General boundary field theory, thermality, and entanglement

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General boundary purposes

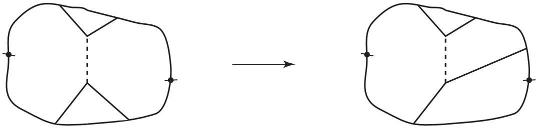
The boundary formalism is useful in quantum gravity:

- Allows generally covariant approach to quantum field theory
- Standard quantum field theory is included in the formalism
- Puts the dependent and independent variables on the same footing by focusing on *processes*

More features

- Deals with actual laboratory physics: finite spacetime regions
- Unifies the ideas of state preparation and state measurement (or destruction) $\rightsquigarrow$ just boundary observation
- Crossing symmetry of QFT becomes manifest

$$A + B \to C + D \implies \begin{array}{c} A \to \bar{B} + C + D \\ A + \bar{C} \to \bar{C} + D \qquad \dots \end{array}$$



Questions

What do we learn from the thermality of event horizons?

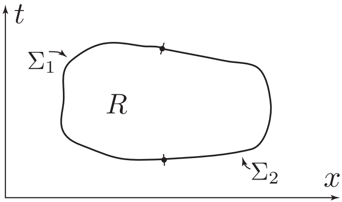
Is horizon thermality local? Is there a quantum version of the equivalence principle?

How does entanglement speak with gravity and the boundary formalism?

Outline

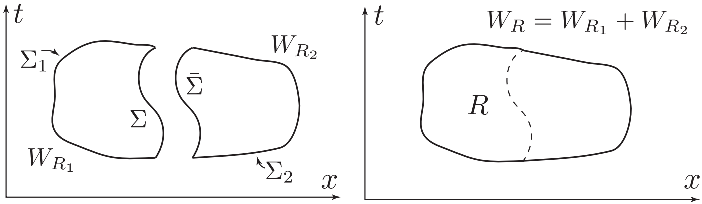
Structure of the general boundary formalism

Fundamental ingredients: 1) Decomposition: Oeckl [1,2,3]



Hilbert space $\mathcal{B}$ of ∂R decomposes: $\mathcal{B} = \mathcal{H}_1 \otimes \mathcal{H}_2$

2) Gluing: glue two manifolds along boundary, amplitudes add



Think of path integrals over regions. Rigorous in context of TQFT.

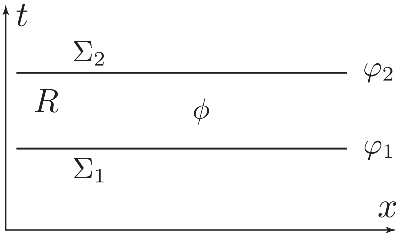
The vacuum

Minkowski vacuum Conrady et al [4]:

$$|0_M\rangle\langle 0_M| = \lim_{T\to\infty} \sum_n e^{-E_n T} |n\rangle\langle n| \equiv \lim_{T\to\infty} W(T),$$

with $|n\rangle$ a Fock basis. In a field basis,

$$W[\varphi_1, \varphi_2, T] = \langle \varphi_2 | W(T) | \varphi_1 \rangle = \int_{\phi|_{t=0}=\varphi_1}^{\phi|_{t=T}=\varphi_2} \mathcal{D}\phi\, e^{-S_E^T[\phi]}.$$



$$\begin{aligned} \text{Vacuum in } \mathcal{H}_1: \quad & \Psi_M^0[\varphi_1] = \lim_{T\to\infty} W[0, \varphi_1, T] \\ \mathcal{H}_2^*: \quad & \Psi_M^0[\varphi_2] = \lim_{T\to\infty} W[\varphi_2, 0, T] \end{aligned}$$

The vacuum

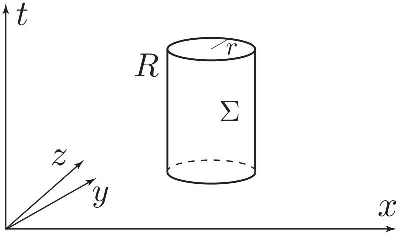
Non-perturbative case $\rightsquigarrow$ metric included as boundary field:

$$W[\varphi, \Sigma] = \int_{\phi|_\Sigma = \varphi} \mathcal{D}\phi \, e^{-S_E^R[\phi]}$$

Minkowski vacuum in $\mathcal{B} = \mathcal{H}_{in} \otimes \mathcal{H}_{out}^* \otimes \mathcal{H}_{side}$,

$$\Psi_M^0[\varphi] = \lim_{r, T \to \infty} W[\varphi_{in}, \varphi, \varphi_{rT}],$$

where $\varphi_{rT} = (0, g_{rT})$.



The Kubo-Martin-Schwinger condition

The KMS condition is standard for identifying thermality.

For a statistical state ρ and observable A

$$\langle A \rangle = \mathrm{Tr}[A\rho],$$

the correlation of A and B is

$$\langle AB \rangle = \mathrm{Tr}[AB\rho],$$

and the time-dependent correlation is

$$\langle A(t)B(0) \rangle = \mathrm{Tr}[e^{iHt}Ae^{-iHt}B\rho].$$

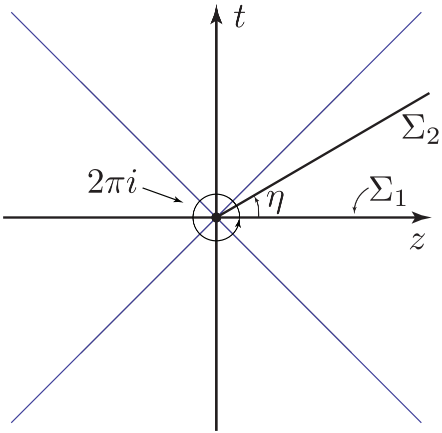
Then the KMS condition is

$$\langle A(t)B(0) \rangle = \langle B(0)A(t+i\beta) \rangle,$$

with β the inverse temperature.

Minkowski vacuum for wedge boundary?

What is the vacuum state in a Rindler wedge? (work w/ E. Bianchi)



Amplitude as above:

$$W_\Sigma[\varphi] = W_\eta[\varphi_1, \varphi_2].$$

Idea: Vacuum should be the path integral over the exterior of the wedge.

Boost through the light cone by analytically continuing:

- pick up $i\frac{\pi}{2}$ on each crossing of the light cone.

Builds on the ideas of Unruh and Weiss [5].

Wedge vacuum

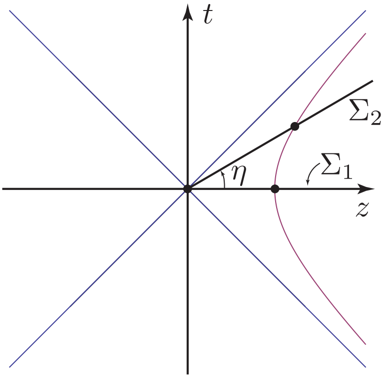
Wedge amplitude $W_\eta[\varphi_1, \varphi_2]$; conjecture $\Psi^0_\eta = W_{-\eta+2\pi i}[\varphi_1, \varphi_2]$.

To check it, do path integral for free scalar field:

$$\langle \varphi_1(x)\varphi_2(x)\rangle_\eta = \frac{\int \mathcal{D}\varphi \Psi^0_\eta \varphi_1 \varphi_2 W_\eta}{\int \mathcal{D}\varphi \Psi^0_\eta W_\eta}.$$

For insertions along accelerated trajectory

$$G(\tau) = \langle \varphi_1(x)\varphi_2(x)\rangle_{\eta=a\tau} \sim \frac{1}{\sinh^2\left(\frac{\eta}{2}\right)} = \frac{1}{\sinh^2\left(\frac{a\tau}{2}\right)}$$



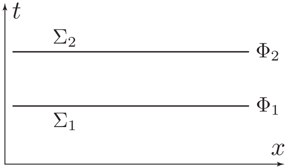
The Unruh effect: $G(\tau) = G(\tau + \frac{2\pi}{a} i)$, a thermal correlation.

Entanglement and the vacuum

The boundary formalism provides a new intepretation of the thermality of the Rindler dynamics.

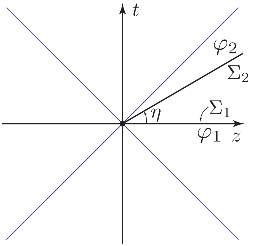
QFT: vacuum state factorizes

$$\Psi_t^0[\Phi_1, \Phi_2] = \psi^0[\Phi_1]\overline{\psi^0[\Phi_2]}$$



Vacuum state Rindler wedge:

$$\Psi_\eta^0[\varphi_1, \varphi_2] \neq f(\varphi_1)g(\varphi_2)$$



- Minkowski vacuum has entanglement between initial and final Rindler times

Outline

Rindler horizons

In the near horizon limit all black hole horizons look like the Rindler horizon. (e.g. Jacobson & Parentani [6])

Analogs of black hole results can be derived for Rindler, e.g.:

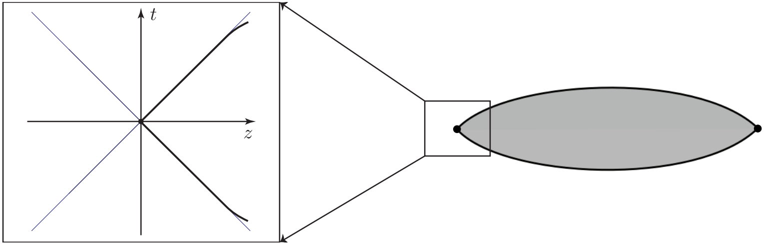
- $\Delta A_H = \frac{8\pi G}{\kappa}\Delta Q$ Bianch & Satz [7]
- $\Delta S_{\text{gen}} \geq 0$ Wall [8,9]
- $\Delta S_{\text{ent}} = \frac{\Delta A_H}{4G}$ Bianch [10]

For example, the last work provides new resolutions to:
1) Entanglement entropy of Rindler horizon divergent,
2) tuning of high energy cutoff Λ, 3) species problem.

These works all use the tools of perturbative quantum field theory and provide an accessible treatment of quantum Rindler horizons.

Quantum equivalence principle

The above references suggest and support a quantum version of the equivalence principle.



Near a corner a finite spacetime region looks like Rindler

Specific realization: assume that near corners physics is locally Lorentz invariant.

- Below we investigate the consequences of this assumption for non-perturbative, general-boundary gravity.

Outline

Milo

Milo

Milo calculaing with generally covariant statistical mechanics!

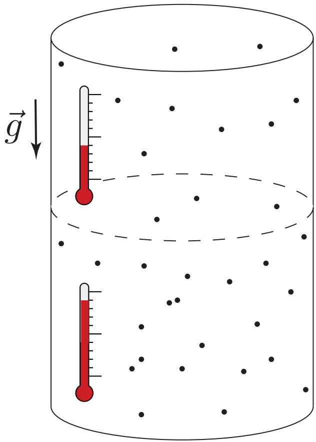





Gravitational equilibrium

Informational notions clarify local equilibrium where $T(t, \vec{x})$:

Gas in const gravitational field:



- Gas is hotter at the bottom

Identical clocks at different altitudes run at different rates, "slouching clocks run slow."

The temperature has to be higher at low altitudes; the faster state transitions compensate exactly the slowing down of proper time.

The upper and lower systems transit the same number of states during interaction interval Δt.

[HMH & Rovelli], gr-qc/1302.0724

Strategy

I want to explain the sense in which the boundary formalism shifts focus from ‘states’ to ‘processes’.

This exploration leads naturally to the identification and inclusion of statistical states in the boundary formalism.

The strategy will be to move “upwards" towards more covariance by proceeding as follows:
non-relativistic formalism $\rightsquigarrow$ relativistic formalism $\rightsquigarrow$ general-covariant boundary formalism

Non-relativistic formalism

With $\mathcal{H}_t \simeq \mathcal{H}$, define the boundary Hilbert space $\mathcal{B}_t = \mathcal{H}_0 \otimes \mathcal{H}_t^*$.
This is the space of kinematical *processes*, e.g. $\Psi = \psi \otimes \phi^*$.

Mechanics

Two notable structures:

$W_t(\psi \otimes \phi^*) := \langle \phi | e^{-iHt} | \psi \rangle$,

and extend by linearity. In a Schrödinger basis $|x, x'\rangle$

$$W_t(x, x') = \langle x' | e^{-iHt} | x \rangle.$$

$\sigma : \psi \mapsto \psi \otimes (e^{-iHt}\psi)^*$.
Ψ in Image(σ) satisfy $W_t(\Psi) = 1$, call them "physical boundary states".

Stat. Mech.

Statistical state

$\rho = \sum_n c_n |n\rangle \otimes \langle n| \in \mathcal{B}_0$,

s.t. $\sum_n c_n = 1$.

There is a corresponding element of $\mathcal{B}_t$,

$\rho_t = \sum_n c_n |n\rangle\langle n| e^{iHt}$

and it is physical, $W_t(\rho_t) = 1$.

$\langle A(t) B(0) \rangle = W_t(\ (B \otimes A)\ \rho_t)$

Non-relativistic formalism II

Which are the solutions of the physical state condition $\langle W_t|\Psi\rangle$?

Due to unitarity $(e^{-iHt}|n\rangle)^*$ is a basis of $\mathcal{H}_t^*$. When $\mathcal{B}_t = \mathcal{H}_0 \otimes \mathcal{H}_t^*$,

$$\mathcal{B}_t \ni \Psi = \textstyle\sum_{nn'} c_{nn'} |n\rangle \otimes (e^{-iHt}|n'\rangle)^*.$$

Therefore,

$$\langle W_t|\Psi\rangle = \textstyle\sum_{nn'} c_{nn'} \langle n'|e^{iHt}e^{-iHt}|n\rangle = \sum_n c_{nn} = 1,$$

is precisely the trace class condition and so:

- They are the pure and statistical states from previous slide.

Notice that $\Psi \in \mathcal{B}_t$ is not generally normalized, instead

$$|\Psi|^2 = \textstyle\sum_{nn'} |c_{nn'}|^2 \leq 1,$$

with equality if Ψ is a pure state.

Relativistic formalism

Parallel structure: main idea is to include the time in the boundary state. $\mathcal{K}$ is Hilbert space of (generalized) states $\psi(x,t)$.

The boundary Hilbert space is $\mathcal{B} = \mathcal{K} \otimes \mathcal{K}^*$ (no t label needed).

Mechanics

Two notable structures:

$W(\psi \otimes \phi^*) := \langle \phi | P | \psi \rangle$,

and extend by linearity. In a Schrödinger basis

$$W(x,t,x',t') = \langle x',t' | P | x,t \rangle \\ = \langle x' | e^{i(t-t')H} | x \rangle.$$

$\sigma : \psi \mapsto \psi \otimes \psi^*$.
Image(σ): $W(\Psi) = 1$.

Stat. Mech.

ψ_n soln t-dep. Schrödinger eq

$$\rho = \sum_n c_n \psi_n \psi_n^*,$$

s.t. $\sum_n c_n = 1$. E.g. ,

$$\rho(x,t,x',t') = \\ \sum_n \; c_n \; \psi_n(x,t) \; \overline{\psi_n(x',t')}.$$

$\langle AB \rangle = W(\; (B \otimes A) \; \rho)$

General boundary

Quantum system: $(\mathcal{B}, \mathcal{A}, W)$, the boundary Hilbert space, an algebra of observables, and a linear map defining the dynamics.

$\Psi \in \mathcal{B}$ represent processes. If $A(\Psi) = a\Psi$, it represents a process where the corresponding boundary observable has value a.

$W(\Psi) = \langle W|\Psi\rangle$ is the amplitude of a process. Its modulus square determines the relative probability of distinct processes.

Physical processes $\Psi \in \mathcal{B}$ have amplitude one, $\langle W|\Psi\rangle = 1$.

If a tensor structure in $\mathcal{B}$ is not given, then there is no *a priori* distinction between pure and mixed states.

Quantum Gravity

Study probability amplitudes for local processes by associating boundary states to a finite portion of spacetime, *and including the quantum dynamics of spacetime itself in the process*.

Quantum equivalence principle & Unruh effect $\rightsquigarrow$ all local boundary states are mixed. But $\mathcal{B}$ can be made bipartite in many different manners, so better to say *non-separable*.

- Local gravitational states are entangled states

Remarkably, the complete absence of physical pure states means that there is no distinction between quantum and statistical fluctuations in quantum gravity. (see also Smolin [11 12])

Results

♣ The Minkowski vacuum in the Rindler wedge can be described by $\Psi^0_\eta[\varphi_1, \varphi_2] \neq f(\varphi_1)g(\varphi_2)$.

♠ Opens the way to a general covariant treatment of quantum statistical mechanics. Rovelli [13]

♦ In quantum gravity finite spacetime regions are the describable physical processes.

It appears that these regions are *always* entangled $\rightsquigarrow$ there is no fundamental distinction between statistical and quantum fluctuations.