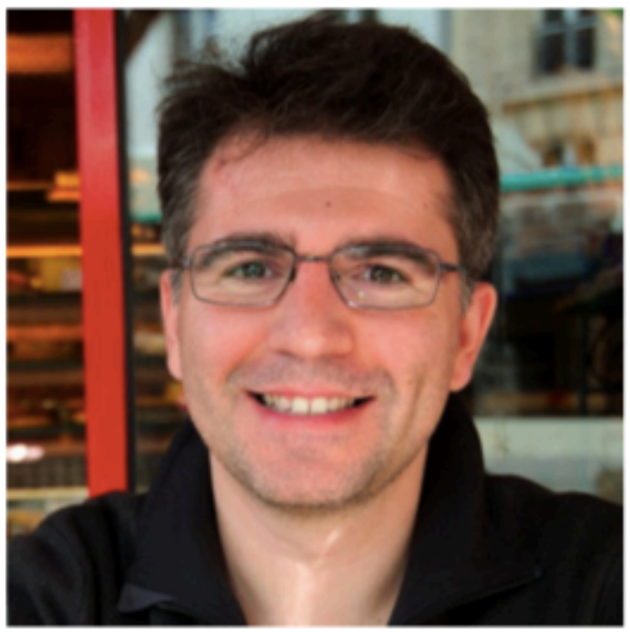
$$S(M, J) = k \frac{A(M, J)}{4\,\hbar G / c^3}$$

In the early 1970's Bekenstein and Hawking argued that black holes have an enormous entropy…

…, but, of course, this entropy is only present when a black hole can form.

Bekenstein-Hawking Entropy and Gravitational Wave Observations: Statistical Equilibrium as a Mechanism for Small Black Hole Spins

Joint work with Penn State collaborators:



Eugenio Bianchi, Anuradha Gupta & Sathya Sathyaprakash

[E. Bianchi, A. Gupta, HMH, & B.S. Sathyaprakash, arXiv:1812.05127]

When do primordial BHs form?

Staller mass BHs are formed due to the QCD transition's drop in pressure. This is after the formation of hadrons and before atoms form

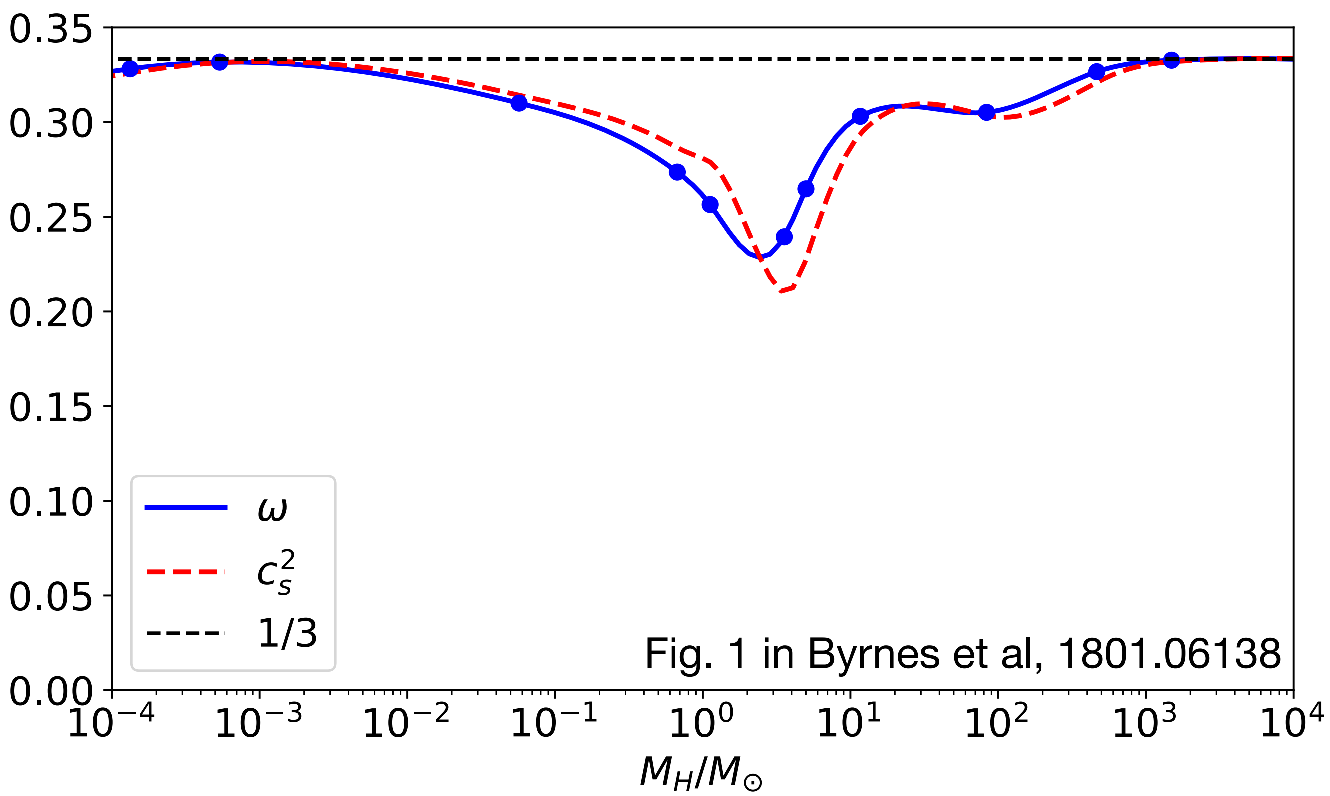


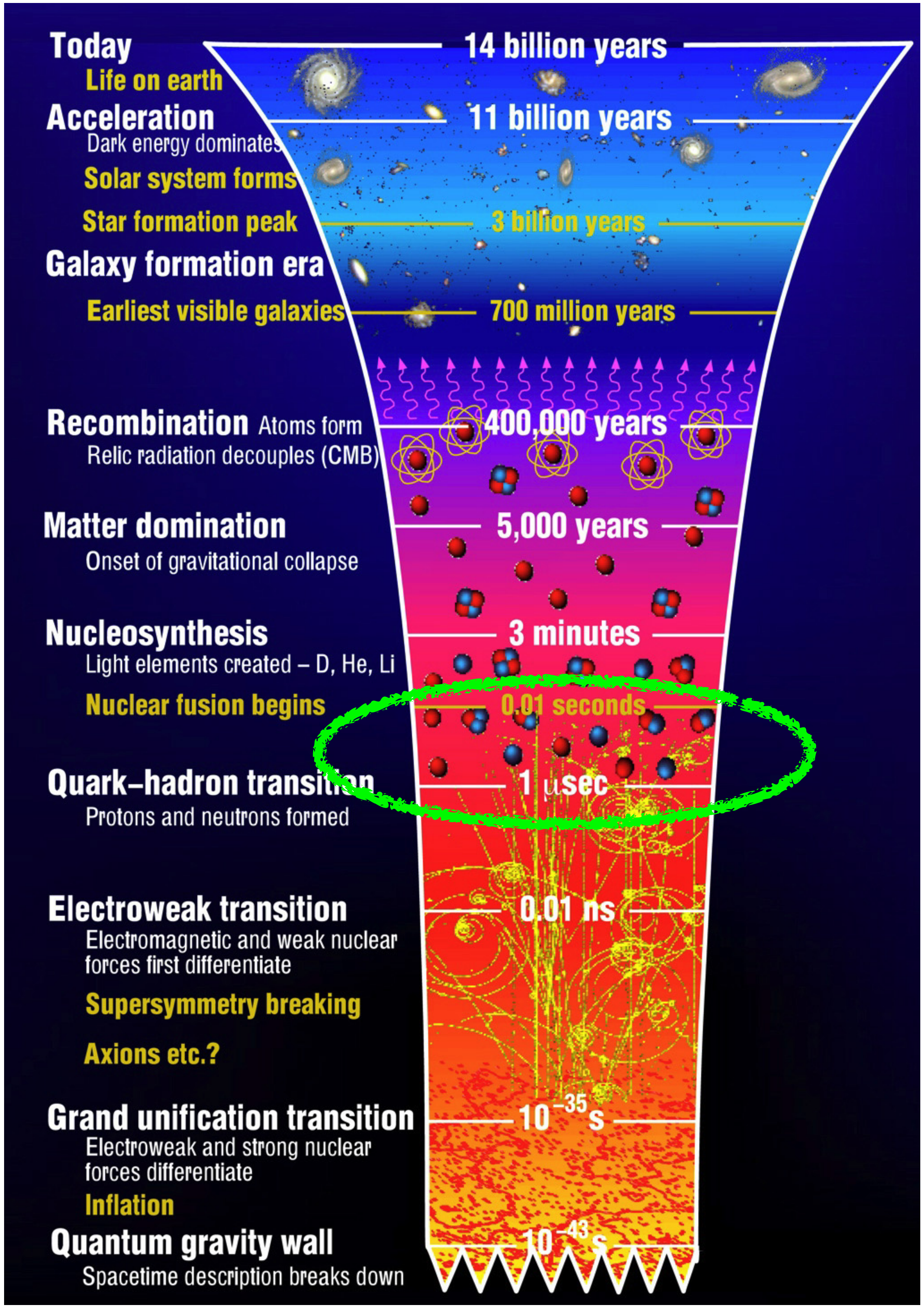
Fig. 1 in Byrnes et al, 1801.06138

Early Universe: during the QCD phase transition, the pressure drops

$$\rho_0 = M_0/r_S^3 \quad r_S = 2GM_0/c^2 \Longrightarrow M_0 = \frac{c^3}{2\sqrt{2}G^{3/2}}\frac{1}{\sqrt{\rho_0}}$$

$$\rho_0 \sim (150\ \mathrm{MeV})^4/\hbar^3 c^5 \quad \Longrightarrow \quad M_0 \sim 25\ M_\odot$$

lattice QCD simulations: BH mass range $\quad 0.1 - 100\ M_\odot$



At fixed mass, rotating black holes have a smaller entropy

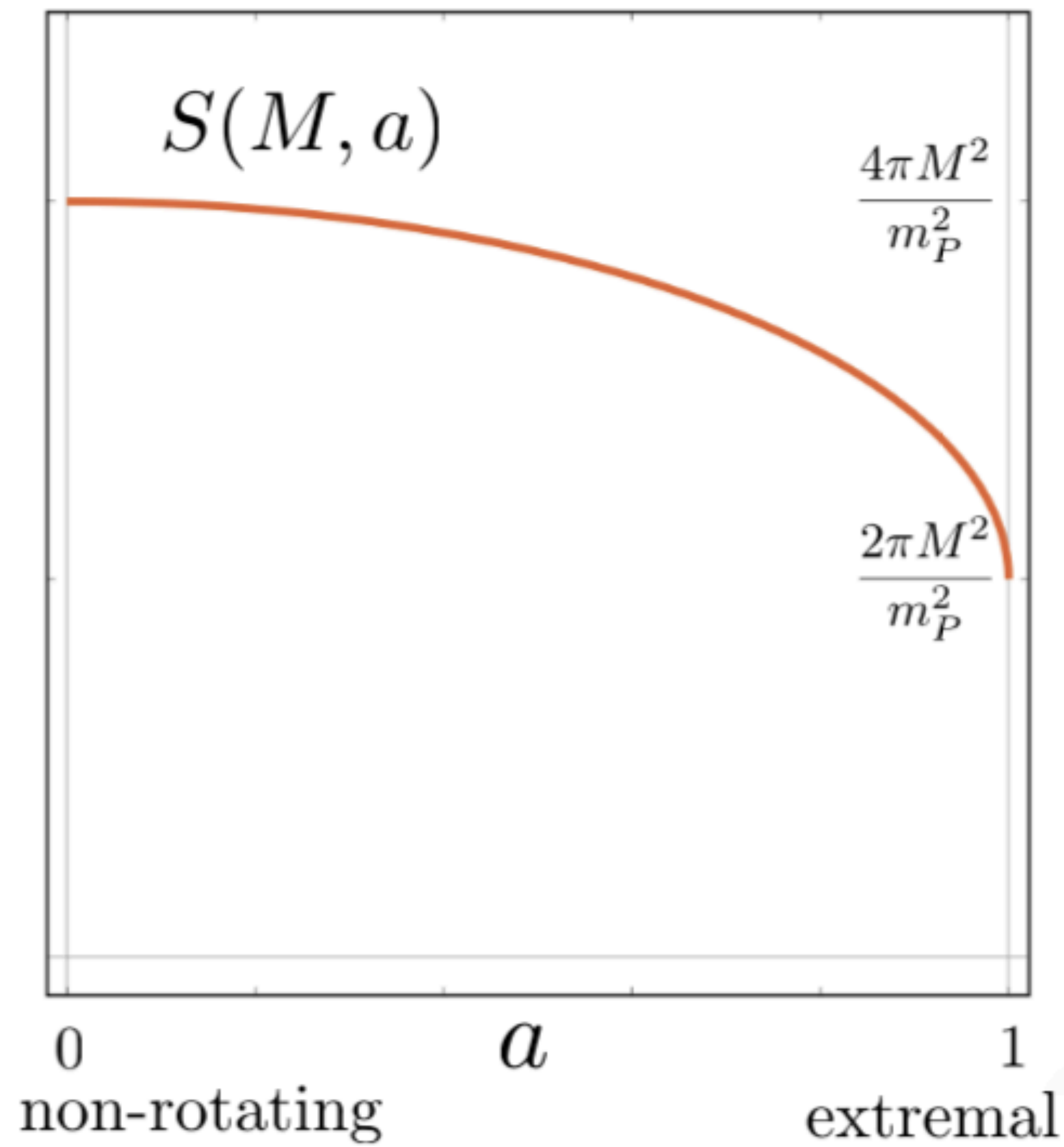
Dimensionless spin parameter

$$a = \frac{|\vec{J}|}{GM^2/c},$$

$a \in [0, 1]$.

Bekenstein-Hawking entropy

$$S(M, a) = (1 + \sqrt{1 - a^2}\,)\frac{2\pi M^2}{m_P^2}$$



Black-hole entropy and the spin distribution of black holes

Number of microstates at fixed (M, a)

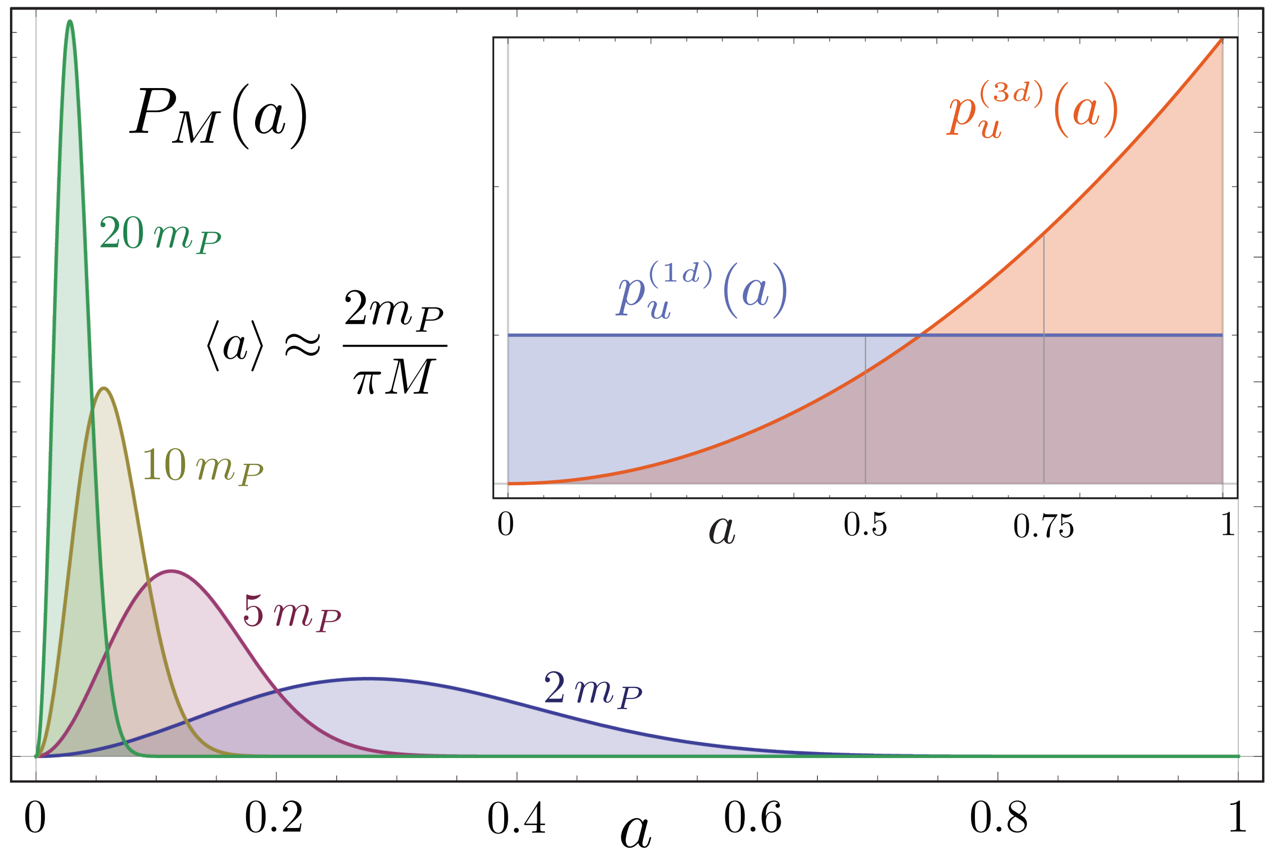
$$\mathcal{N} \sim e^{S(M,a)}$$

In thermal equilibrium

(at fixed energy M),

the probability that a BH

has spin a is



$$P_M(a) = \frac{e^{A(M,a)/4\ell_P^2}\, a^2}{\int_0^1 e^{A(M,a')/4\ell_P^2}\, a'^2\, da'}$$

Prediction: BHs in *microcanonical equilibrium* have small spins

Observation of black-hole spins in GW events

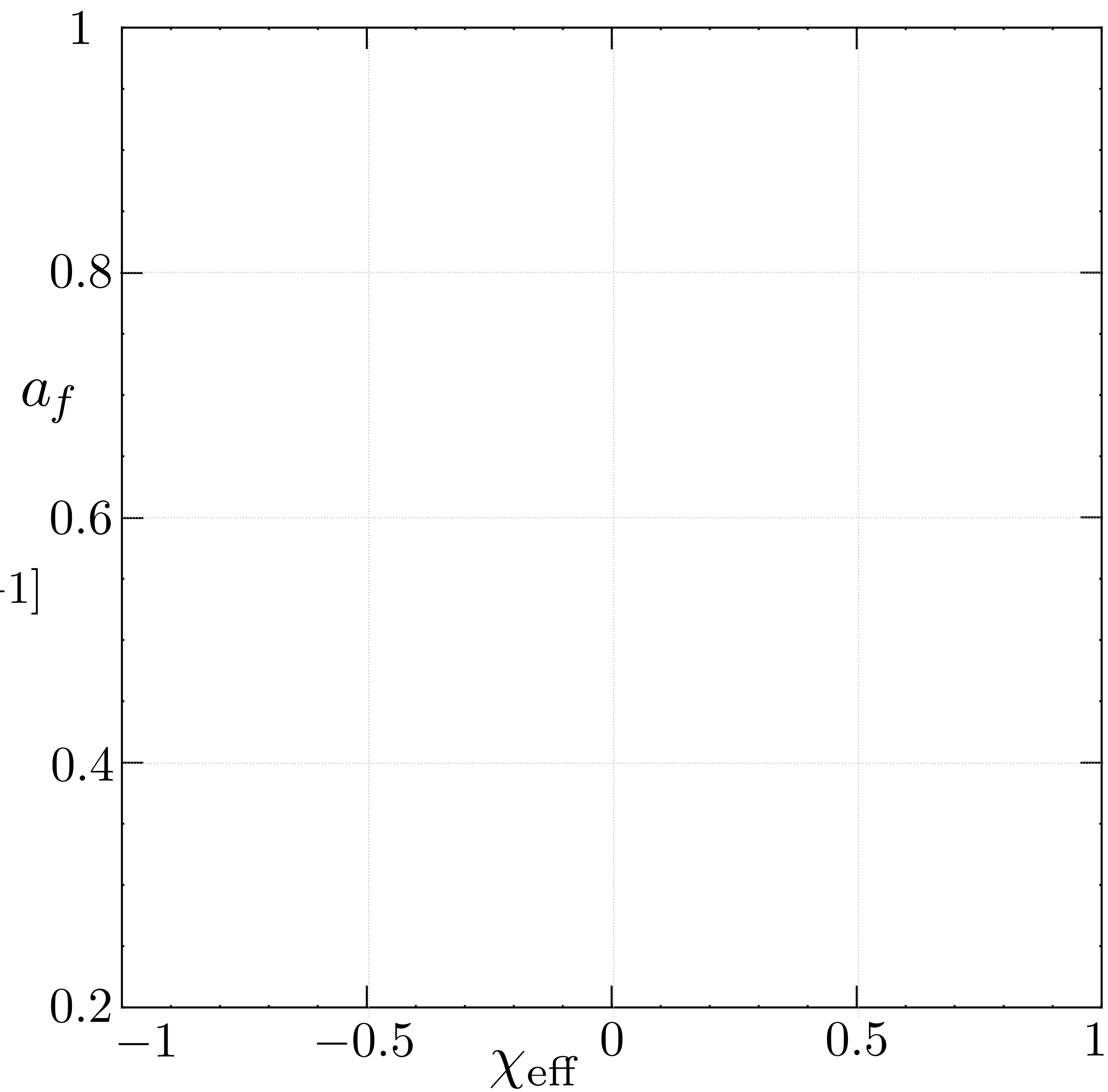
$$(M_1, \vec{a}_1) \; + \; (M_2, \vec{a}_2) \; + \; \vec{L} \; \longrightarrow \; (M_f, \vec{a}_f) \; + \; GW$$

Final spin

$$a_f \; \in [0, 1]$$

Effective initial spin

$$\chi_{\rm eff} = \frac{M_1 \, \vec{a}_1 + M_2 \, \vec{a}_2}{M_1 + M_2} \cdot \frac{\vec{L}}{|\vec{L}|} \quad \in \; [-1, +1]$$



Observation of black-hole spins in GW events

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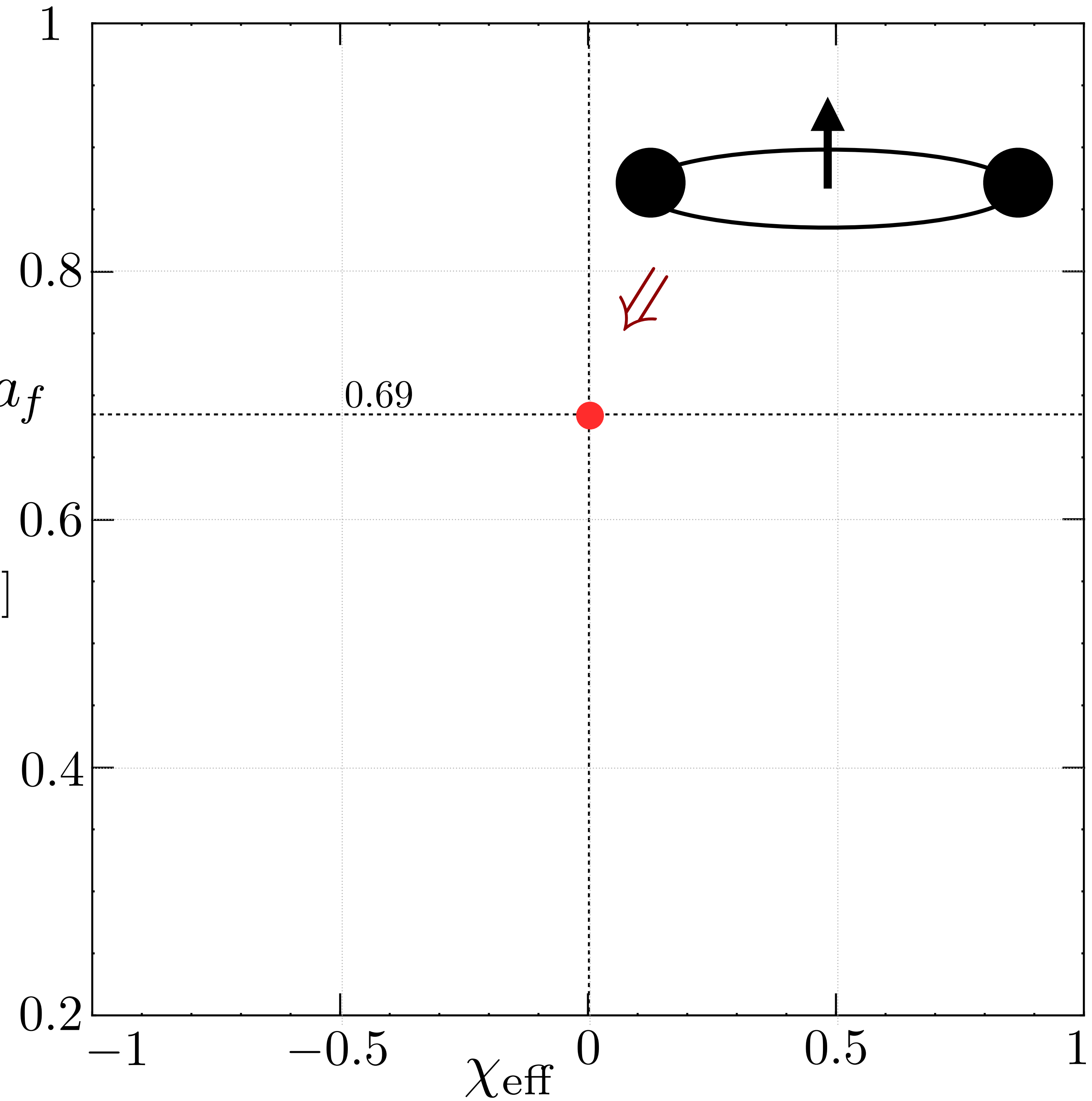


Observation of black-hole spins in GW events

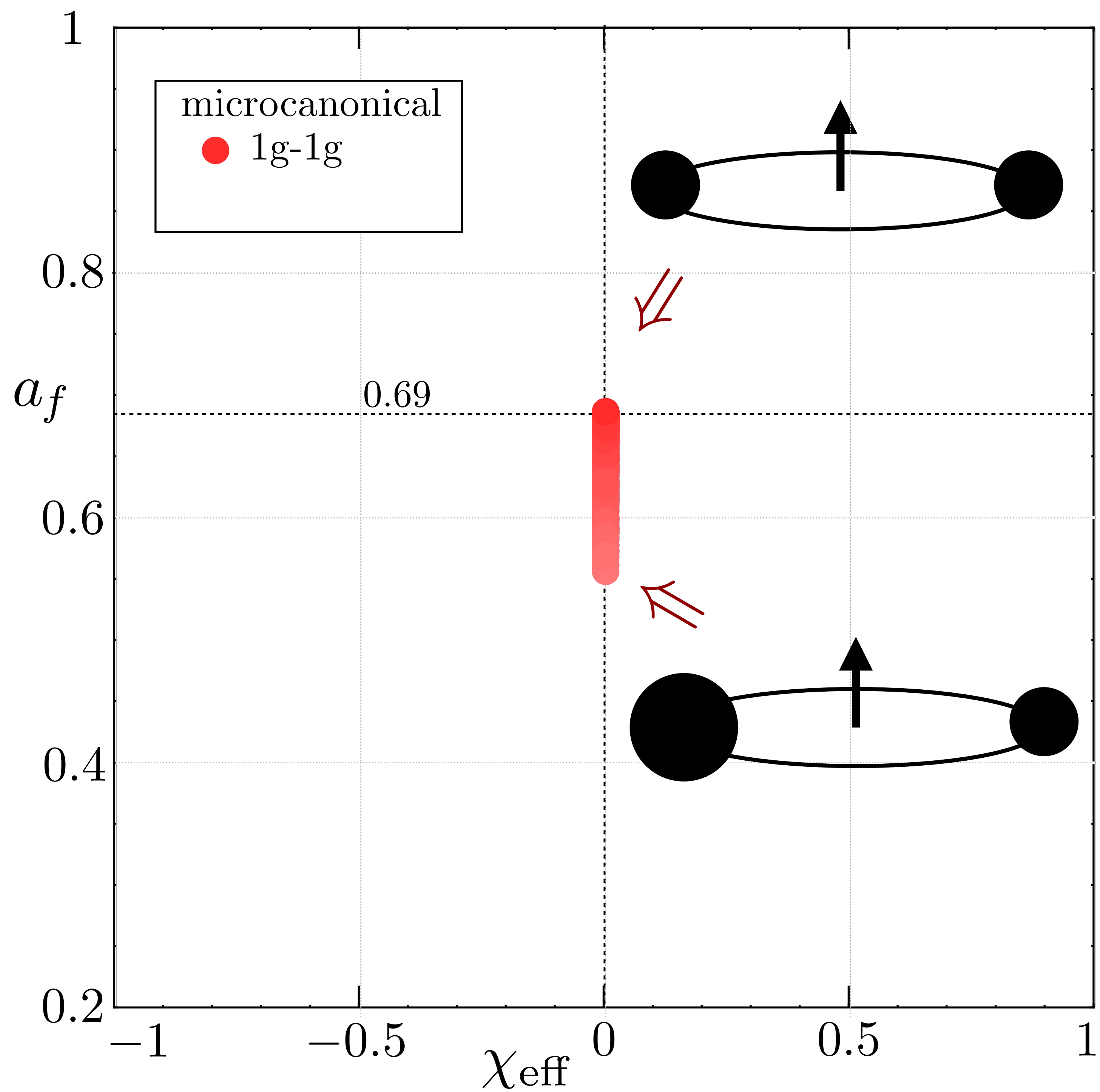
$$(M_1, \vec{a}_1) + (M_2, \vec{a}_2) + \vec{L} \longrightarrow (M_f, \vec{a}_f) + GW$$

Final spin

$$a_f \in [0, 1]$$

A population with final spin computed from fit of numerical-relativity [LIGO-T1600168]

$$a_f \simeq 0.69 - \left(\frac{M_1 - M_2}{M_1 + M_2}\right)^2 \times 0.56$$



Observation of black-hole spins in GW events

$$(M_1, \vec{a}_1) \ + \ (M_2, \vec{a}_2) \ + \ \vec{L} \ \longrightarrow \ (M_f, \vec{a}_f) \ + \ GW$$

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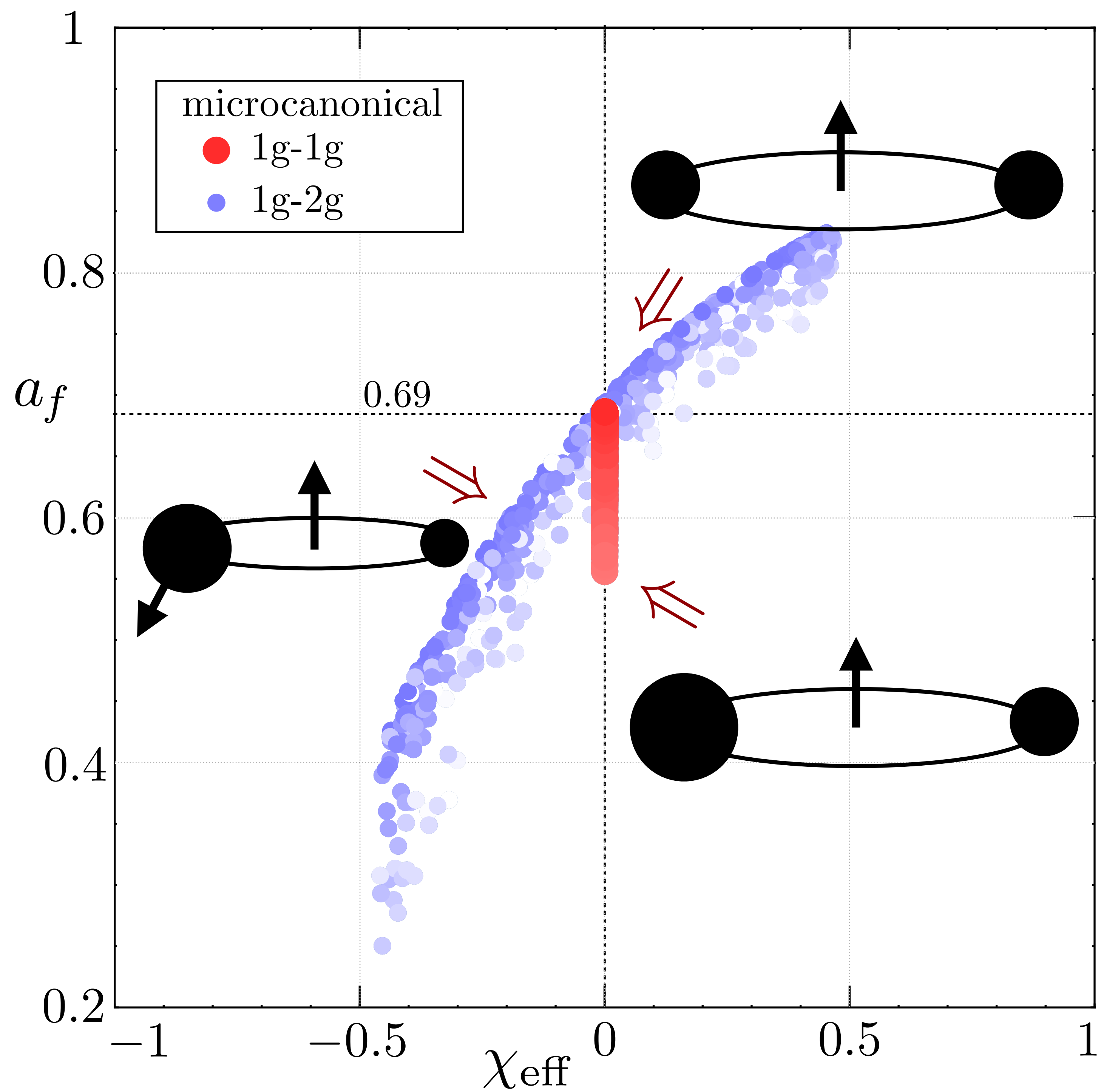

Observation of black-hole spins in GW events

$$(M_1, \vec{a}_1) + (M_2, \vec{a}_2) + \vec{L} \longrightarrow (M_f, \vec{a}_f) + GW$$

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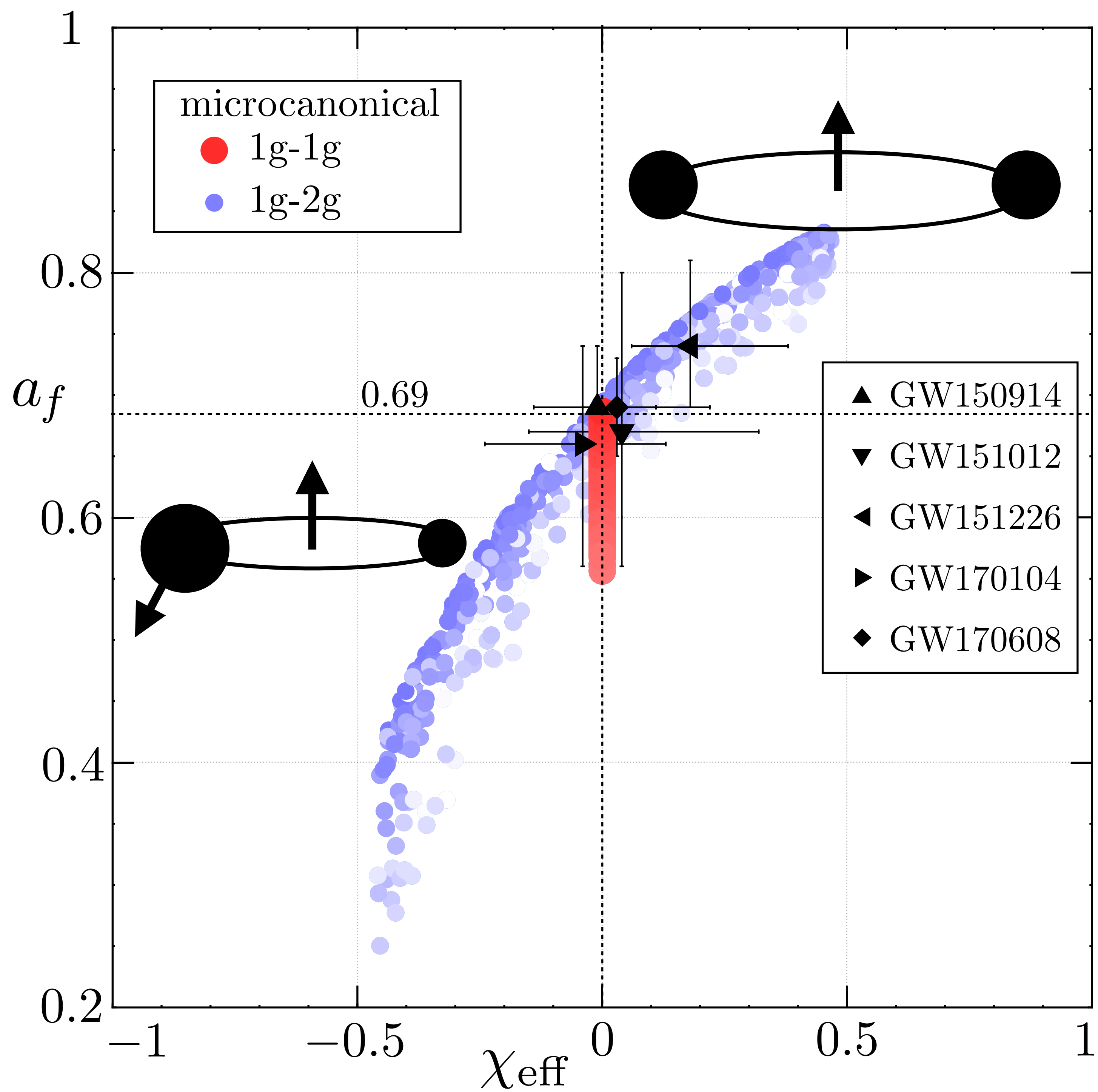

Observation of black-hole spins in GW events

$$(M_1, \vec{a}_1) \;+\; (M_2, \vec{a}_2) \;+\; \vec{L} \;\longrightarrow\; (M_f, \vec{a}_f) \;+\; GW$$

Final spin

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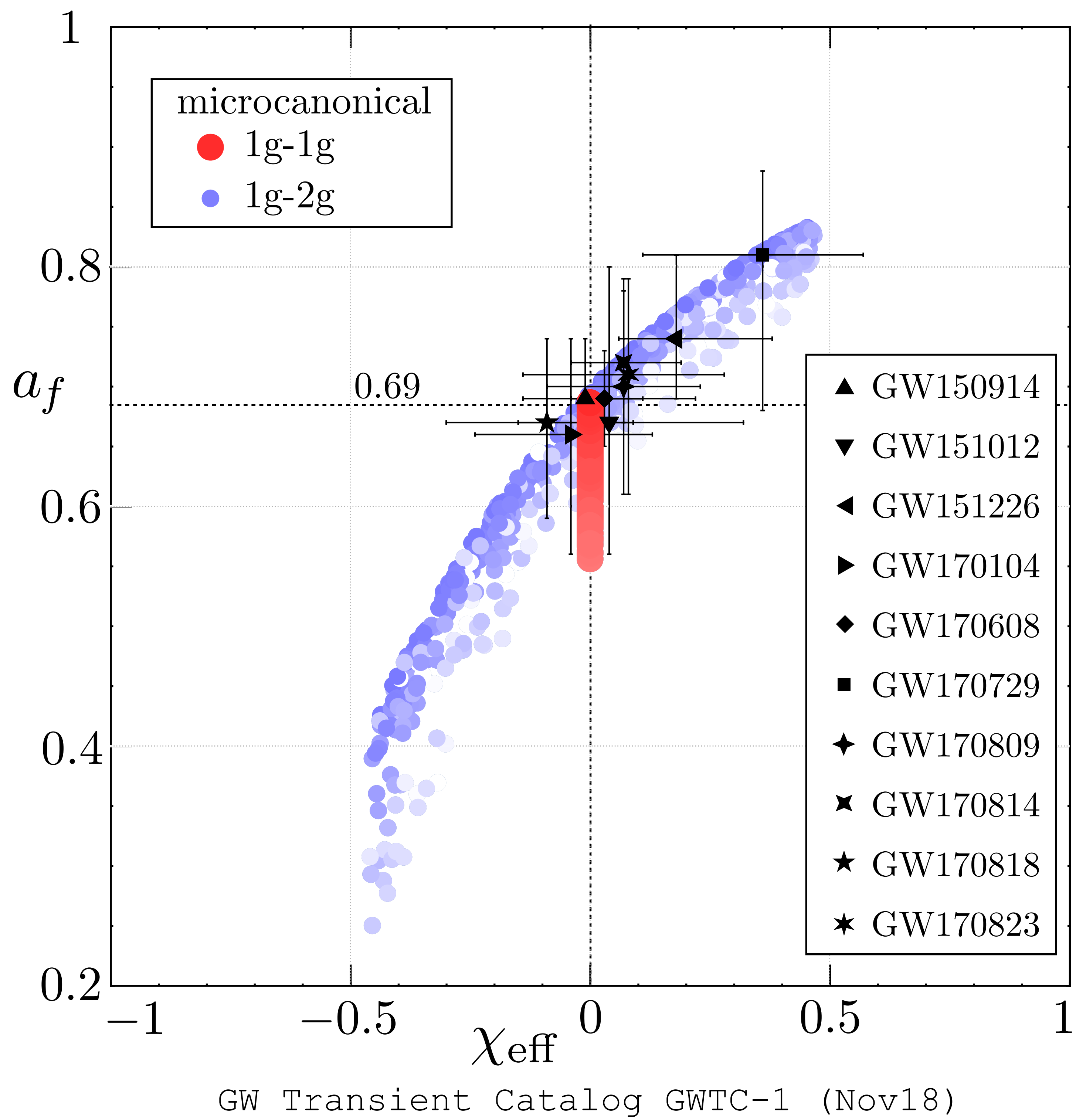

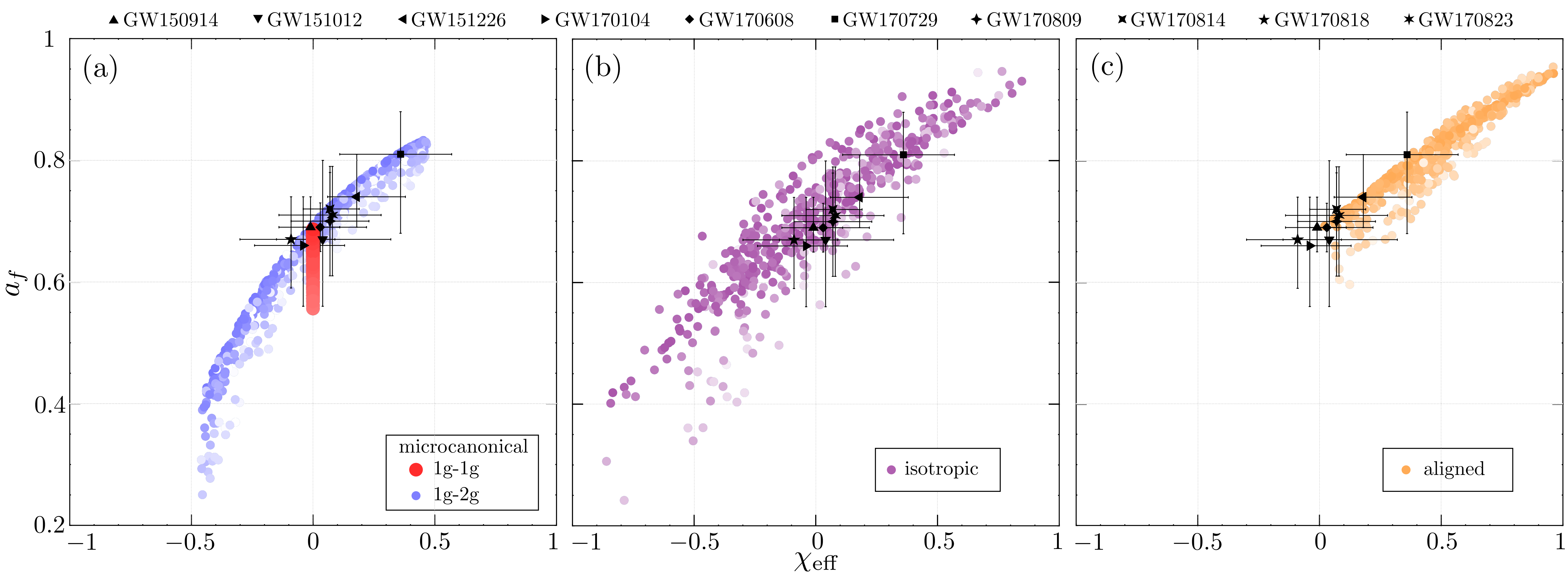

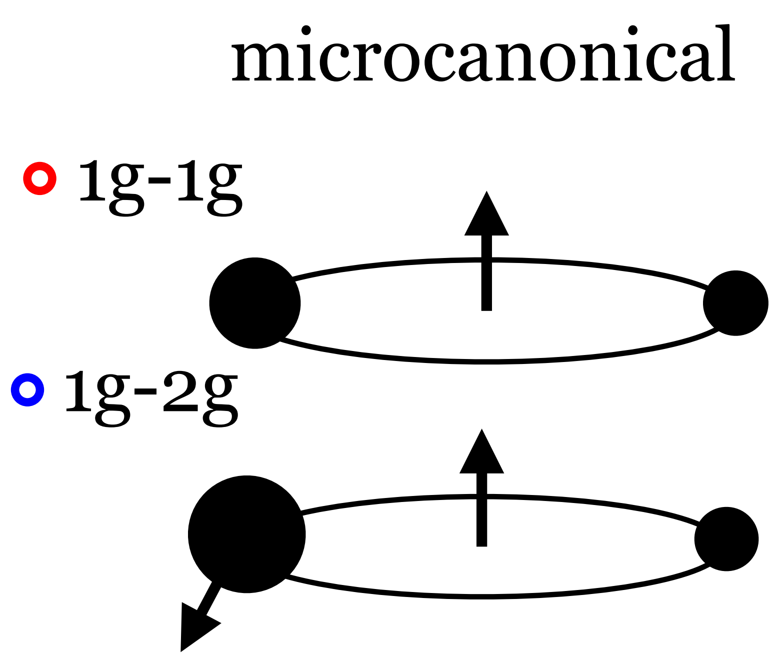

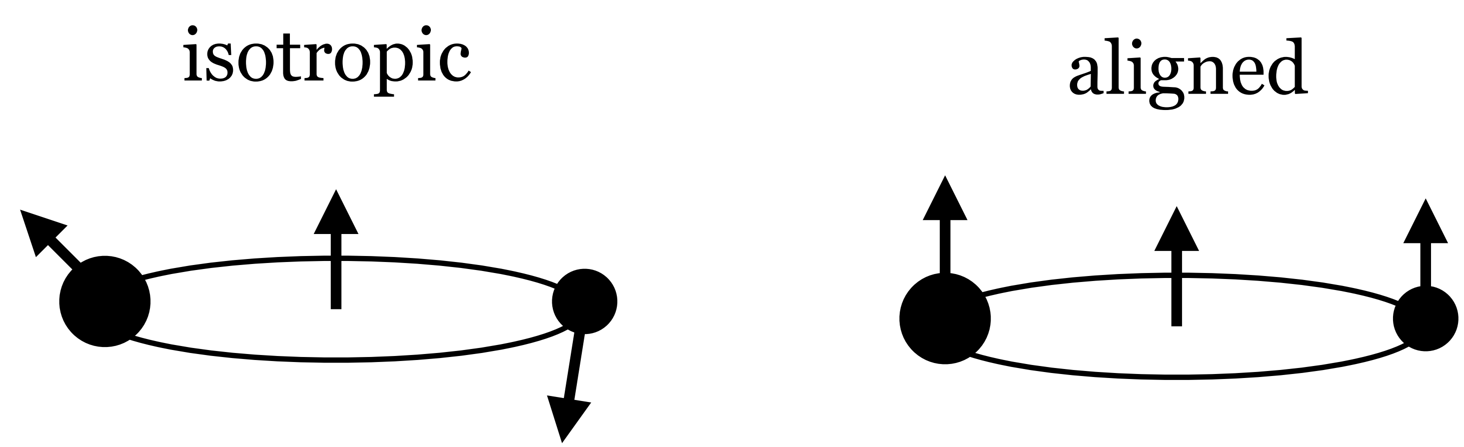

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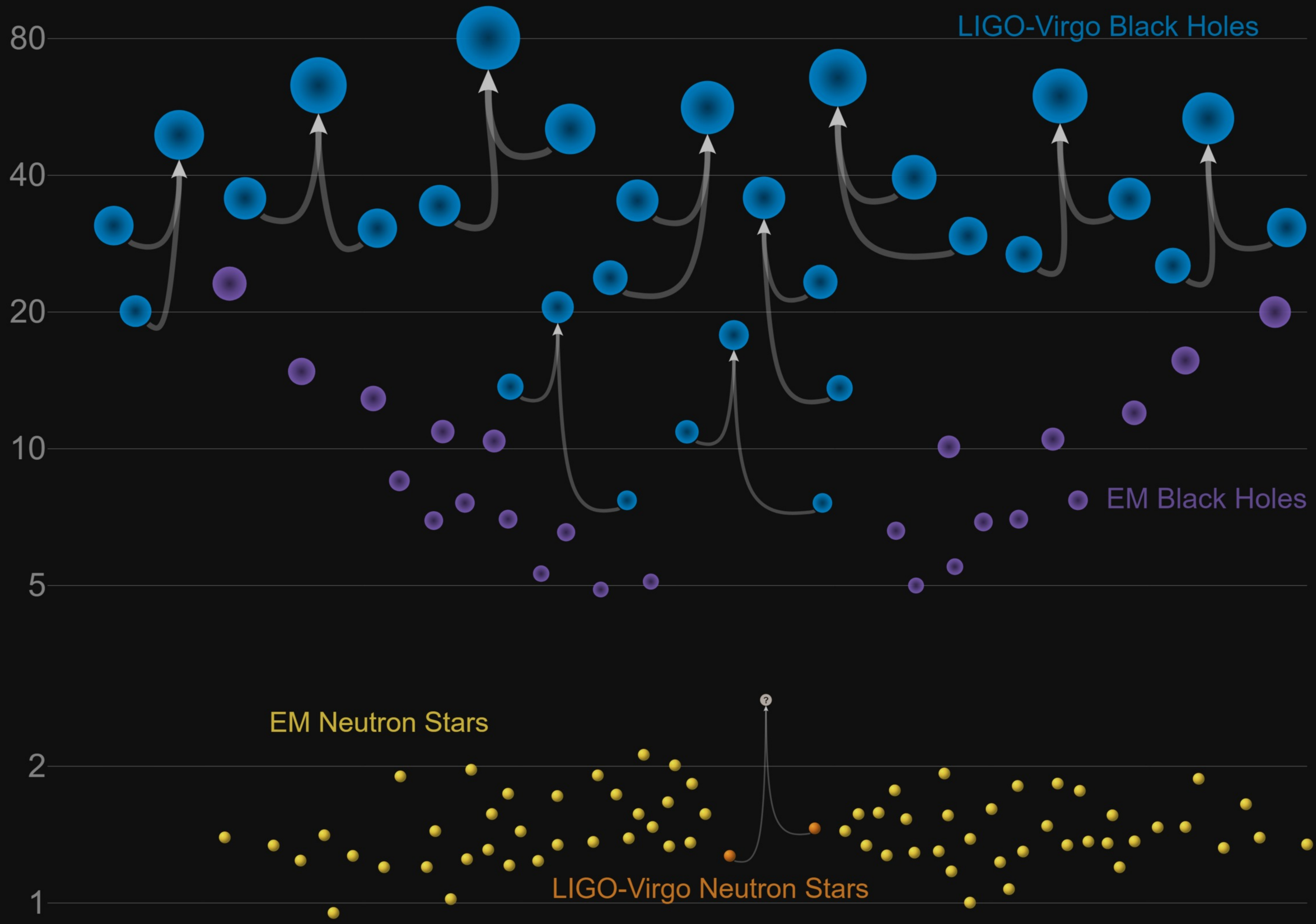
Belczynski et al., "The origin of low spin of black holes in LIGO/Virgo mergers," 1706.07053
Rodriguez et al., "Illuminating Black Hole Binary Formation Channels with Spins in Advanced LIGO," Astrophys. J. 832 (2016)
Piran and Hotokezaka, "On The Origin of LIGO's Merging Binary Black Holes," 1807.01336

In solar masses



LIGO-Virgo | Frank Elavsky | Northwestern

Watch out…

Watch out…

…I’m actually starting to believe my own story.

Could it be that the black holes that the gravitational wave instruments are measuring are black holes formed in the early universe?

The End

Quantum gravity and the microcanonical ensemble

[Gibbons Hawking '77]
[Brown York '93][Sen '12]
[Bianchi Haggard '18]

- QG with asymptotically-flat b.c.: microstates $|M, j, \alpha\rangle$ of fixed mass and spin

  orthonormal basis of $\mathcal{H}_{Mj}$

- *Microcanonical ensemble*: microstates of given energy *M* are uniformly populated

$$\rho_M = \frac{1}{\sum_{j'} \dim \mathcal{H}_{Mj'}} \sum_j \sum_\alpha |M, j, \alpha\rangle\langle M, j, \alpha|$$

- Microcanonical ensemble = mixture of microstates of fixed mass and spin

$$\rho_M = \sum_j p_M(j)\, \rho_{Mj} \qquad \text{where} \qquad \rho_{Mj} = \frac{1}{\dim \mathcal{H}_{Mj}} \sum_\alpha |M, j, \alpha\rangle\langle M, j, \alpha|$$

- Probability of finding spin *j* in the microcanonical ensemble

$$p_M(j) = \frac{\dim \mathcal{H}_{Mj}}{\sum_{j'} \dim \mathcal{H}_{Mj'}}$$

- Semiclassical one-loop calculation of the number of microstates

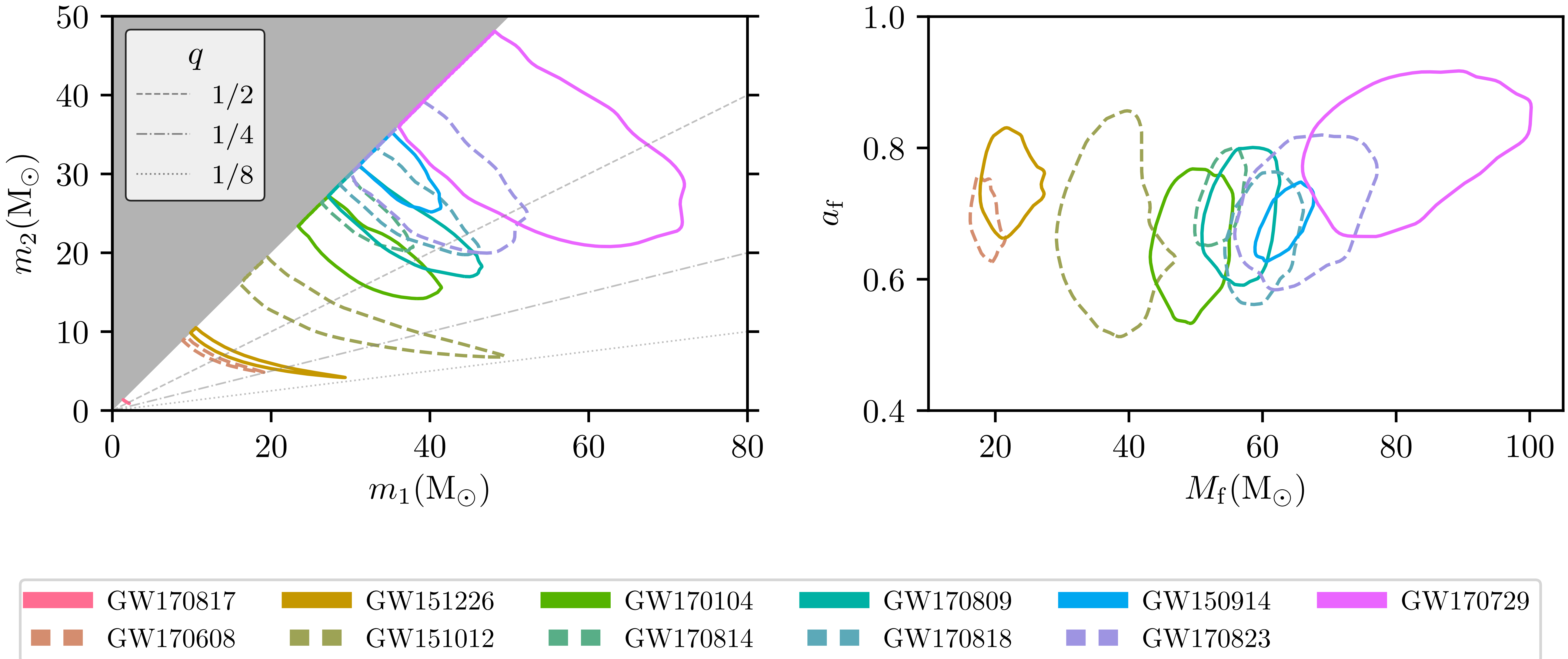
$$\dim \mathcal{H}_{Mj} \sim \sqrt{S(M, a_j)^{\frac{212}{45} - \frac{3}{2}}}\; e^{\,S(M,a_j)}\, a_j^2$$

where $a_j = \sqrt{j(j+1)}\, m_p^2/M^2$ and $S(M, a_j) = \dfrac{A(M, a_j)}{4\ell_P^2}$

$$\Longrightarrow \quad \boxed{P_M(a) = \frac{e^{A(M,a)/4\ell_P^2}\, a^2}{\int_0^1 e^{A(M,a')/4\ell_P^2}\, a'^2\, da'}}$$

Event	$m_1/\mathrm{M}_\odot$	$m_2/\mathrm{M}_\odot$	$\mathcal{M}/\mathrm{M}_\odot$	χ_{eff}	$M_{\mathrm{f}}/\mathrm{M}_\odot$	a_{f}	$E_{\mathrm{rad}}/(\mathrm{M}_\odot c^2)$	$\ell_{\mathrm{peak}}/(\mathrm{erg\ s^{-1}})$	d_L/Mpc	z	$\Delta\Omega/\mathrm{deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4} \times 10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	180
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.5^{+0.5}_{-0.5}$	$3.2^{+0.8}_{-1.7} \times 10^{56}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7} \times 10^{56}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.9} \times 10^{56}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$	924
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.05}_{-0.1}$	$3.5^{+0.4}_{-1.3} \times 10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	396
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5} \times 10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9} \times 10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5} \times 10^{56}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1 \times 10^{56}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7} \times 10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9} \times 10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

GW Transient Catalog GWTC-1 (Nov18)
LIGO/VIRGO Collab. 1811.12907 [astro-ph.HE]



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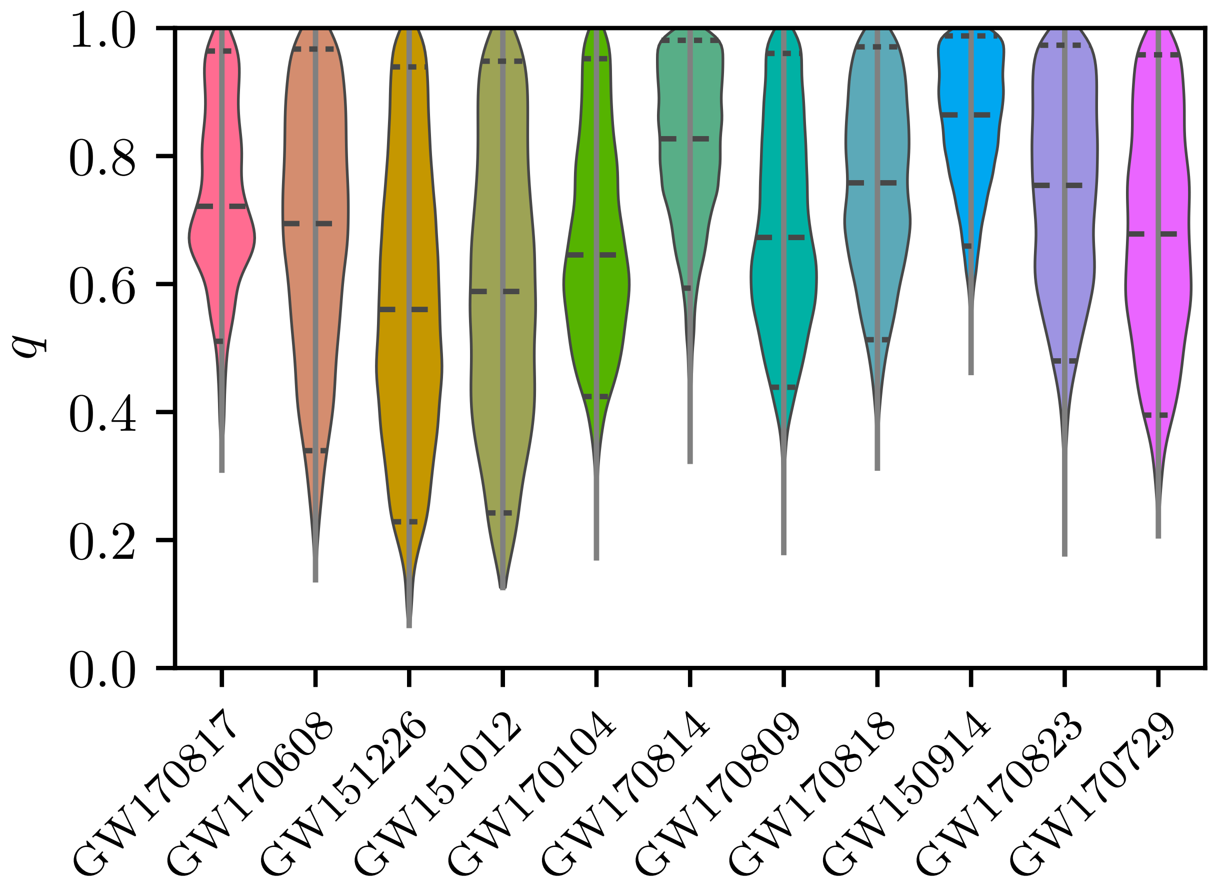

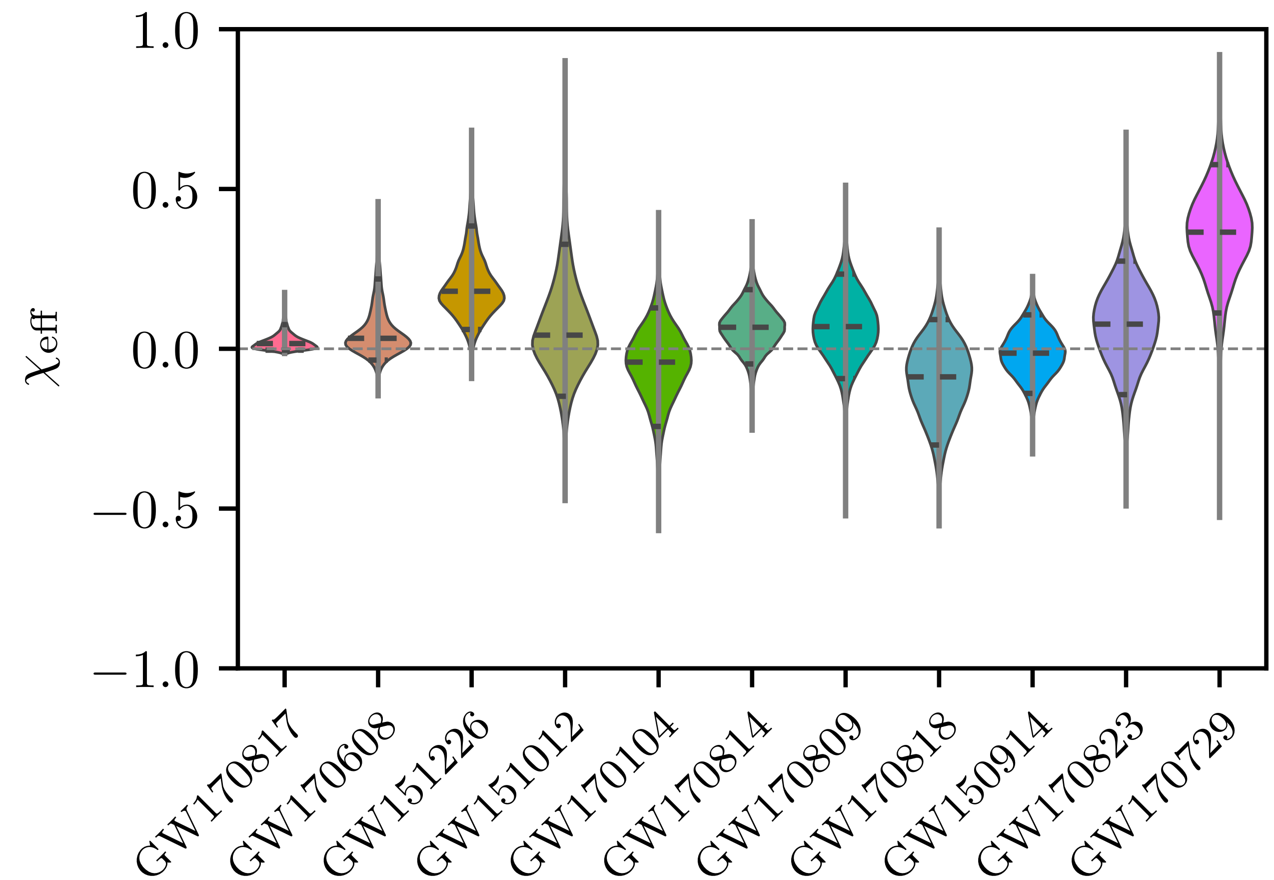


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