General boundary field theory, thermality, and entanglement

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Questions

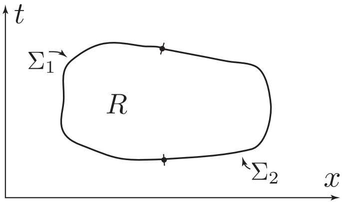
What do we learn from the thermality of event horizons?

Is horizon thermality local? Is there a quantum version of the equivalence principle?

How does entanglement speak with gravity and the boundary formalism?

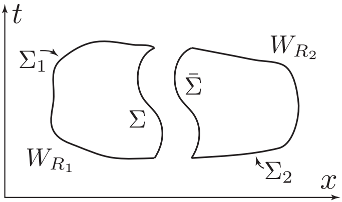
Structure of the general boundary formalism

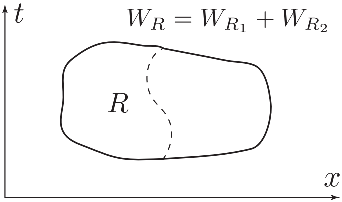
Fundamental ingredients: 1) Decomposition: Oeckl [1,2,3]



Hilbert space $\mathcal{B}$ of ∂R decomposes:
$\mathcal{B} = \mathcal{H}_1 \otimes \mathcal{H}_2$

2) Gluing: glue two manifolds along boundary, amplitudes add





Think of path integrals over regions. Rigorous in context of TQFT.

The Kubo-Martin-Schwinger condition

The KMS condition is standard for identifying thermality.

For a statistical state ρ and observable A

$$\langle A \rangle = \mathrm{Tr}[A\rho],$$

the correlation of A and B is

$$\langle AB \rangle = \mathrm{Tr}[AB\rho],$$

and the time-dependent correlation is

$$\langle A(t)B(0) \rangle = \mathrm{Tr}[e^{iHt}Ae^{-iHt}B\rho].$$

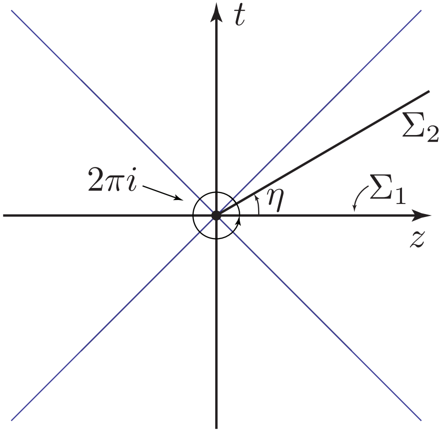
Then the KMS condition is

$$\langle A(t)B(0) \rangle = \langle B(0)A(t+i\beta) \rangle,$$

with β the inverse temperature.

Minkowski vacuum for wedge boundary?

What is the vacuum state in a Rindler wedge? (work w/ E. Bianchi)



The amplitude is:

$W_\Sigma[\varphi] = W_\eta[\varphi_1, \varphi_2]$.

Idea: Vacuum should be the path integral over the exterior of the wedge.

Boost through the light cone by analytically continuing:

- pick up $i\frac{\pi}{2}$ on each crossing of the light cone.

Builds on the ideas of Unruh and Weiss [4].

Wedge vacuum

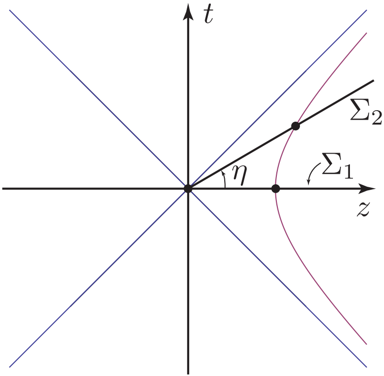
Wedge amplitude $W_\eta[\varphi_1, \varphi_2]$; conjecture $\Psi^0_\eta = W_{-\eta+2\pi i}[\varphi_1, \varphi_2]$.

To check it, do path integral for free scalar field:

$$\langle \varphi_1(x)\varphi_2(x)\rangle_\eta = \frac{\int \mathcal{D}\varphi \Psi^0_\eta \varphi_1 \varphi_2 \, W_\eta}{\int \mathcal{D}\varphi \Psi^0_\eta \, W_\eta}.$$

For insertions along accelerated trajectory

$$G(\tau) = \langle \varphi_1(x)\varphi_2(x)\rangle_{\eta = a\tau}$$
$$\sim \frac{1}{\sinh^2\left(\frac{\eta}{2}\right)} = \frac{1}{\sinh^2\left(\frac{a\tau}{2}\right)}$$



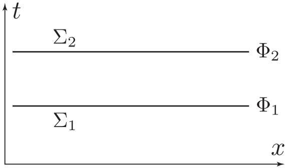
The Unruh effect: $G(\tau) = G(\tau + \frac{2\pi}{a} i)$, a thermal correlation.

Entanglement and the vacuum

The boundary formalism provides a new intepretation of the thermality of the Rindler dynamics.

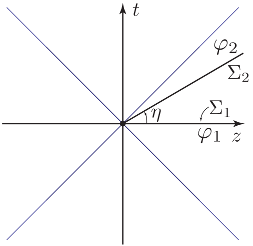
QFT: vacuum state factorizes

$$\Psi_t^0[\Phi_1, \Phi_2] = \psi^0[\Phi_1]\overline{\psi^0[\Phi_2]}$$



Vacuum state Rindler wedge:

$$\Psi_\eta^0[\varphi_1, \varphi_2] \neq f(\varphi_1)g(\varphi_2)$$



- Minkowski vacuum has entanglement between initial and final Rindler times

Results

♣ The Minkowski vacuum in the Rindler wedge can be described by $\Psi^0_\eta[\varphi_1, \varphi_2] \neq f(\varphi_1)g(\varphi_2)$.

Rindler horizons

In the near horizon limit all black hole horizons look like the Rindler horizon. (e.g. Jacobson & Parentani [5])

Analogs of black hole results can be derived for Rindler, e.g.:

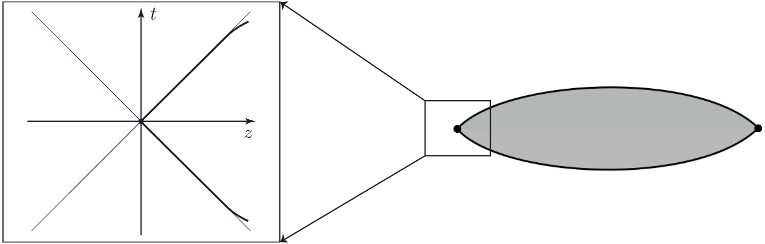
- $\Delta A_H = \frac{8\pi G}{\kappa}\Delta Q$ Bianch & Satz [6]
- $\Delta S_{\text{gen}} \geq 0$ Wall [7,8]
- $\Delta S_{\text{ent}} = \frac{\Delta A_H}{4G}$ Bianch [9]

For example, the last work provides new resolutions to:
1) Entanglement entropy of Rindler horizon divergent,
2) tuning of high energy cutoff Λ, 3) species problem.

These works all use the tools of perturbative quantum field theory and provide an accessible treatment of quantum Rindler horizons.

Quantum equivalence principle

The above references suggest and support a quantum version of the equivalence principle.



Near a corner a finite spacetime region looks like Rindler

Specific realization: assume that near corners physics is locally Lorentz invariant.

- Below we investigate the consequences of this assumption for non-perturbative, general-boundary gravity.

Non-relativistic formalism

With $\mathcal{H}_t \simeq \mathcal{H}$, define the boundary Hilbert space $\mathcal{B}_t = \mathcal{H}_0 \otimes \mathcal{H}_t^*$.

Mechanics

Two notable structures:

$W_t(\psi \otimes \phi^*) := \langle\phi|e^{-iHt}|\psi\rangle$,

and extend by linearity.

$\sigma : \psi \mapsto \psi \otimes (e^{-iHt}\psi)^*$.

$\Psi = \sigma(\psi)$ satisfy $W_t(\Psi) = 1$, call 'em "physical boundary states".

Stat. Mech.

Statistical state

$\rho = \sum_n c_n|n\rangle \otimes \langle n| \in \mathcal{B}_0$,

s.t. $\sum_n c_n = 1$.

Corresponding element of $\mathcal{B}_t$,

$\rho_t = \sum_n c_n|n\rangle\langle n|e^{iHt}$

and it is physical, $W_t(\rho_t) = 1$.

Non-relativistic formalism II

Which are the solutions of the physical state condition $\langle W_t|\Psi\rangle$?

Can show,

$$\langle W_t|\Psi\rangle = \sum_{nn'} c_{nn'}\langle n'|e^{iHt}e^{-iHt}|n\rangle = \sum_n c_{nn} = 1,$$

the trace class condition and so:

- They are the pure and statistical states from previous slide.

Notice that $\Psi \in \mathcal{B}_t$ is not generally normalized, instead

$$|\Psi|^2 = \sum_{nn'} |c_{nn'}|^2 \leq 1,$$

with equality if Ψ is a pure state.

Parallel structure: main idea is to include the time in the boundary state. $\mathcal{K}$ is Hilbert space of (generalized) states $\psi(x,t)$.

The boundary Hilbert space is $\mathcal{B} = \mathcal{K} \otimes \mathcal{K}^*$ (no t label needed).

Mechanics

Two notable structures:

$W(\psi \otimes \phi^*) := \langle \phi | P | \psi \rangle,$

and extend by linearity.
In a Schrödinger basis

$$\begin{aligned} W(x,t,x',t') &= \langle x',t' | P | x,t \rangle \\ &= \langle x' | e^{i(t-t')H} | x \rangle. \end{aligned}$$

$\sigma : \psi \mapsto \psi \otimes \psi^*.$
Image(σ): $W(\Psi) = 1.$

Stat. Mech.

ψ_n soln t-dep Schrödinger eq

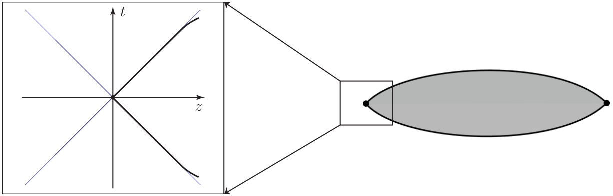
$$\rho = \sum_n c_n \psi_n \psi_n^*,$$

s.t. $\sum_n c_n = 1.$

E.g.,

$$\rho(x,t,x',t') = \sum_n c_n \, \psi_n(x,t) \, \overline{\psi_n(x',t')}.$$

Finite regions: the quantum equivalence principle & Unruh effect



$\rightsquigarrow$ all local boundary states are mixed. But $\mathcal{B}$ can be made bipartite in many different manners, so better to say *non-separable*.

- Local gravitational states are always entangled states

Remarkably, the complete absence of physical pure states means that there is no distinction between quantum and statistical fluctuations in quantum gravity. (see also Smolin [10, 11])

Results

♣ The Minkowski vacuum in the Rindler wedge can be described by $\Psi^0_\eta[\varphi_1, \varphi_2] \neq f(\varphi_1)g(\varphi_2)$.

♦ In quantum gravity finite spacetime regions are the describable physical processes.

These regions are *always* entangled $\rightsquigarrow$ there is no fundamental distinction between statistical and quantum fluctuations.

Spherical causal domain

Results leading us to reconsider the Cauchy development of spherical regions:

- Foliation leads to repacking of the Rindler trajectories

- Thermal properties subtle: detectors thermalize but region is not in equilibrium

- $m = 0$: Spherical entangling surface $\rightsquigarrow |0\rangle \sim e^{-\pi C_{in}}$, Sorkin-Johnston vacuum [12, 13, 14] in these regions?

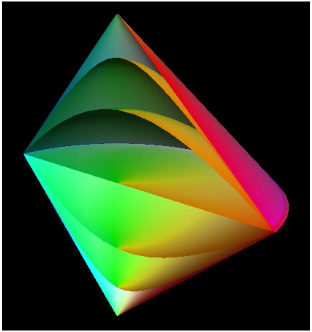


Figure: 2+1 spacetime cutaway visualization of spherical causal domain and hyperbolic foliation

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♦ In quantum gravity finite spacetime regions are the describable physical processes.

These regions are *always* entangled $\rightsquigarrow$ there is no fundamental distinction between statistical and quantum fluctuations.

♠ Revealing new ideas about finite spacetime regions: thermality, vacua, and horizon entanglement