# Death and Resurrection of the Zeroth Principle of Thermodynamics

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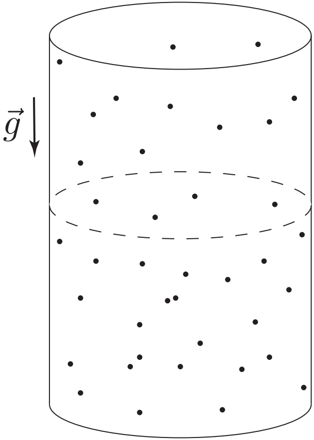
April 3rd, 2013
Quantum Fields, Gravity and Information

[PRD **87**, 084001], gr-qc/1302.0724

# Equilibrium

Gas in const gravitational field:

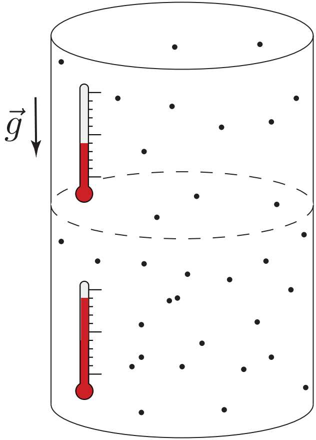



$0^{\text{th}}$ principle: At equilibrium $T$ is constant throughout.

False! Need to account for relativistic effects.

# Equilibrium

Gas in const gravitational field:




Instead:

$$T_1\left(1+\frac{\Phi(h_1)}{c^2}\right)=T_2\left(1+\frac{\Phi(h_2)}{c^2}\right),$$

the Tolman-Ehrenfest law.

- Gas is hotter at the bottom

Small effect: at surface of Earth

$$\frac{\nabla T}{T}=10^{-18}\mathrm{cm}^{-1}.$$

# Equilibrium

We have to separate two previously identical notions:

i) temperature $T$ as measured by a thermometer and

ii) the label, call it $\tau_o$, that says two bodies are in equilibrium

What killed $T$ as this label?
In microcanonical:

$$S = k \ln N(E); \qquad \frac{1}{kT} \equiv \frac{dS(E)}{dE}.$$

Get equilibrium maximizing $N = N_1 N_2$ under energy transfer $dE$.
Gives $T_1 = T_2$.

Conservation of energy is tricky in GR, intuitively $dE$ "weighs."

# Questions

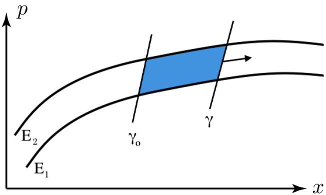
- Is there a more general statistical argument that governs equilibrium in a relativistic context?
- Can we get the Tolman law from generalization of maximizing # micorostates, without a model for $dE$?
- Is it possible to understand equilibrium in a generally covariant context (thermal energy also flowing to gravity)? Is there a general principle that retains its meaning in the absence of a background spacetime?

We attack these questions by considering *processes*, or histories and by associating an information content to an history.

# Histories

$$\dot{x} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial x}$$

**Semiclassical ensemble:**




**Overlap:**

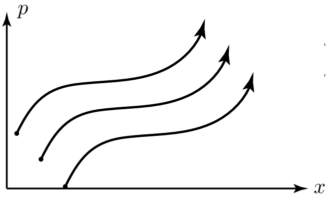
$$P(t) = |\langle \psi(0) | \psi(t) \rangle|^2$$

**Timescale:**

$$\frac{d^2 P}{dt^2} = \frac{1}{\hbar^2}\left(\langle H \rangle^2 - \langle H^2 \rangle\right) = -\frac{(\Delta E)^2}{\hbar^2}$$

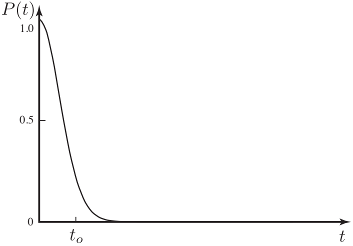
$$\Longrightarrow \boxed{t_o = \frac{\hbar}{\Delta E}}$$

**Classical:**




$$\frac{dV(t)}{dt} = \Delta E \implies t_o = \frac{\hbar}{\Delta E}$$

**Quantum:**



# All things thermal

Specialize to thermal states and dynamics. Then $U \sim \Delta E \sim kT$. The mean time to move from state to state is

$$t_o = \frac{\hbar}{kT},$$

a fundamental time step. Universal!

Introduce "thermal time,"

$$\tau = \frac{t}{t_o} = \frac{kT}{\hbar} t,$$

time measured in steps of $t_o$; or the # of (distinct) states transited in time $t$. Parameter of Tomita flow, Connes-Rovelli [1,2].

♦ Unveils informational meaning of temperature ($\hbar = k = 1$): $T = \frac{\tau}{t}$ *is the number of states transited per unit time.*

# A new postulate

Now, we can associate information to an history: it is the # of states $N$ the system has transited during the history's duration.

- Agrees with Shannon's notion of information—a # of states.

Allow two systems to interact; System 2 has access to info

$$I_1 = \log N_1$$

about System 1 and similarly in reverse. Introduce the information *flow* $\delta I = I_2 - I_1$.

Equilibrium: time-reversal invariant $\implies$ all net flows vanish. So we postulate,

$$\delta I = 0,$$

as a condition for equilibrium. Check merit with applications.

# Applications

Postulated $\delta I = 0 \implies N_1 = N_2$, but for # of states to be equal in some interval, the transit rates must be equal,

$$\tau_1 = \tau_2 \qquad \text{(recall } \tau = \frac{kT}{\hbar} t\text{)}$$

Non-relativistic equilibrium: time is universal, $t$ dependence of thermal time cancels and so,

$$\tau_1 = \tau_2 \implies T_1 = T_2.$$

Relativistic equilibrium: (proper) time is a local quantity $ds$, then,

$$d\tau = \frac{kT}{\hbar} ds$$

Spacetime should be stationary, i.e. have a timelike Killing field $\xi$.

# Applications cont

Relativistic equilib.: thermal time $d\tau = (kT/\hbar)ds$, Killing field $\xi$.

Proper time along $\xi$-orbits is $ds = |\xi| dt$, $t$ an affine parameter.

Equilibrium: $\tau_1 = \tau_2$ during interaction interval $\Delta t$ gives,

$$|\xi|_1 T_1 = |\xi|_2 T_2 \qquad \text{or} \qquad \boxed{|\xi| T = \text{const.},}$$

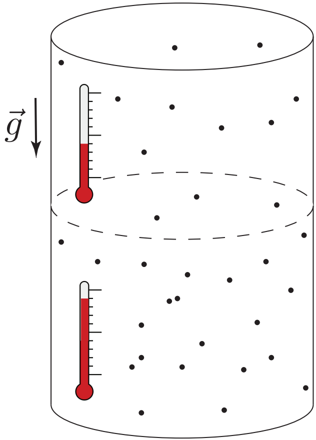
this is the covariant form of the Tolman-Ehrenfest law!

Take $ds^2 = g_{00}(\vec{x}) dt^2 - g_{ij}(\vec{x}) x^i x^j$, $\xi = \partial/\partial t$; note $|\xi| = \sqrt{g_{00}}$ and in the Newtonian limit, $g_{00} = 1 + 2\Phi/c^2$, you recover the expression on slide 2.

- Can derive Wien's displacement law as well.

# Equilibrium: thermal time is constant

Gas in const gravitational field:




- Gas is hotter at the bottom

Identical clocks at different altitudes run at different rates, "slouching clocks run slow."

The temperature has to be higher at low altitudes; the faster state transitions compensate exactly the slowing down of proper time.

The upper and lower systems transit the same number of states during interaction interval $\Delta t$.

# Results

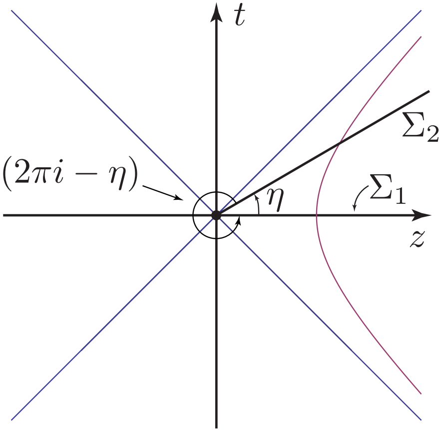
♣ "Two histories are in equilibrium if the net information flow between them vanishes, namely, if they transit the same number of states during the interaction period."

♠ Equivalently, the thermal time $\tau$ elapsed for the two systems is the same. This is time measured in elementary time steps $t_o$.

♦ Temperature is the rate at which systems move from state to state.

# Conclusion

Work in progress with E. Bianchi: general boundary Unruh effect.




(Oeckl [3]) Amplitude:

$Z_\Sigma[\phi] = Z_\eta[\phi_1, \phi_2]$.

We argue for the vacuum

$\Psi^0_\eta[\phi_1, \phi_2] = Z_{2\pi i - \eta}[\phi_1, \phi_2]$.

And the KMS property becomes manifest! Implies the Unruh effect.

The vacuum

$\Psi^0_\eta[\phi_1, \phi_2] \neq f[\phi_1]g[\phi_2]$

doesn't factorize.
"Thermality is entanglement in time."

Should fit well with Olson and Ralph's works [4,5].