Bard School
Computing Lab

Cosmic Microwave Background

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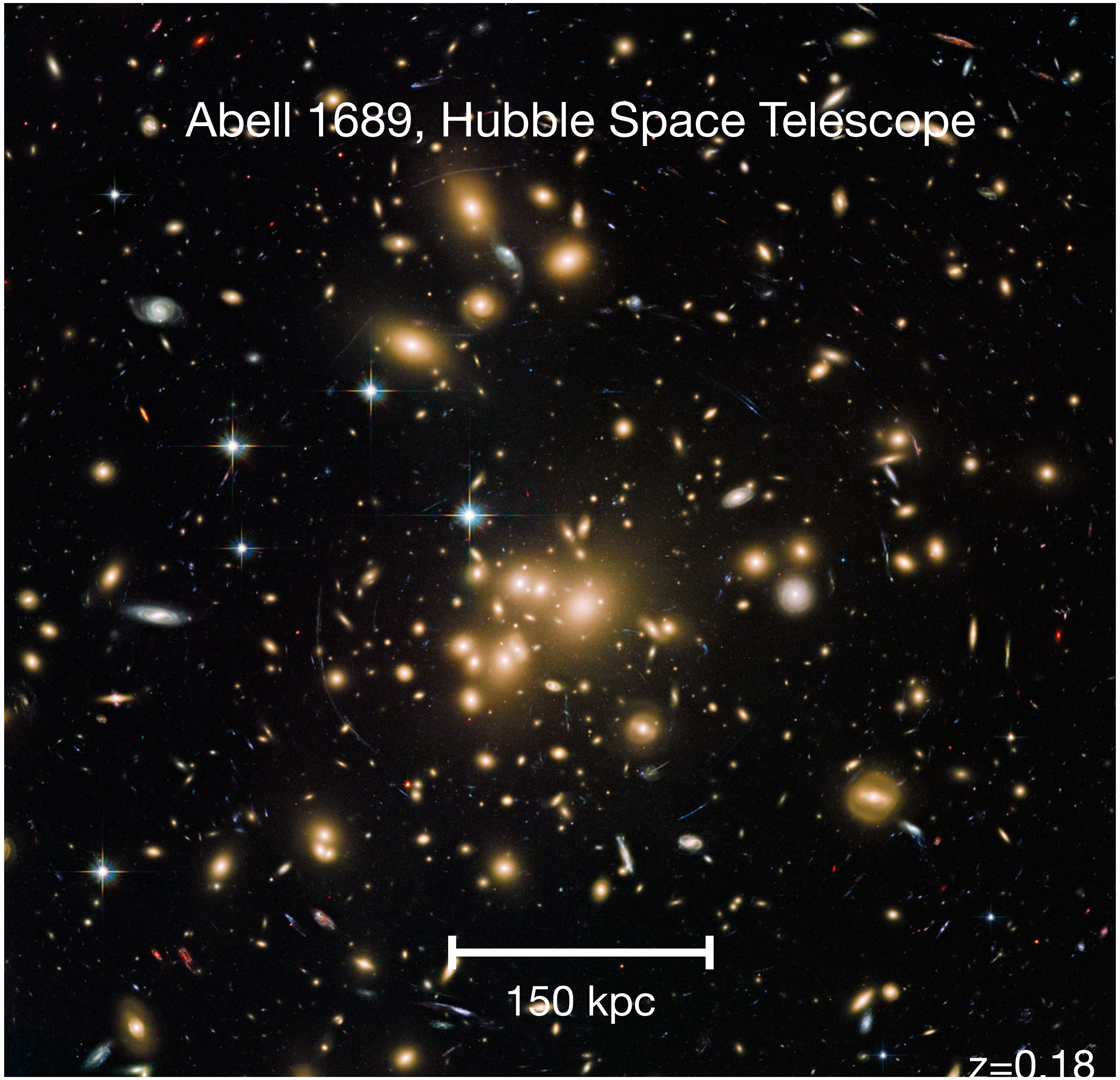
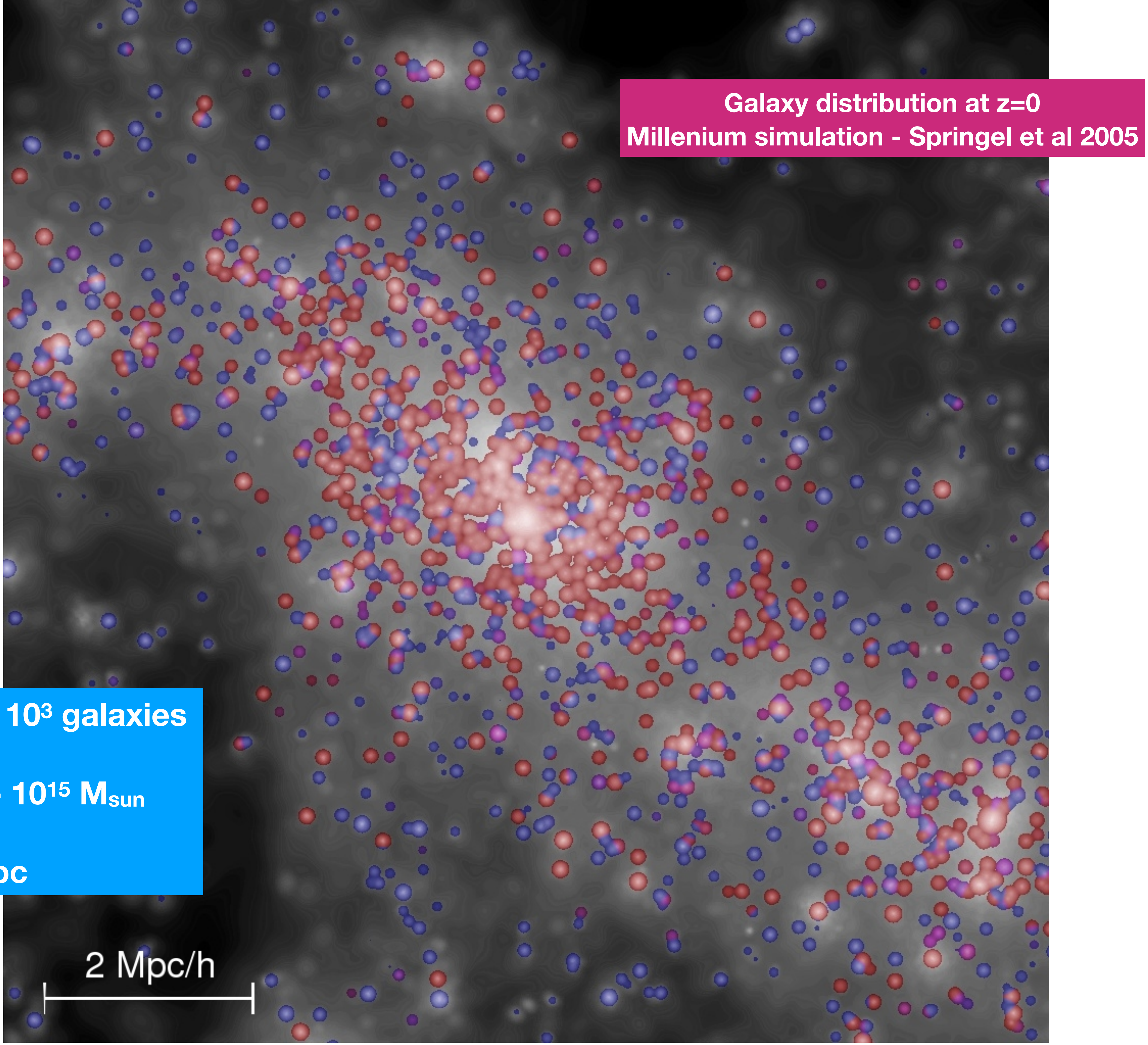
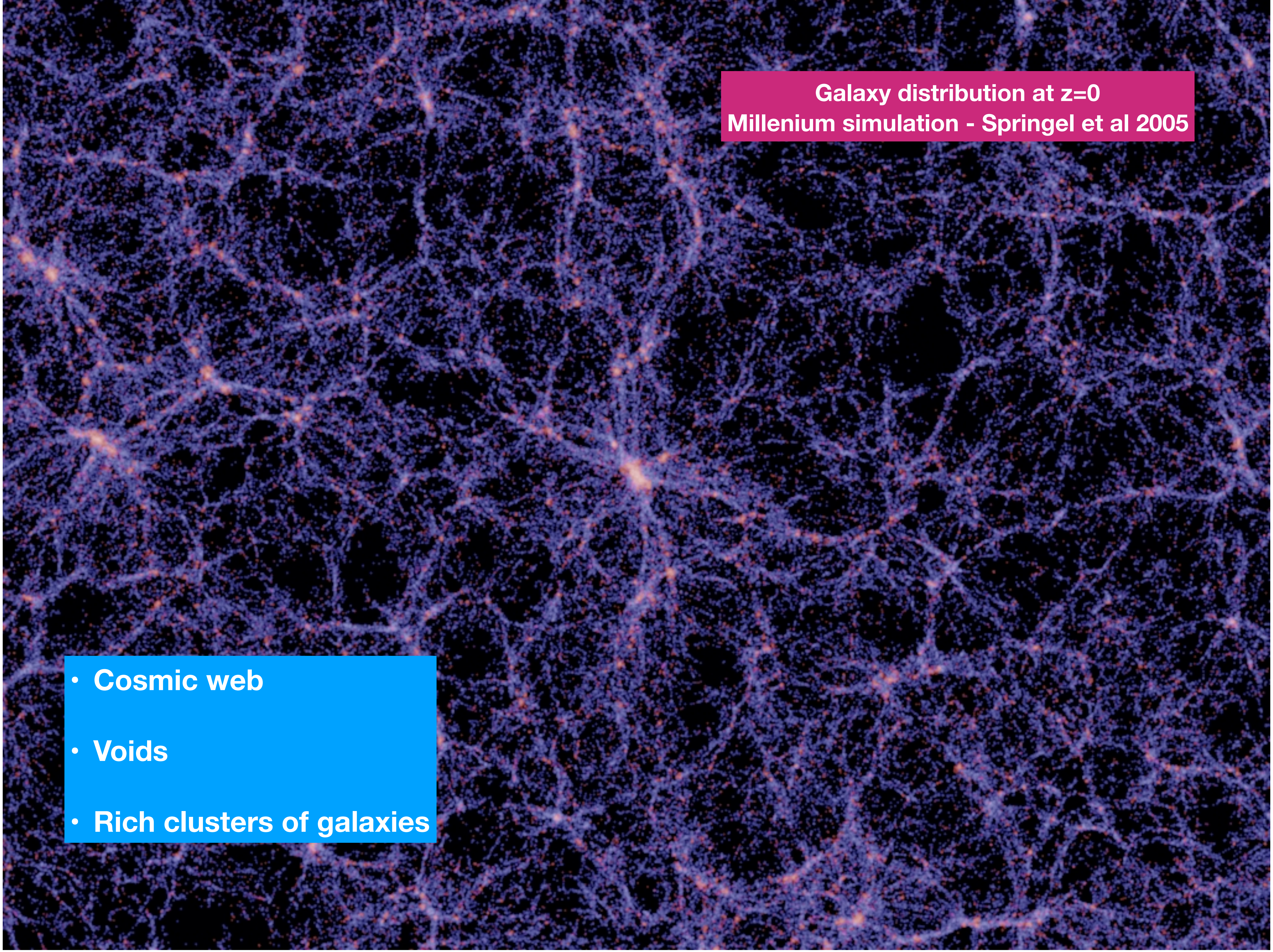
Day 1:

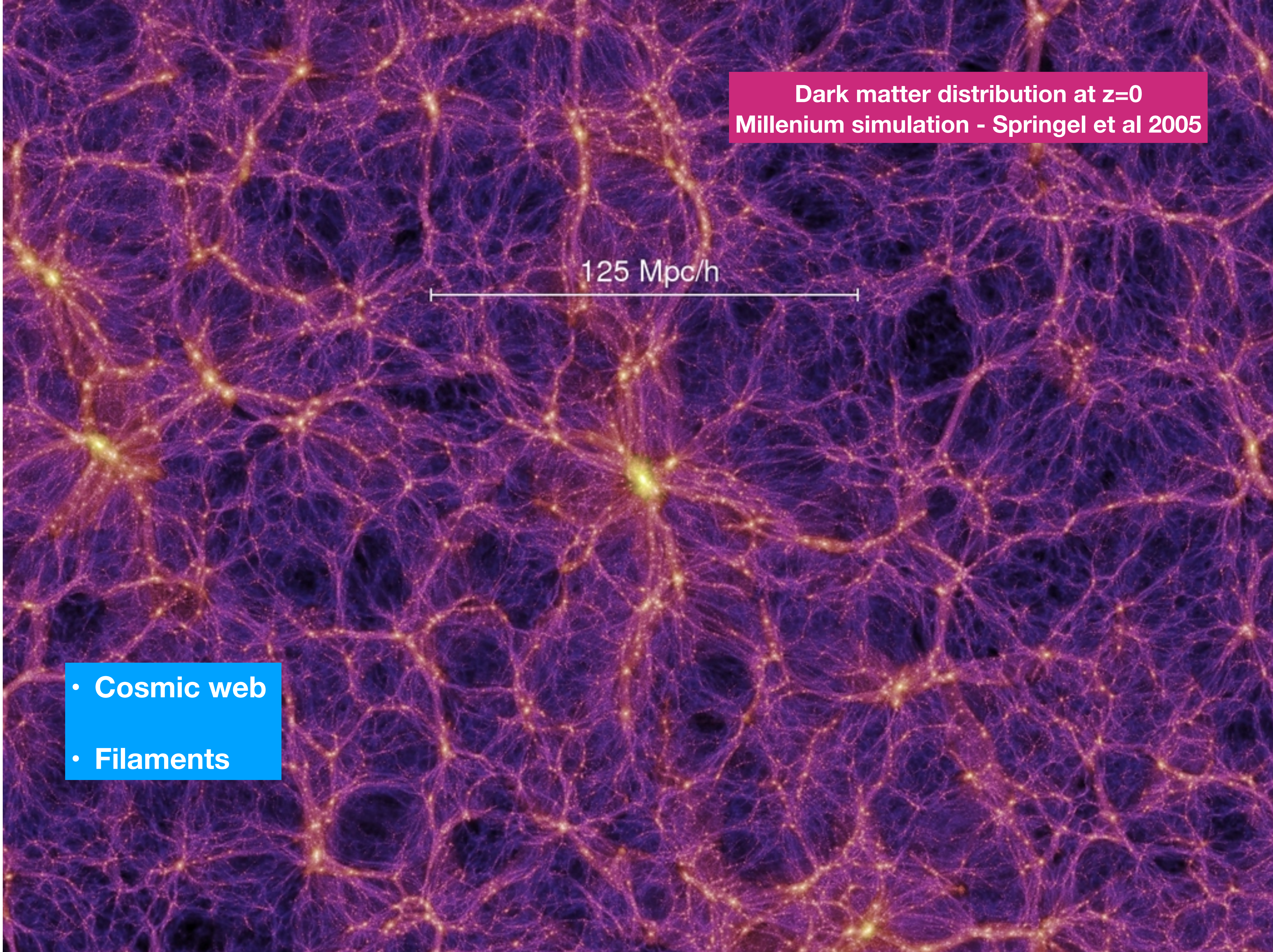
- Intro + questions: 15’
- CLASS: 30’
- Jupiter Notebook and Plotting: 30’
- Physical Understanding based on our numerics: 45’

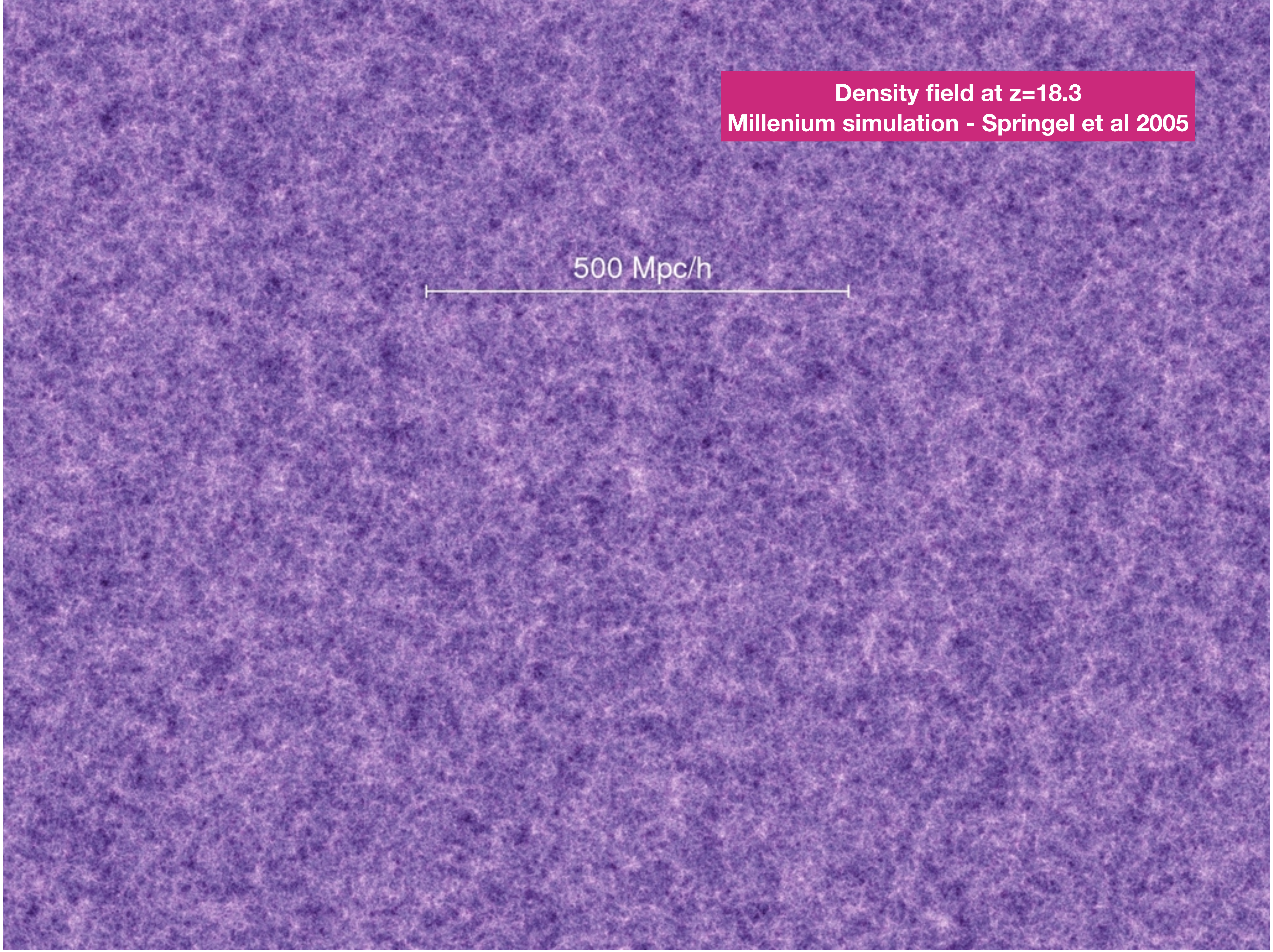

Abell 1689, Hubble Space Telescope
150 kpc
z=0.18



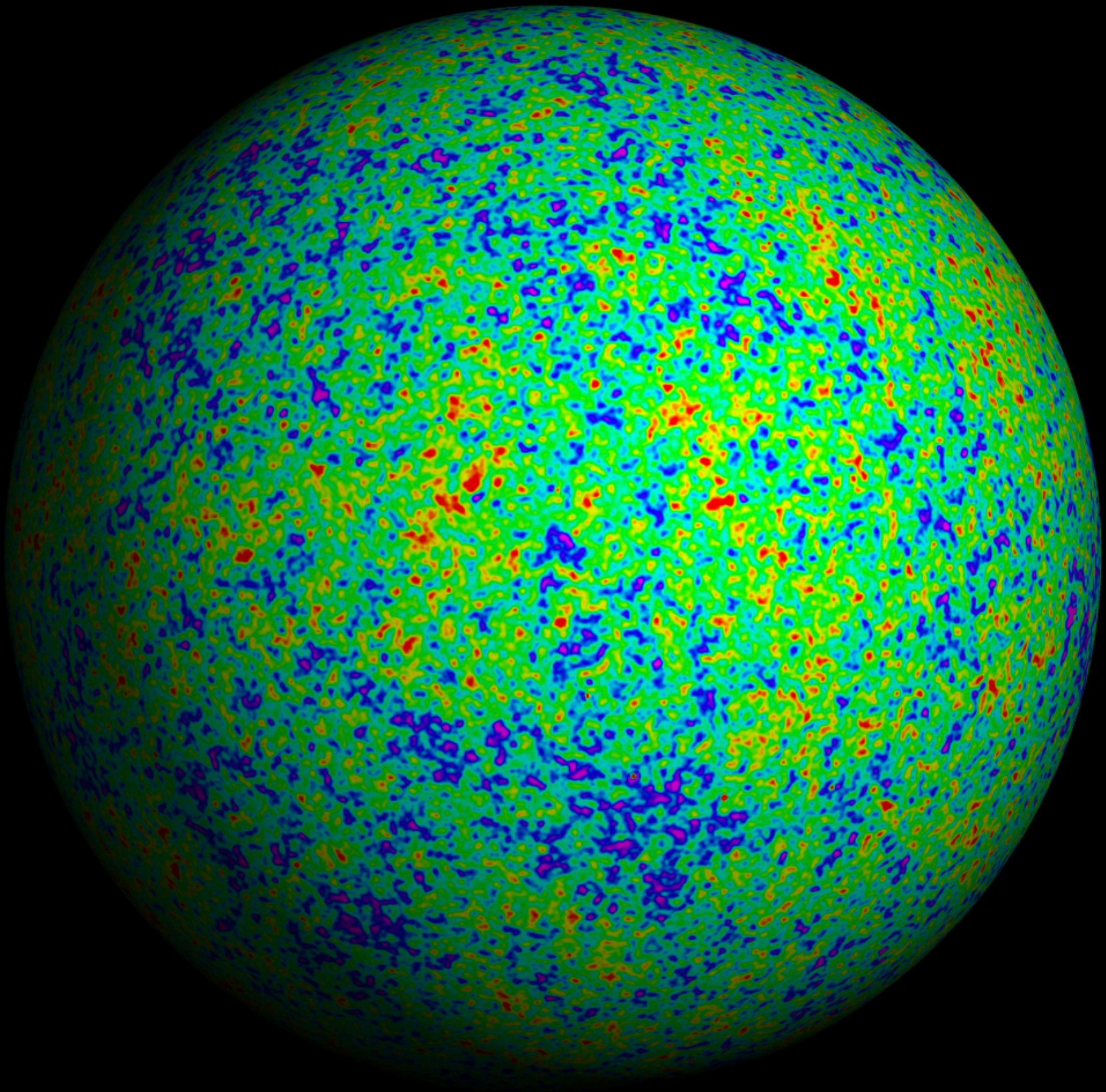
- 10^2 - 10^3 galaxies
- 10^{14} - 10^{15} M_{sun}
- ~1Mpc


Galaxy distribution at z=0
Millenium simulation - Springel et al 2005
• Cosmic web
• Voids
• Rich clusters of galaxies

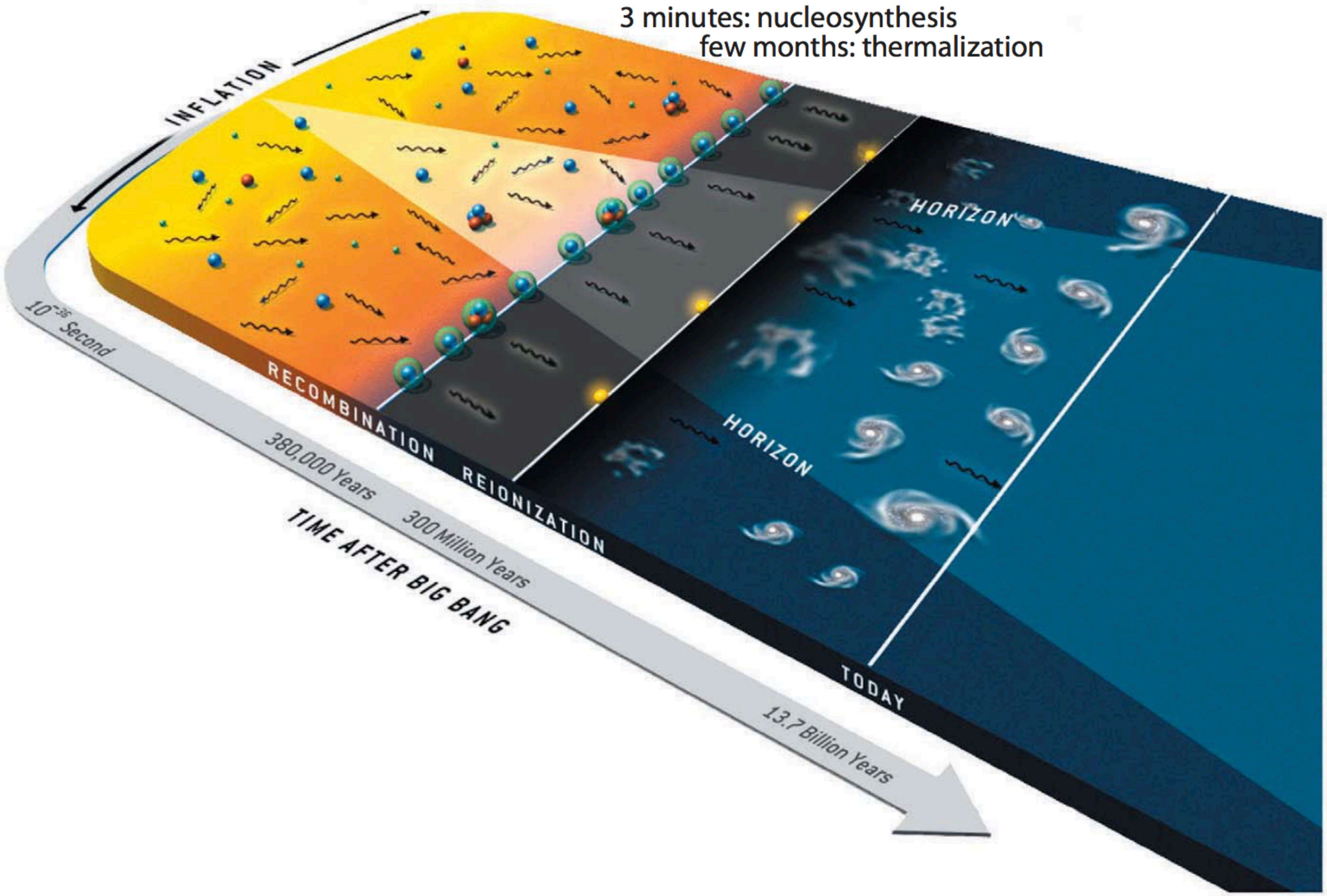

Dark matter distribution at z=0
Millenium simulation - Springel et al 2005
125 Mpc/h
• Cosmic web
• Filaments


Density field at z=18.3
Millenium simulation - Springel et al 2005
500 Mpc/h

What is the CMB?



Timeline



nucleosynthesis, thermalization (radiation black body spectrum), recombination and reionization

Figure from Hu

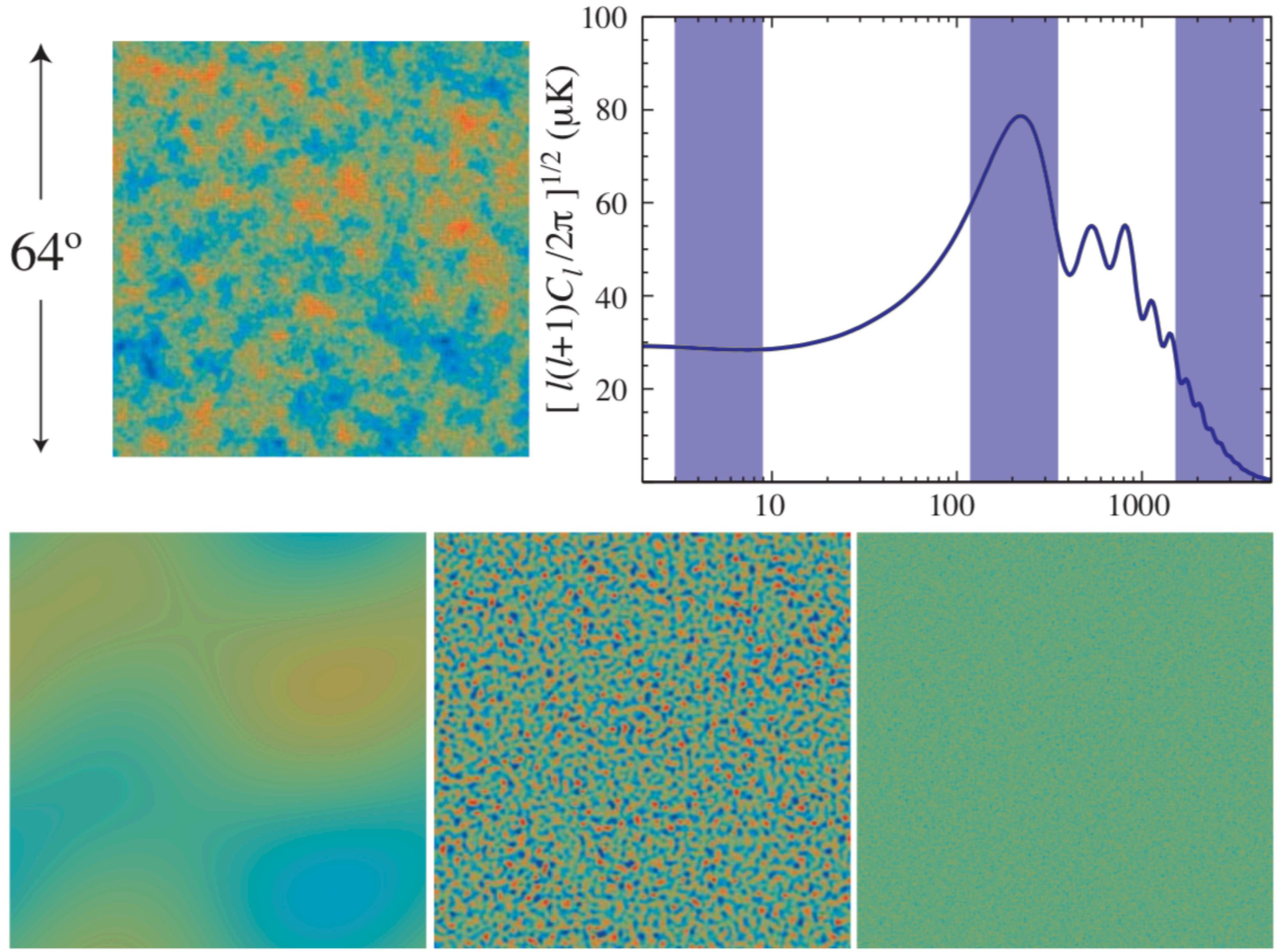


Fig. 6. From temperature maps to power spectrum. The original temperature fluctuation map (top left) corresponding to a simulation of the power spectrum (top right) can be band filtered to illustrate the power spectrum in three characteristic regimes: the large-scale gravitational regime of COBE, the first acoustic peak where most of the power lies, and the damping tail where fluctuations are dissipated. Adapted from Hu and White (2004).

Figure from Hu

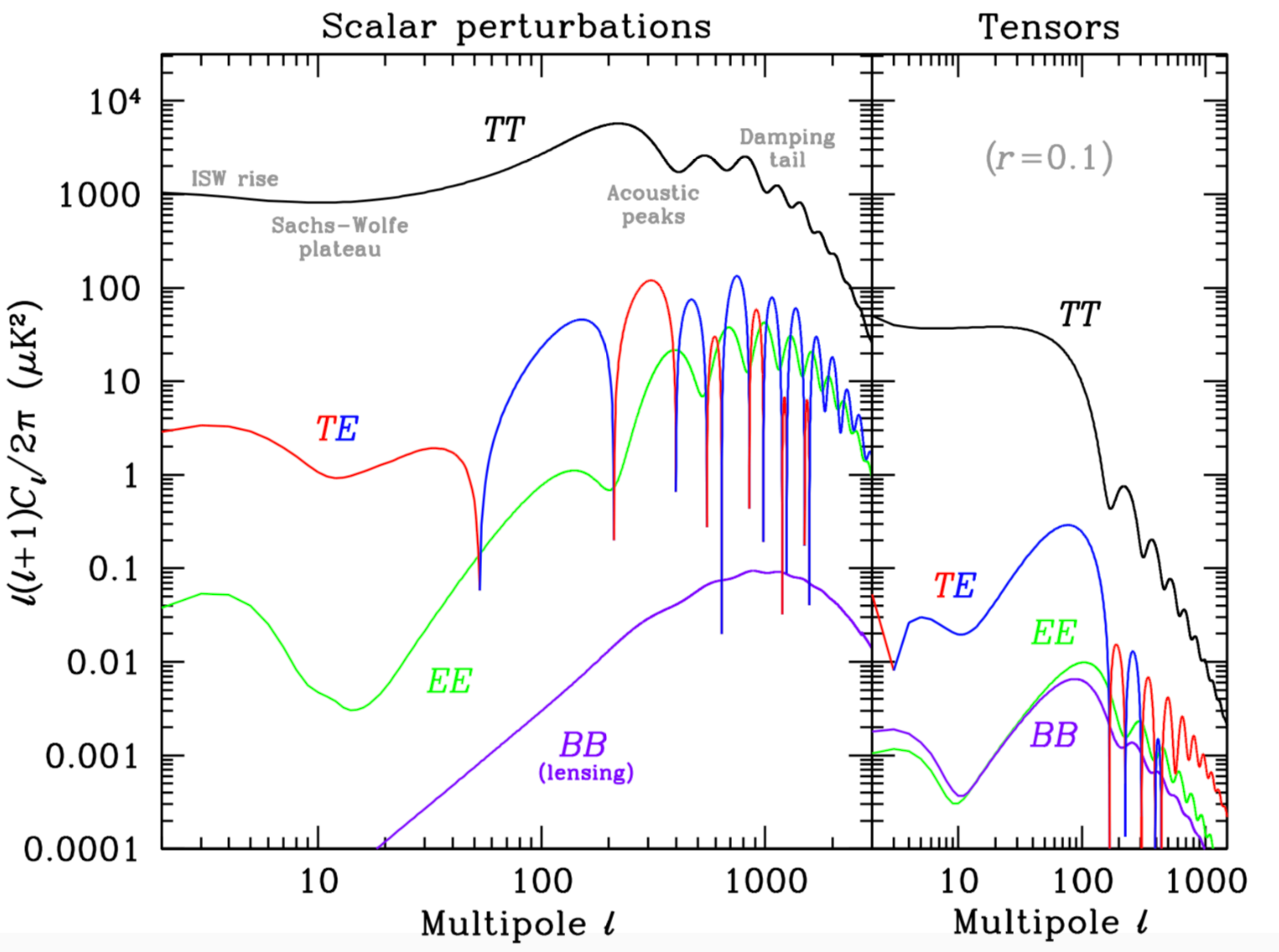


Figure PDG - Scott and Smooth

Objectives of the Lab:

- Compute CMB
- Compute primordial power spectra
- Compare with observations

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At the end of the week:
you are able to use state-of-the art softwares to **compute cosmological perturbations** and able to make **plots,** so you have the tools to be able to ***understand*** **CMB.**

Directories for the lab

- Create a directory called **bard_cmb_lab**
  - subdirectories:
    - **codes**
    - **notebooks**
    - **course_material**

Course Material

Put the documents you like in **course_material**, for instance:

- Julien Lesgourgues:
  - TASI Lecture
- Eiichiro Komatsu:
  - LAL Lectures I, II, III (slides)
- Daniel Baumann:
  - TASI Lecture on Inflation
- Wayne Hu:
  - PhD Thesis
  - Lecture Notes
  - Slides

Course Material

Cosmological Perturbation Theory in the Synchronous and Conformal Newtonian Gauges

1995

Chung-Pei Ma[1]
Theoretical Astrophysics 130-33, California Institute of Technology, Pasadena, CA 91125
and
Edmund Bertschinger[2]
Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

A LINE OF SIGHT INTEGRATION APPROACH TO COSMIC MICROWAVE BACKGROUND ANISOTROPIES

Uroš Seljak
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1996

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Objectives of the Lab:

- **Compute CMB**
- Compute primordial power spectra
- Compare with observations

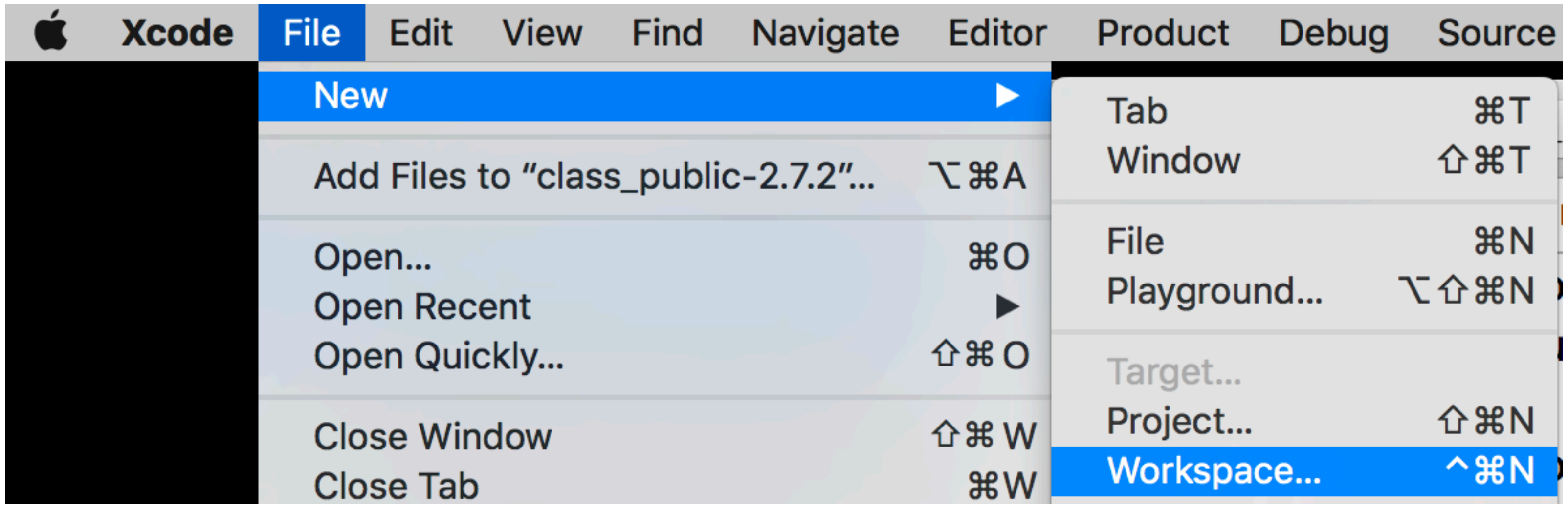
CLASS

We use ***CLASS*** to compute CMB temperature spectra

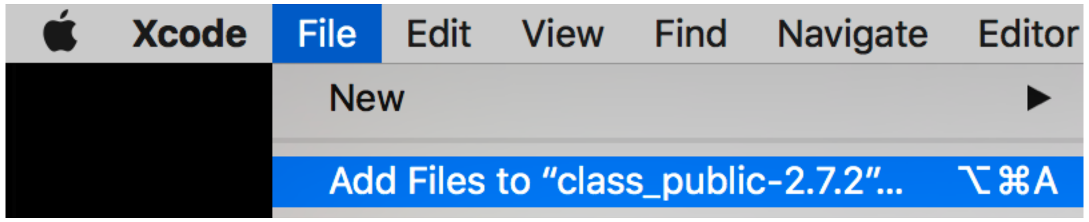
- Boltzmann Code by Julien Lesgourgues
- Go to: http://class-code.net
- Download latest version: class_public-X.X.X.tar.gz (X.X.X. = version number)
- Unzip the directory into **codes**
  - This creates a directory **class_public-X.X.X**

Navigating ***CLASS***

- Mac users: **Xcode** to navigate and edit the files
- Linux users: Something equivalent to **Xcode**
- Open **Xcode**
- Create workspace



- Add files to workspace



  - add the ***CLASS*** files by selecting the **class_public-X.X.X** directory

Running *CLASS*

- In Terminal, make sure you are in **class_public-X.X.X** directory

  $ pwd

  /Users/boris/Bard-School/bard_cmb_lab/codes/class_public-2.7.2

- Run the code

  $ ./class explanatory.ini

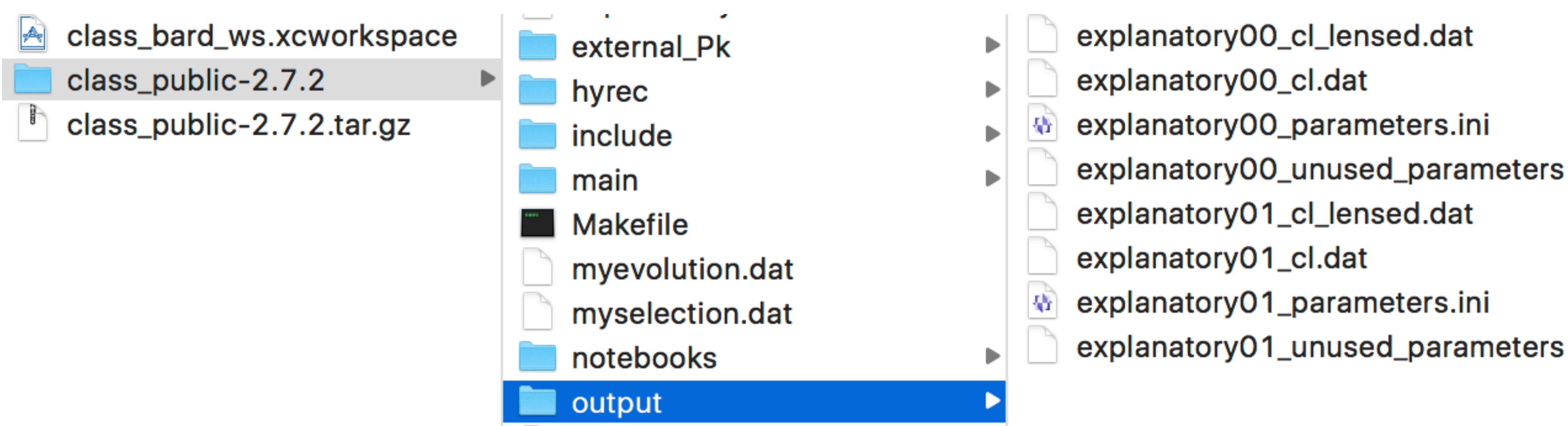
- Et voilà!

***CLASS* has computed CMB spectra**

```
vpn65:class_public-2.7.2 boris$ ./class explanatory.ini
Reading input parameters
 -> matched budget equations by adjusting Omega_Lambda = 6.878622e-01
Running CLASS version v2.7.2
Computing background
 -> age = 13.795359 Gyr
 -> conformal age = 14165.045412 Mpc
 -> radiation/matter equality at z = 3402.964903
    corresponding to conformal time = 112.800022 Mpc
Computing thermodynamics with Y_He=0.2453
 -> recombination at z = 1089.184869
    corresponding to conformal time = 280.591903 Mpc
    with comoving sound horizon = 144.703150 Mpc
    angular diameter distance = 12.735871 Mpc
    and sound horizon angle 100*theta_s = 1.042196
 -> baryon drag stops at z = 1059.089246
    corresponding to conformal time = 286.504945 Mpc
    with comoving sound horizon rs = 147.384053 Mpc
 -> reionization with optical depth = 0.092664
    corresponding to conformal time = 4255.316282 Mpc
Computing sources
Computing primordial spectra (analytic spectrum)
No non-linear spectra requested. Nonlinear module skipped.
Computing transfers
Computing unlensed linear spectra
Computing lensed spectra (fast mode)
Writing output files in output/explanatory01_...
```

First look at the results

- With file browser go to **class_public-X.X.X/output** directory



- Files “explanatory00_X” where there before

- Files “explanatory01_X” are the results of our computation

First look at the results

explanatory01_cl_lensed.dat

class_public-2.7.2 〉 output 〉 explanatory01_cl_lensed.dat 〉 No Selection

```
# dimensionless total lensed [l(l+1)/2pi] C_l's
# for l=2 to 2500, i.e. number of multipoles equal to 2499
#
# -> if you prefer output in CAMB/HealPix/LensPix units/order, set 'format' to 'camb' in input file
# -> if you don't want to see such a header, set 'headers' to 'no' in input file
# -> for CMB lensing (phi), these are C_l^phi-phi for the lensing potential.
#    Remember the conversion factors:
#    C_l^dd (deflection) = l(l+1) C_l^phi-phi
#    C_l^gg (shear/convergence) = 1/4 (l(l+1))^2 C_l^phi-phi
#
# 1:l      2:TT                    3:EE                    4:TE                    5:BB                    6:phiphi                7:TPhi                  8:Ephi
     2       1.495442756351e-10      7.309265058191e-15      4.815171146232e-13      2.388859482474e-19      8.628903626840e-09      4.854772022277e-10     -2.515854151933e-12
     3       1.409375666013e-10      1.238839452051e-14      6.068719460549e-13      4.780041028657e-19      5.411291734610e-09      3.469573536427e-10     -2.211507031240e-12
     4       1.324638938256e-10      1.446592354116e-14      6.277975858901e-13      7.971813420052e-19      3.776588568344e-09      2.639785239648e-10     -1.764116066973e-12
     5       1.257528921080e-10      1.337973190801e-14      5.879550143704e-13      1.196704831667e-18      2.821984241646e-09      2.093815914787e-10     -1.309529286417e-12
     6       1.208087559436e-10      1.032478440834e-14      5.173987896631e-13      1.676914962609e-18      2.205212286906e-09      1.709428816447e-10     -8.948468928589e-13
     7       1.173342229262e-10      6.843838593597e-15      4.365330659196e-13      2.238194067705e-18      1.778233640037e-09      1.425663810180e-10     -5.443860561625e-13
     8       1.150094184720e-10      4.045255169564e-15      3.576614995729e-13      2.880953919699e-18      1.468351542188e-09      1.208855530841e-10     -2.689334591755e-13
     9       1.135899960901e-10      2.339962949682e-15      2.880577371393e-13      3.605622503705e-18      1.235838574991e-09      1.038809857398e-10     -7.009986560394e-14
    10       1.128612981992e-10      1.576066389180e-15      2.317780346650e-13      4.412630585967e-18      1.056623414340e-09      9.029295137819e-11      5.745597693390e-14
```

First look at the results

explanatory01_cl.dat

class_public-2.7.2 〉 output 〉 explanatory01_cl.dat 〉 No Selection

```
# dimensionless total [l(l+1)/2pi] C_l's
# for l=2 to 3000, i.e. number of multipoles equal to 2999
#
# -> if you prefer output in CAMB/HealPix/LensPix units/order, set 'format' to 'camb' in input file
# -> if you don't want to see such a header, set 'headers' to 'no' in input file
# -> for CMB lensing (phi), these are C_l^phi-phi for the lensing potential.
#    Remember the conversion factors:
#    C_l^dd (deflection) = l(l+1) C_l^phi-phi
#    C_l^gg (shear/convergence) = 1/4 (l(l+1))^2 C_l^phi-phi
#
# 1:l     2:TT                    3:EE                    4:TE                    5:BB                    6:phiphi                7:TPhi                  8:Ephi
    2       1.495437883302e-10      7.309026172210e-15      4.815171198141e-13      0.000000000000e+00      8.628903626840e-09      4.854772022277e-10     -2.515854151933e-12
    3       1.409365941764e-10      1.238791651609e-14      6.068719691435e-13      0.000000000000e+00      5.411291734610e-09      3.469573536427e-10     -2.211507031240e-12
    4       1.324622779182e-10      1.446512635824e-14      6.277976520638e-13      0.000000000000e+00      3.776588568344e-09      2.639785239648e-10     -1.764116066973e-12
    5       1.257504771186e-10      1.337853519779e-14      5.879551646444e-13      0.000000000000e+00      2.821984241646e-09      2.093815914787e-10     -1.309529286417e-12
    6       1.208053896009e-10      1.032310747876e-14      5.173990840392e-13      0.000000000000e+00      2.205212286906e-09      1.709428816447e-10     -8.948468928589e-13
    7       1.173297567759e-10      6.841600365731e-15      4.365335860683e-13      0.000000000000e+00      1.778233640037e-09      1.425663810180e-10     -5.443860561625e-13
    8       1.150037082777e-10      4.042374146263e-15      3.576623509890e-13      0.000000000000e+00      1.468351542188e-09      1.208855530841e-10     -2.689334591755e-13
    9       1.135829021418e-10      2.336357197454e-15      2.880590507426e-13      0.000000000000e+00      1.235838574991e-09      1.038809857398e-10     -7.009986560394e-14
   10       1.128526855248e-10      1.571653533766e-15      2.317799678422e-13      0.000000000000e+00      1.056623414340e-09      9.029295137819e-11      5.745597693390e-14
```

First look at the results

explanatory01_parameters.ini

class_public-2.7.2 〉 output 〉 explanatory01_parameters.ini

```
# List of input/precision parameters actually read
# (all other parameters set to default values)
# Obtained with CLASS v2.7.2 (for developers: svn version 6142M)
#
# This file can be used as the input file of another run
#
h = 0.67556
T_cmb = 2.7255
omega_b = 0.022032
N_ur = 3.046
omega_cdm = 0.12038
Omega_dcdmdr = 0.0
N_ncdm = 0
Omega_k = 0.
Omega_fld = 0
Omega_scf = 0
YHe = BBN
recombination = RECFAST
reio_parametrization = reio_camb
z_reio = 11.357
reionization_exponent = 1.5
reionization_width = 0.5
helium_fullreio_redshift = 3.5
helium_fullreio_width = 0.5
annihilation = 0.
decay = 0.
output = tCl,pCl,lCl
modes = s
lensing = yes
ic = ad
gauge = synchronous
P_k_ini type = analytic_Pk
k_pivot = 0.05
A_s = 2.215e-9
n_s = 0.9619
alpha_s = 0.
l_max_scalars = 2500
headers = yes
format = class
write background = no
write thermodynamics = no
write primordial = no
write parameters = yeap
input_verbose = 1
background_verbose = 1
thermodynamics_verbose = 1
perturbations_verbose = 1
transfer_verbose = 1
primordial_verbose = 1
spectra_verbose = 1
nonlinear_verbose = 1
lensing_verbose = 1
output_verbose = 1
root = output/explanatory01_
#
```

First look at the results

explanatory01_unused_parameters.ini

class_public-2.7.2 〉 output 〉 explanatory01_unused_parameters

```
# List of input/precision parameters passed
# but not used (just for info)
#
Gamma_dcdm = 0.0
ncdm_psd_filenames = psd_FD_single.dat
ncdm_psd_parameters = 0.3 ,0.5, 0.05
m_ncdm = 0.04, 0.04, 0.04
use_ppf = yes
c_gamma_over_c_fld = 0.4
p/rho = w0_fld + wa_fld (1-a/a0) (Chevalier-Linder-Polarski),
fluid_equation_of_state = CLP
8c1) equation of state of the fluid in 'CLP' case (p/rho = w0_fld +
w0_fld = -0.9
wa_fld = 0.
cs2_fld = 1
w0_fld = -0.9
Omega_EDE = 0.
cs2_fld = 1
```

Making our own parameter file

- Copy the file **class_public-X.X.X/output/explanatory01_parameters.ini**
- Paste in the CLASS directory: **class_public-X.X.X**
- Rename to **bard_parameters.ini**
- Open in file editor

Making our own parameter file

- Change the name of the output files:

```
root = output/bard_
```

- Run:

$ ./class bard_parameters.ini

- This creates the same output files but with name **bard_XX**

Adding the primordial power spectrum as an output

- In **bard_parameters.ini** edit:

```
write primordial = yes
```

- Run:

$ ./class bard_parameters.ini

- This creates file: **bard_primordial_Pk.dat**

class_public-2.7.2 〉 output 〉 bard_primordial_Pk.dat 〉 No Selection

```
# Dimensionless primordial spectrum, equal to [k^3/2pi^2] P(k)
#    1:k [1/Mpc]              2:P_scalar(k)
       7.059631444046e-06       3.105030656706e-09
       8.887549422811e-06       3.077909828390e-09
       1.118876181695e-05       3.051025886407e-09
       1.408581657788e-05       3.024376761680e-09
       1.773299243576e-05       2.997960403200e-09
       2.232451480453e-05       2.971774777877e-09
       2.810489899340e-05       2.945817870375e-09
       3.538197153870e-05       2.920087682964e-09
```

Adding the transfer functions as an output

- In **bard_parameters.ini** edit:

```
output = tCl,pCl,lCl,mTk
```

- Run:

```
$ ./class bard_parameters.ini
```

- This creates file: **bard_tk.dat**

class_public-2.7.2 〉 output 〉 bard_tk.dat 〉 No Selection

```
# Transfer functions T_i(k) for adiabatic (AD) mode (normalized to initial curvature=1) at redshift z=0
# for k=1.045e-05 to 1.69353 h/Mpc,
# number of wavenumbers equal to 605
# d_i   stands for (delta rho_i/rho_i)(k,z) with above normalization
# d_tot stands for (delta rho_tot/rho_tot)(k,z) with rho_Lambda NOT included in rho_tot
# (note that this differs from the transfer function output from CAMB/CMBFAST, which gives the same
#  quantities divided by -k^2 with k in Mpc^-1; use format=camb to match CAMB)
#
#    1:k (h/Mpc)              2:d_g                    3:d_b                    4:d_cdm                  5:d_ur                   6:d_tot
       1.045004358465e-05     -1.317129351251e-03      -9.883609224469e-04      -9.883609554133e-04      -1.317108952610e-03      -9.884575034553e-04
       2.333031646003e-05     -6.551353640712e-03      -4.926270239946e-03      -4.926271058685e-03      -6.550847734500e-03      -4.926748139830e-03
       3.625588232309e-05     -1.576336400374e-02      -1.189678668305e-02      -1.189679145546e-02      -1.576042236398e-02      -1.189792593308e-02
       4.926151484748e-05     -2.894671118007e-02      -2.196251604685e-02      -2.196253229886e-02      -2.893672937247e-02      -2.196457976160e-02
       6.238256282054e-05     -4.609655202415e-02      -3.521963893260e-02      -3.521968068369e-02      -4.607102923353e-02      -3.522286557651e-02
       7.565526530538e-05     -6.720687375041e-02      -5.179963245287e-02      -5.179972265134e-02      -6.715205880879e-02      -5.180422702591e-02
       8.911707687973e-05     -9.226671524325e-02      -7.187171419628e-02      -7.187188757631e-02      -9.216209999330e-02      -7.187783793011e-02
       1.028070072767e-04     -1.212551981648e-01      -9.564606636541e-02      -9.564637286380e-02      -1.210718222249e-01      -9.565382446367e-02
       1.167659797859e-04     -1.541351670999e-01      -1.233779351116e-01      -1.233784440293e-01      -1.538336873629e-01      -1.233873620196e-01
       1.310372128073e-04     -1.908451534422e-01      -1.553727119090e-01      -1.553735170313e-01      -1.903736399834e-01      -1.553837535082e-01
       1.456666289547e-04     -2.312893479908e-01      -1.919921390810e-01      -1.919933650238e-01      -2.305807701044e-01      -1.920046311150e-01
```

Adding the computation of tensor modes

- In **bard_parameters.ini** edit:

```
modes = s,t
```

- Run:

$ ./class bard_parameters.ini

- This creates files: **bard_cls.dat, bard_clt.dat**

- and adds tensor column to **bard_primordial_Pk.dat**

Adding the background functions as an output

- In **bard_parameters.ini** edit:

```
write background = yes
```

- Run:

```
$ ./class bard_parameters.ini
```

- This creates files: **bard_background.dat**

class_public-2.7.2 〉 output 〉 bard_background.dat 〉 No Selection

```
# Table of selected background quantities
# All densities are multiplied by (8piG/3) (below, shortcut notation (.) for this factor)
# Densities are in units [Mpc^-2] while all distances are in [Mpc].
#    1:z                   2:proper time [Gyr]     3:conf. time [Mpc]      4:H [1/Mpc]             5:comov. dist.          6:ang.diam.dist.
       1.000000000000e+14      7.558503632224e-26      4.634785000235e-09      2.157545946648e+22      1.416504541243e+04      1.416504541243e-10
       9.930486593843e+13      7.664693049753e-26      4.667526809499e-09      2.127654528182e+22      1.416504541243e+04      1.426420073032e-10
       9.861456399050e+13      7.772374322409e-26      4.700268618762e-09      2.098177236191e+22      1.416504541243e+04      1.436405013543e-10
       9.792906056653e+13      7.881568409264e-26      4.733010428026e-09      2.069108333218e+22      1.416504541243e+04      1.446459848638e-10
       9.724832231036e+13      7.992296563846e-26      4.765752237290e-09      2.040442161294e+22      1.416504541243e+04      1.456585067579e-10
       9.657231609767e+13      8.104580338272e-26      4.800313035958e-09      2.012173140839e+22      1.416504541243e+04      1.466781163052e-10
       9.590100903443e+13      8.218441587444e-26      4.833054845221e-09      1.984295769572e+22      1.416504541243e+04      1.477048631193e-10
       9.523436845525e+13      8.333902473306e-26      4.867615643889e-09      1.956804621446e+22      1.416504541243e+04      1.487387971611e-10
```

Jupyter Notebook

- Go to directory **notebooks**:

$ cd /Users/boris/Bard-School/bard_cmb_lab/notebooks

- Open **bard-notebook.ipynb** with **Jupyter**

$ jupyter notebook bard-notebook-module-1.ipynb

• First exercise: Plotting the Planck data (Figure 1 of Planck 2015 XIII)

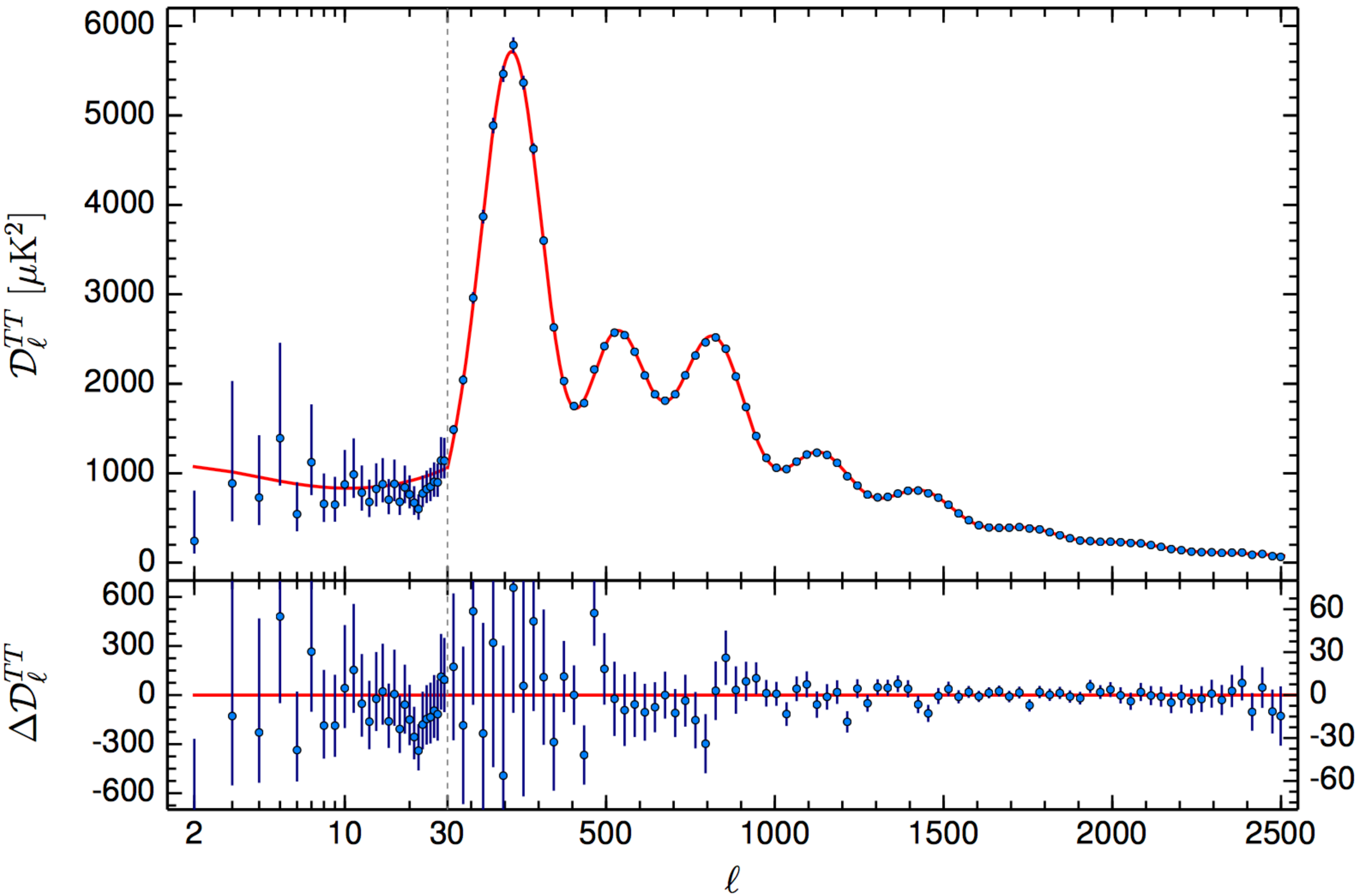
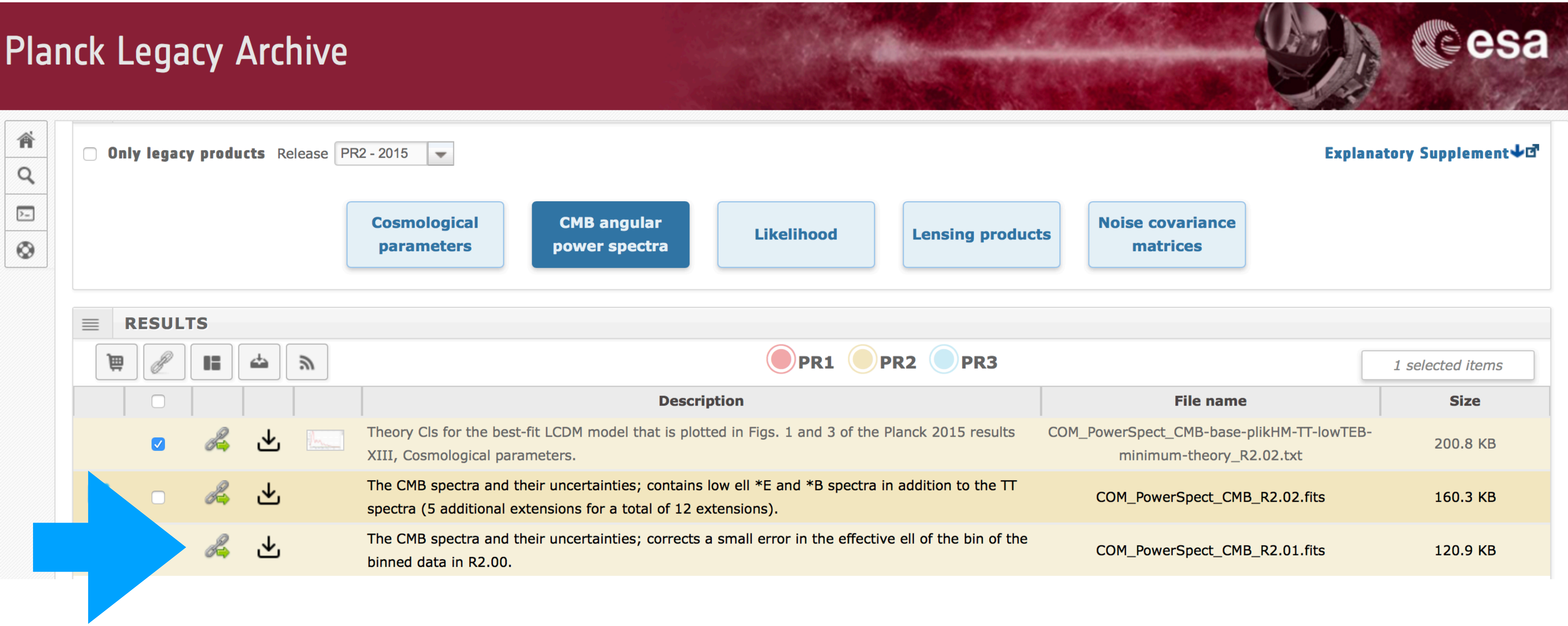


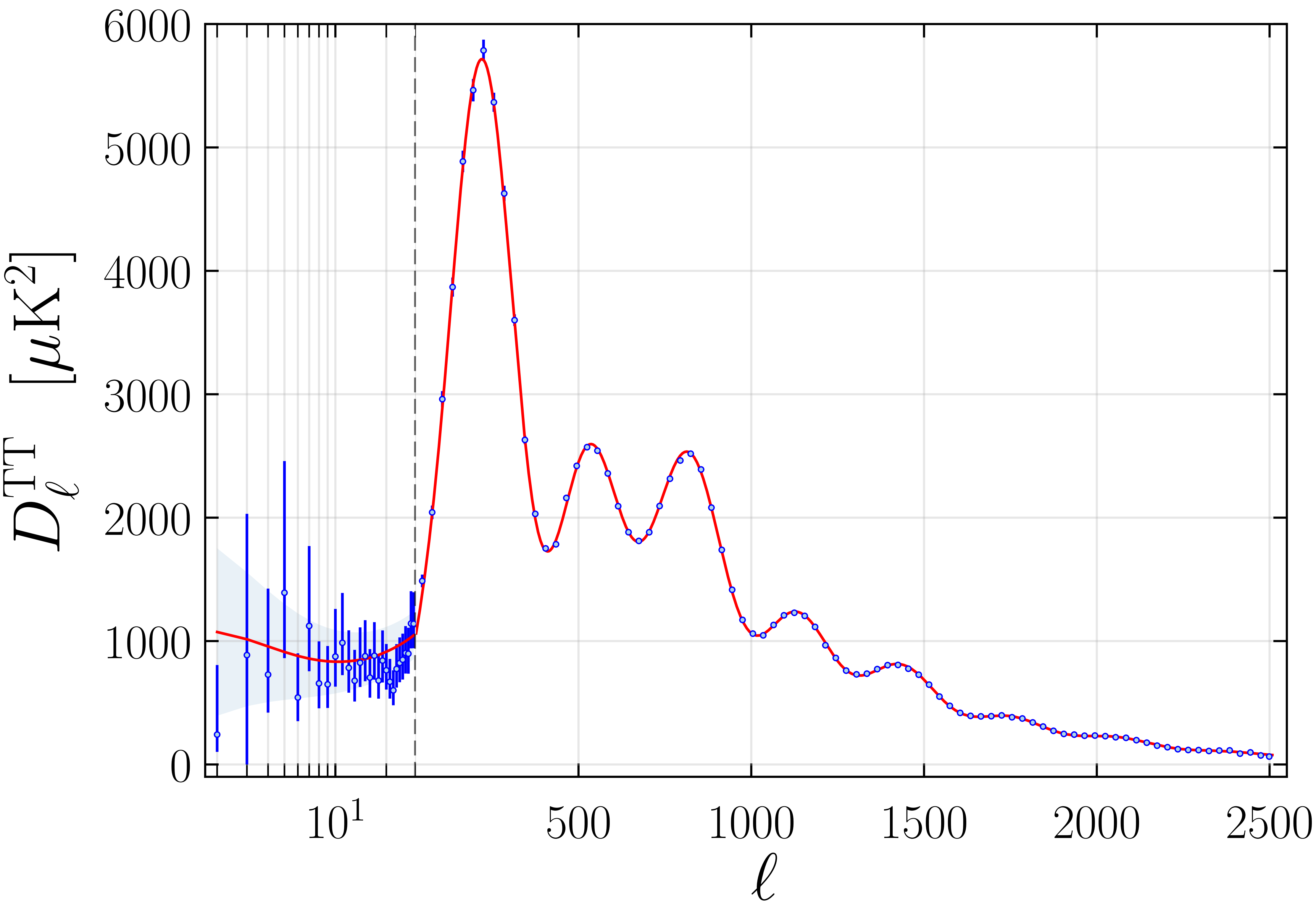
Fig. 1. *Planck* 2015 temperature power spectrum. At multipoles $\ell \geq 30$ we show the maximum likelihood frequency-averaged temperature spectrum computed from the `Plik` cross-half-mission likelihood, with foreground and other nuisance parameters determined from the MCMC analysis of the base ΛCDM cosmology. In the multipole range $2 \leq \ell \leq 29$, we plot the power spectrum estimates from the `Commander` component-separation algorithm, computed over 94 % of the sky. The best-fit base ΛCDM theoretical spectrum fitted to the *Planck* TT+lowP likelihood is plotted in the upper panel. Residuals with respect to this model are shown in the lower panel. The error bars show $\pm 1\,\sigma$ uncertainties.

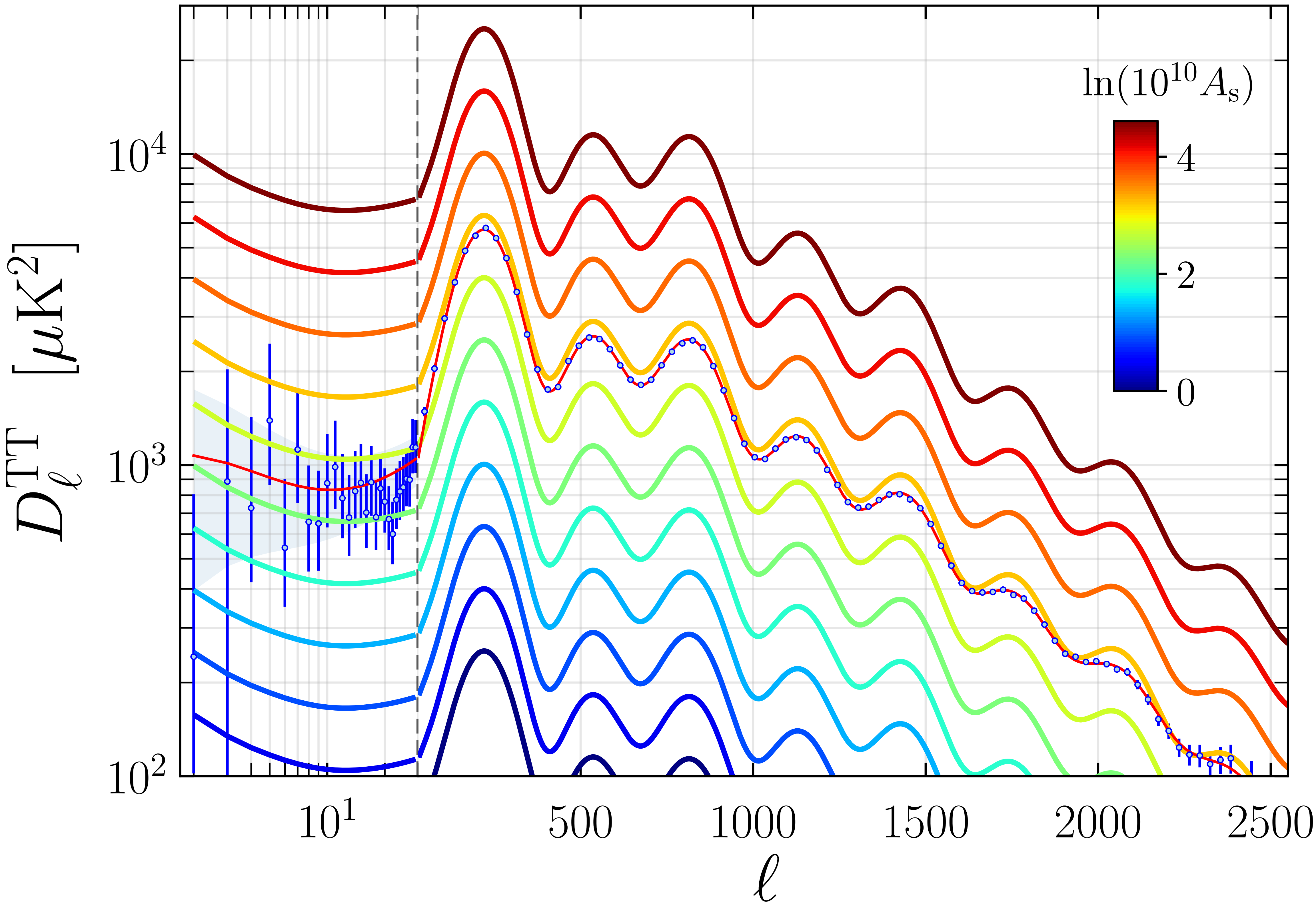
Planck Data



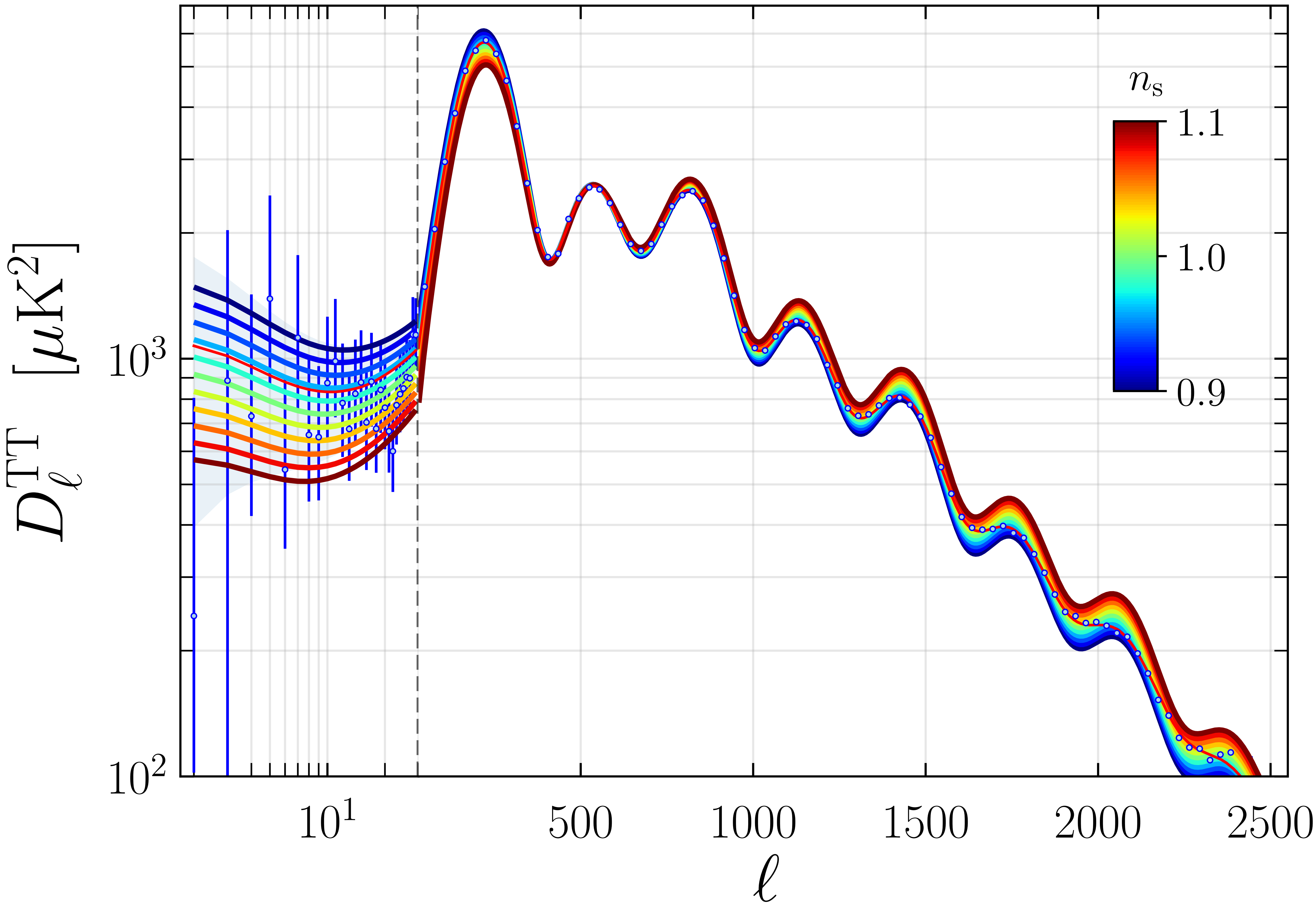
Planck Legacy Archive
esa
Only legacy products
Release
PR2 - 2015
Explanatory Supplement
Cosmological parameters
CMB angular power spectra
Likelihood
Lensing products
Noise covariance matrices
RESULTS
PR1
PR2
PR3
1 selected items
Description
File name
Size
Theory Cls for the best-fit LCDM model that is plotted in Figs. 1 and 3 of the Planck 2015 results XIII, Cosmological parameters.
COM_PowerSpect_CMB-base-plikHM-TT-lowTEB-minimum-theory_R2.02.txt
200.8 KB
The CMB spectra and their uncertainties; contains low ell *E and *B spectra in addition to the TT spectra (5 additional extensions for a total of 12 extensions).
COM_PowerSpect_CMB_R2.02.fits
160.3 KB
The CMB spectra and their uncertainties; corrects a small error in the effective ell of the bin of the binned data in R2.00.
COM_PowerSpect_CMB_R2.01.fits
120.9 KB

From the notebook:

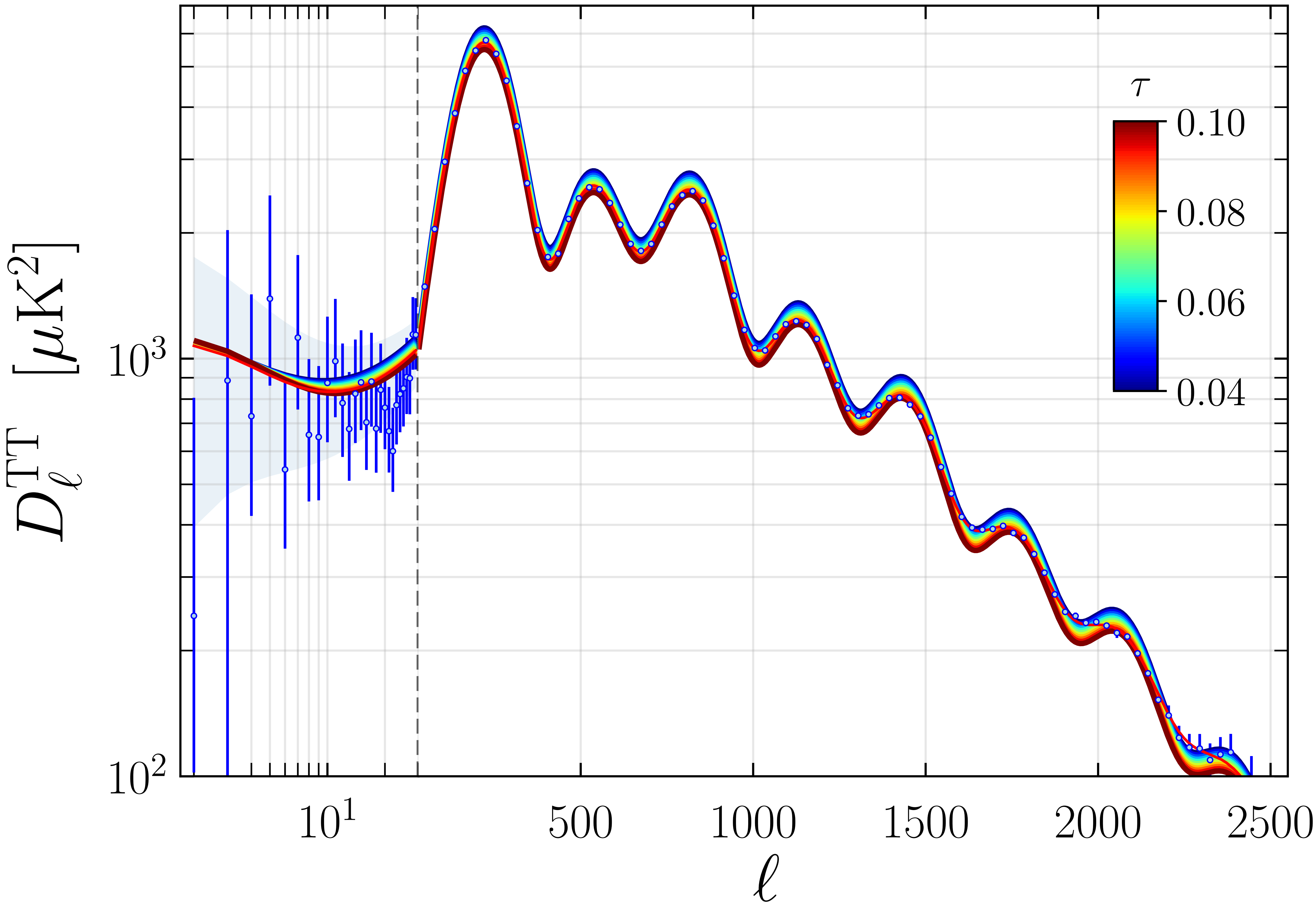




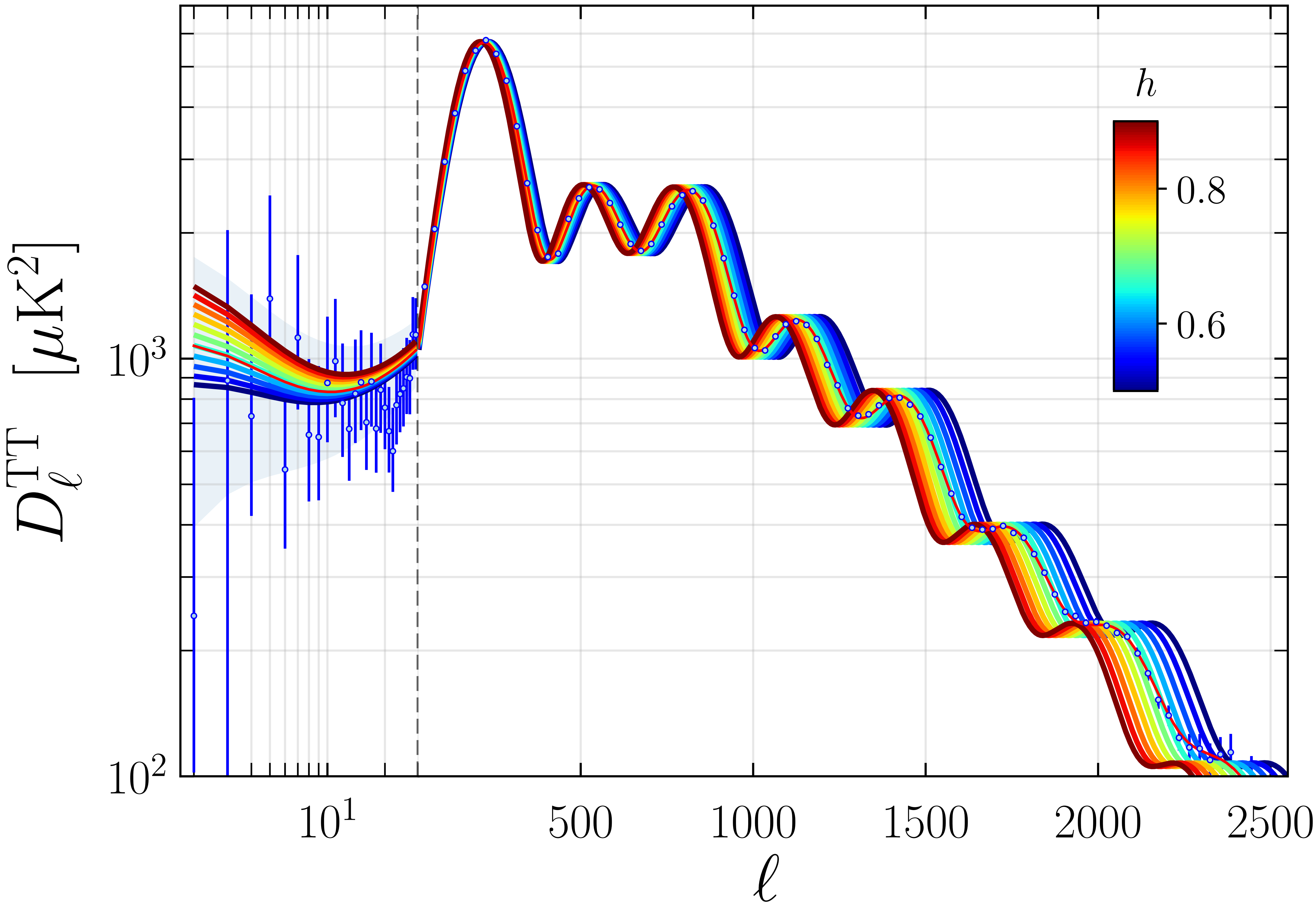
D_ℓ^{TT} [μK^2]
10^4
10^3
10^2
10^1
500
1000
1500
2000
2500
ℓ
$\ln(10^{10}A_s)$
4
2
0



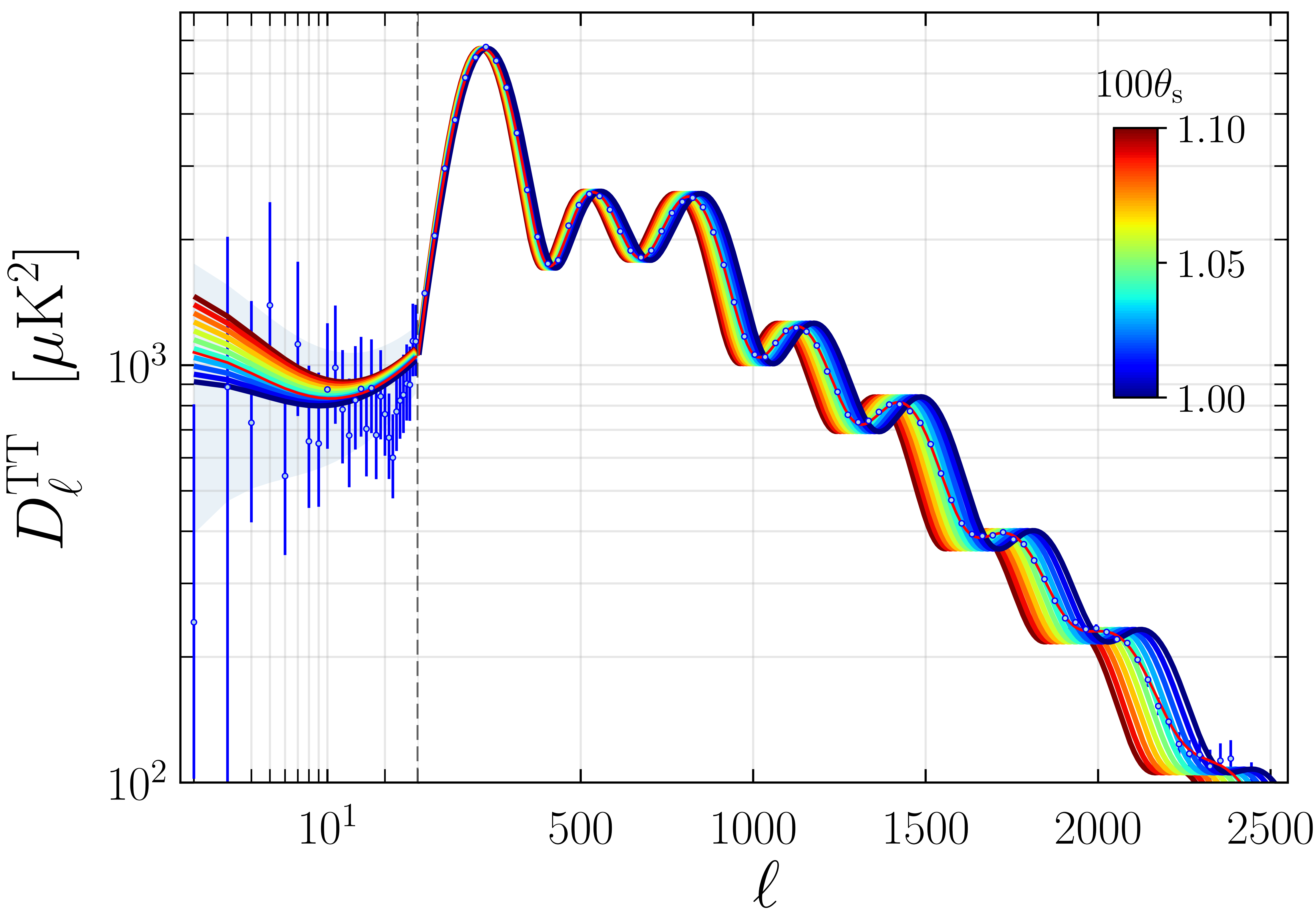
D_ℓ^{TT} [μK^2]
10^3
10^2
10^1
500
1000
1500
2000
2500
ℓ
n_s
1.1
1.0
0.9



D_ℓ^{TT} $[\mu\mathrm{K}^2]$
10^3
10^2
10^1
500
1000
1500
2000
2500
ℓ
τ
0.10
0.08
0.06
0.04

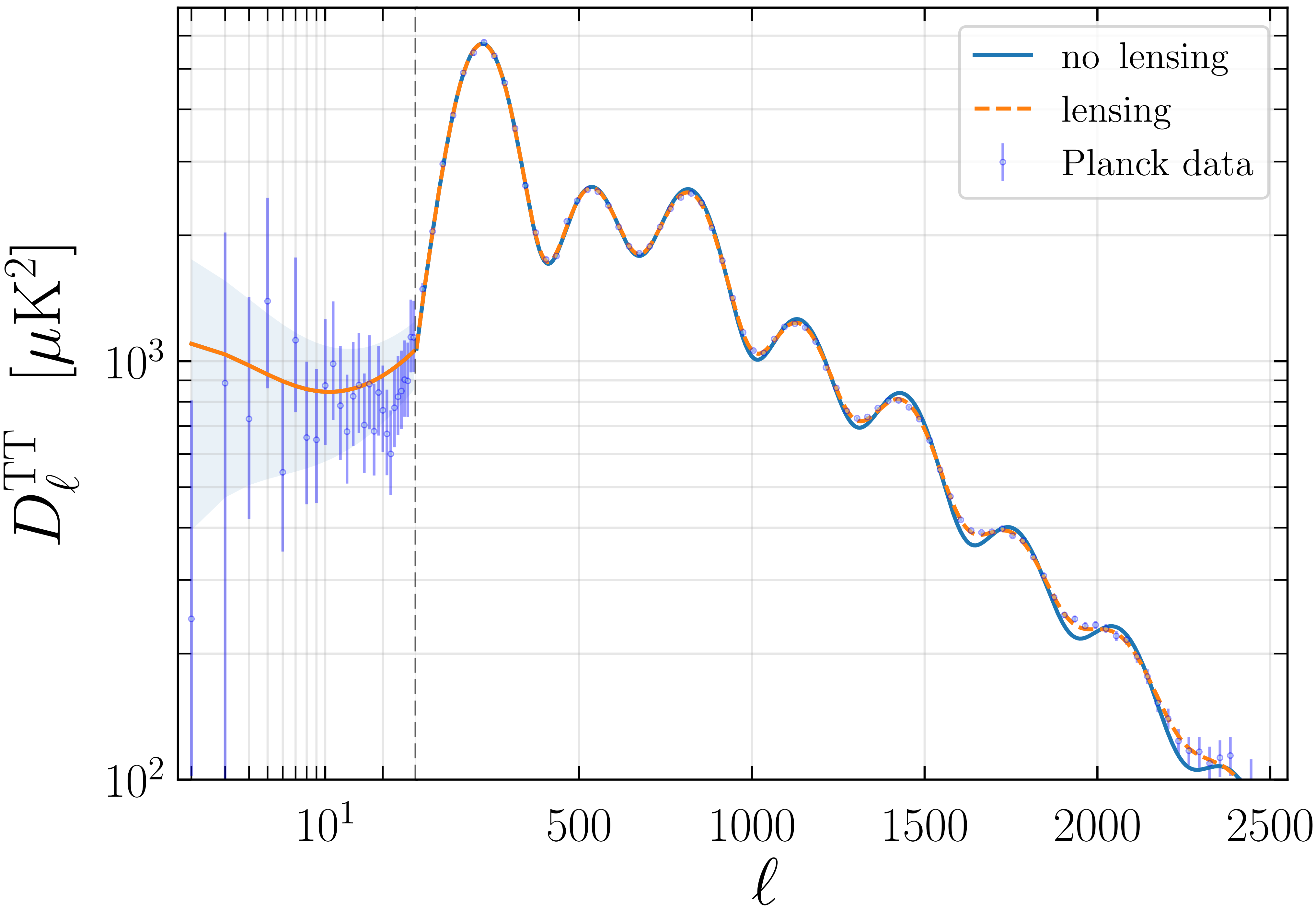


D_ℓ^{TT} $[\mu\mathrm{K}^2]$
10^3
10^2
10^1
500
1000
1500
2000
2500
ℓ
h
0.8
0.6


D_ℓ^{TT} [μK^2]
10^3
10^2
10^1
500
1000
1500
2000
2500
ℓ
$100\theta_s$
1.10
1.05
1.00

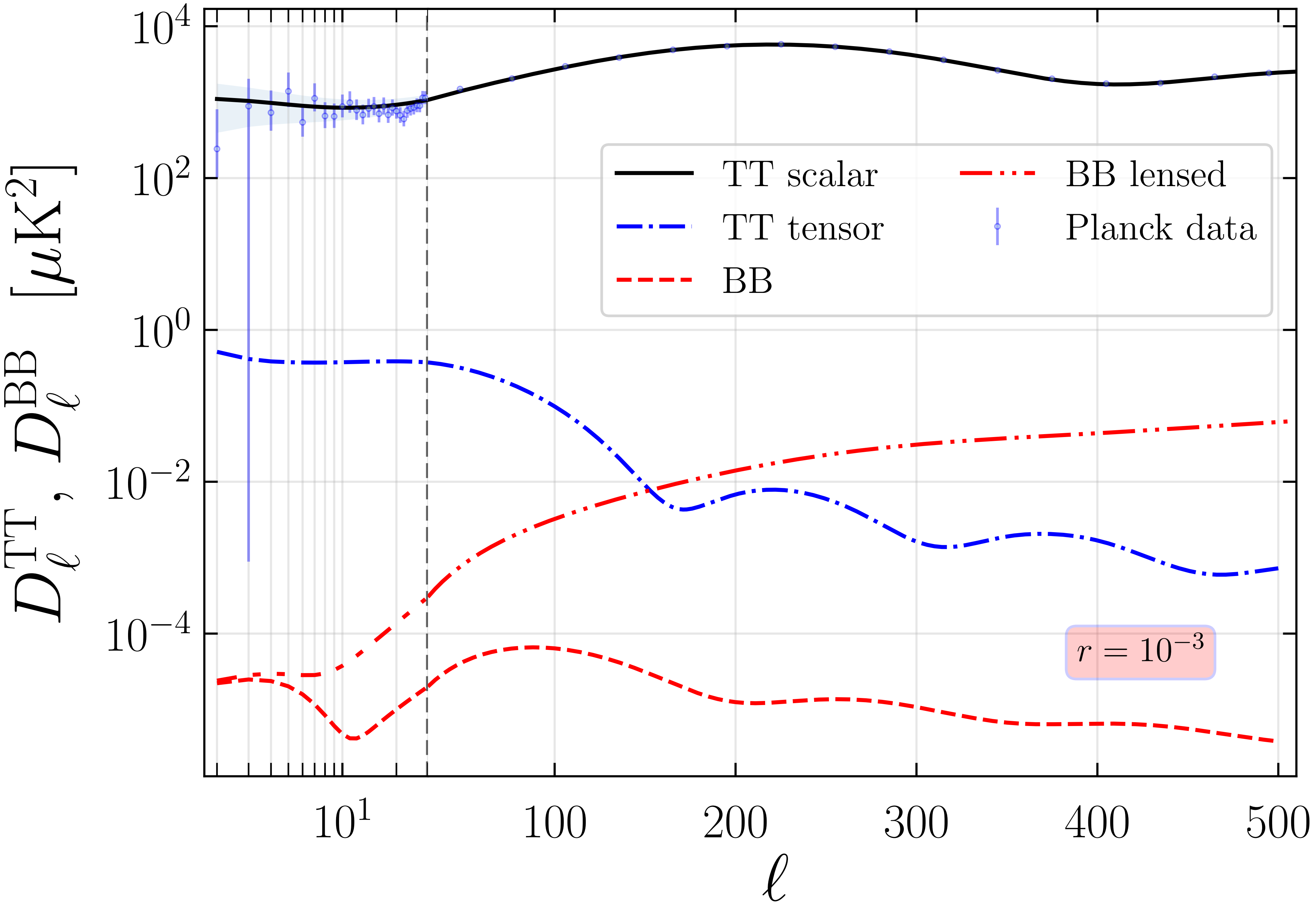
Lensing vs no lensing

- Change the parameter file to compute scalar modes only



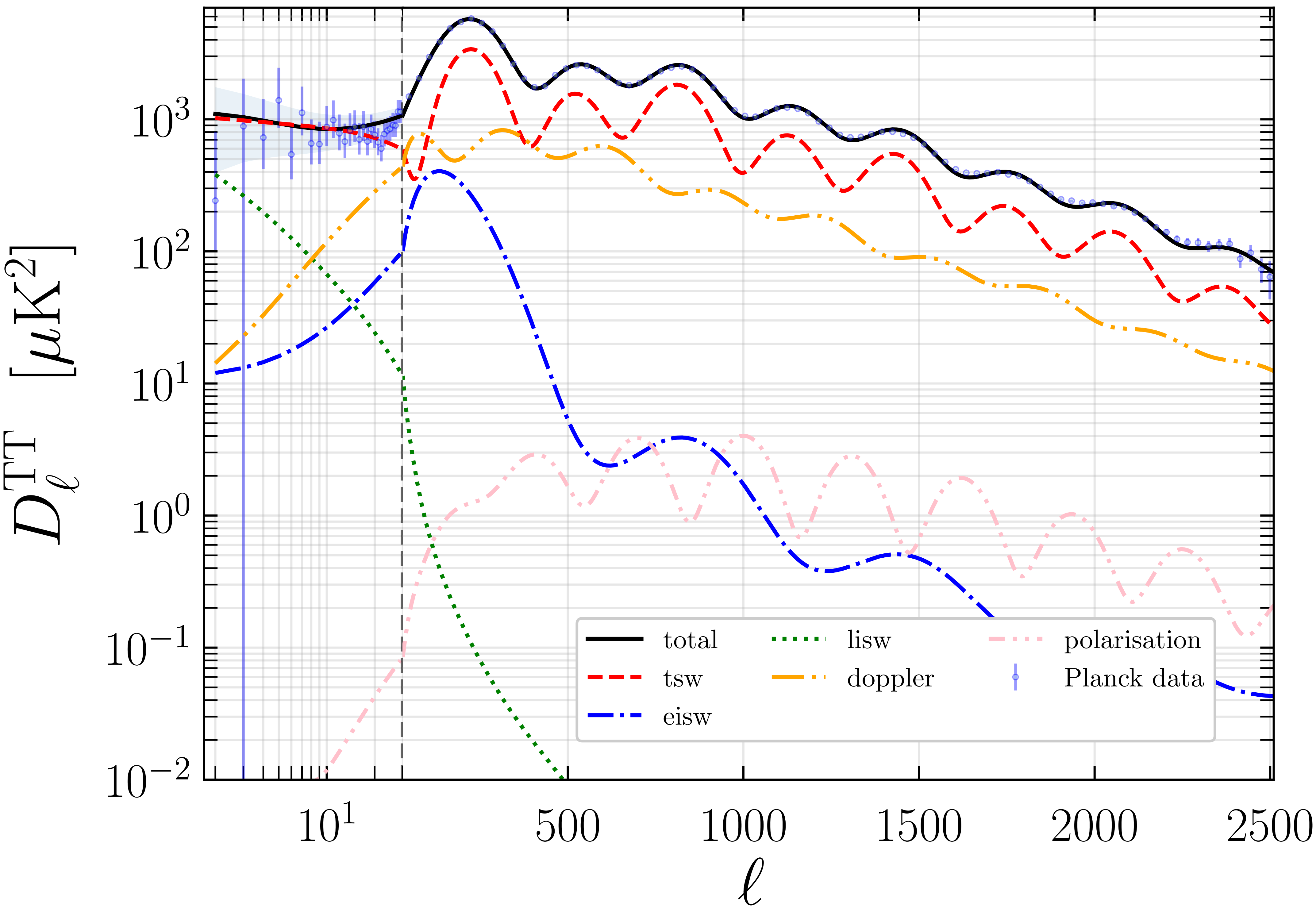
Tensor mode

- Add r=1e-3 (tensor-to-scalar ratio) in the param file



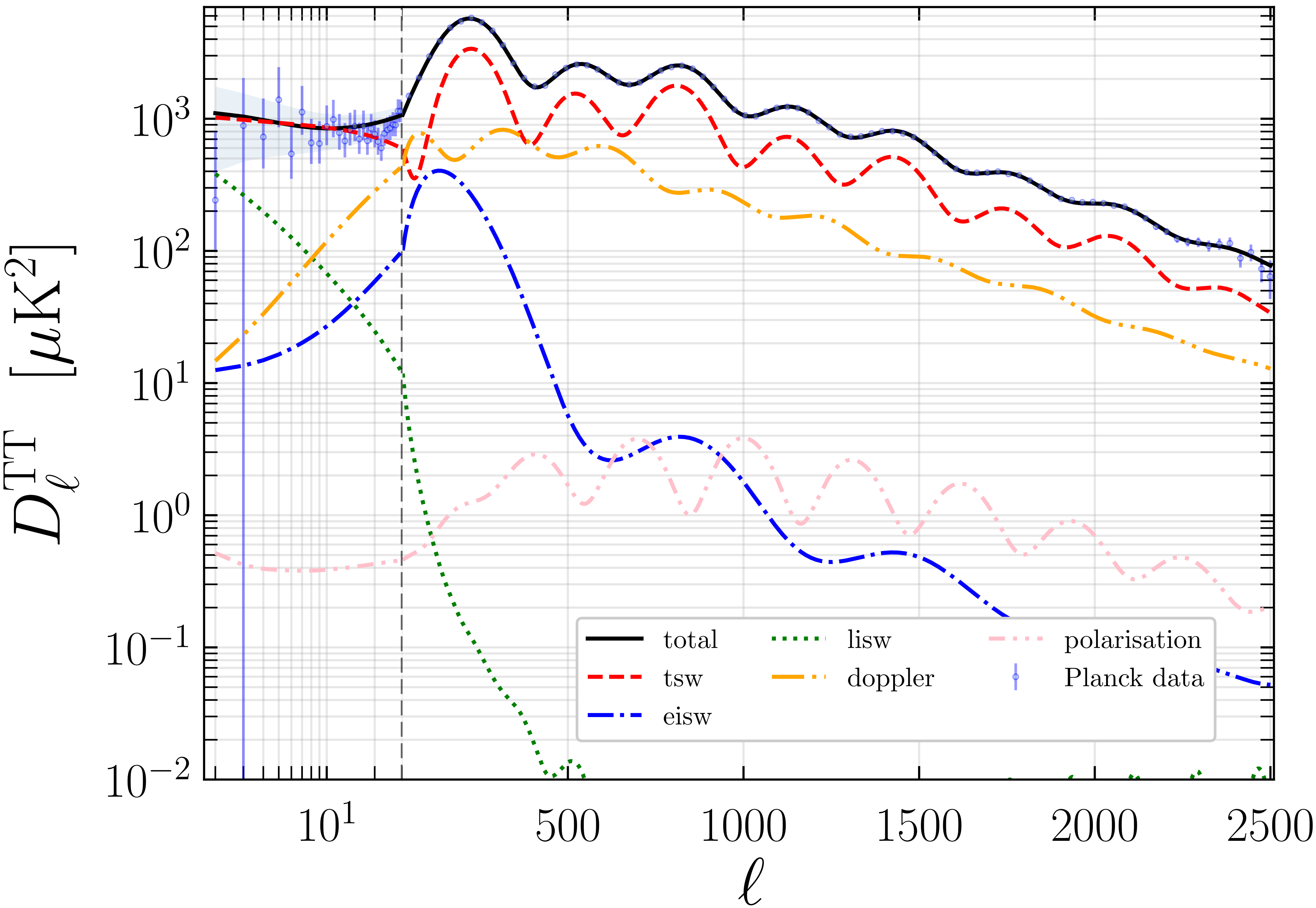
Contributions

unlensed



Contributions

lensed



Boris Bolliet, JBCA Manchester - Bard College, 10-15 June 2019

Doppler:

Photons and baryons are coupled at early time
->baryon velocity

Sachs-Wolf:

power spectrum of theta + Psi

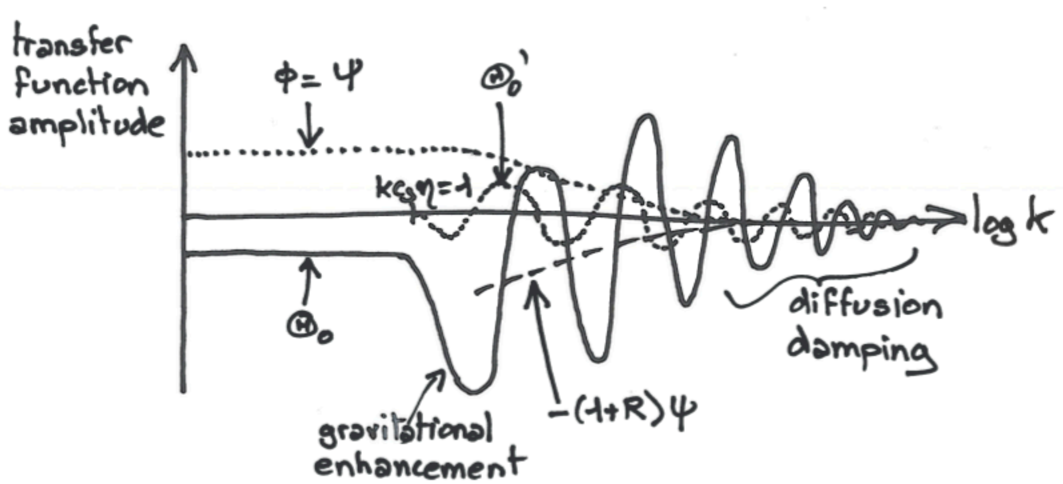


Fig. 5. Transfer functions at the time of decoupling.

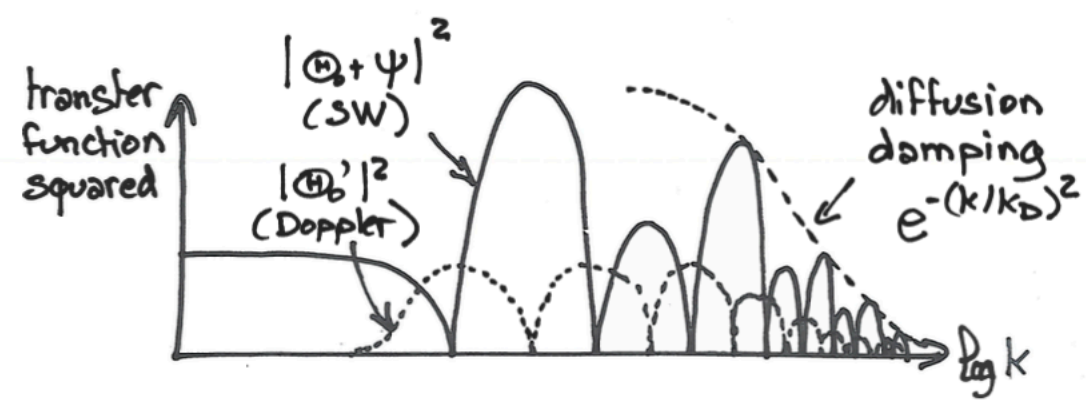
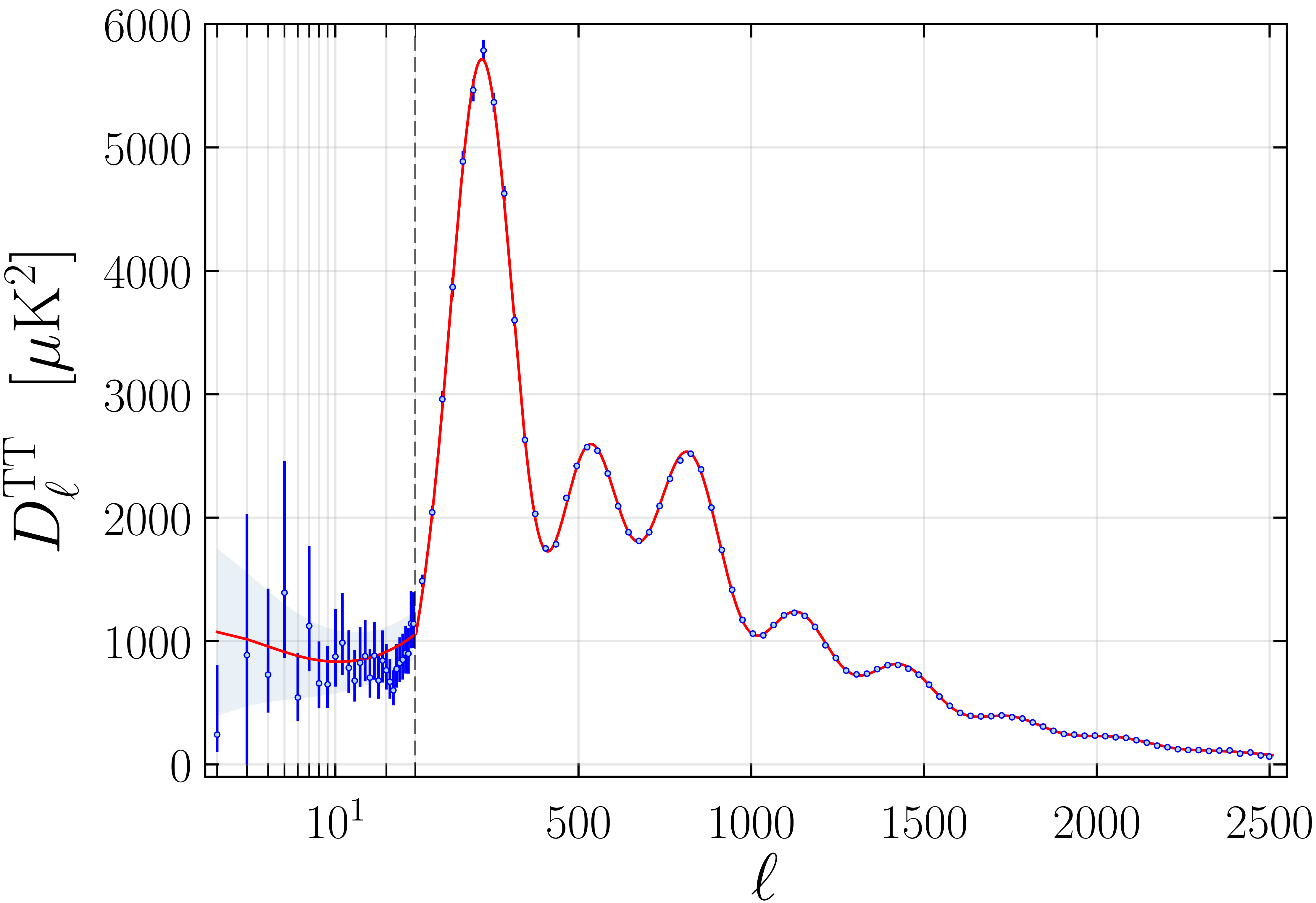


Fig. 6. Squared transfer functions at the time of decoupling.

From the notebook:



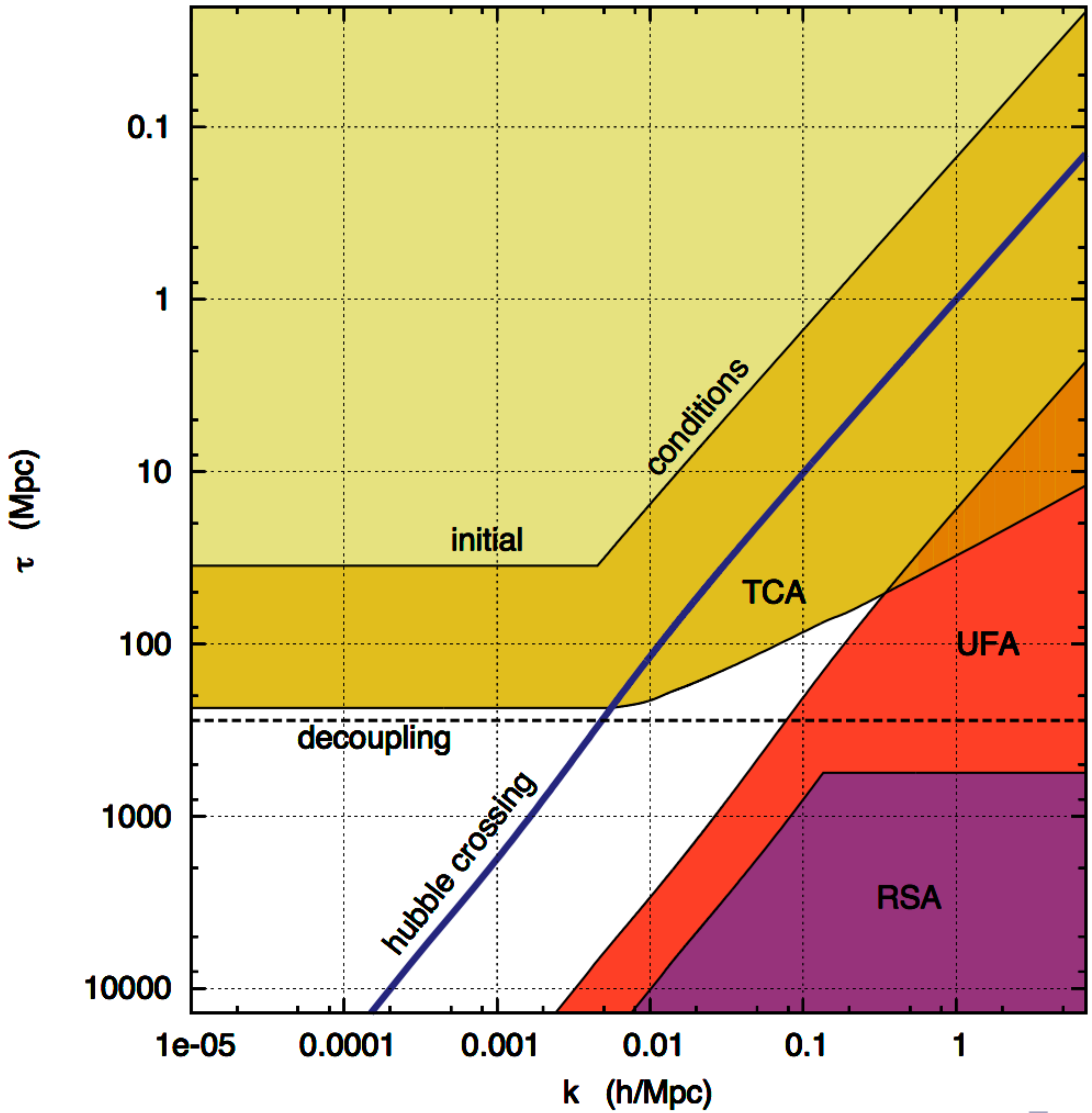


Figure: Lesgourgues

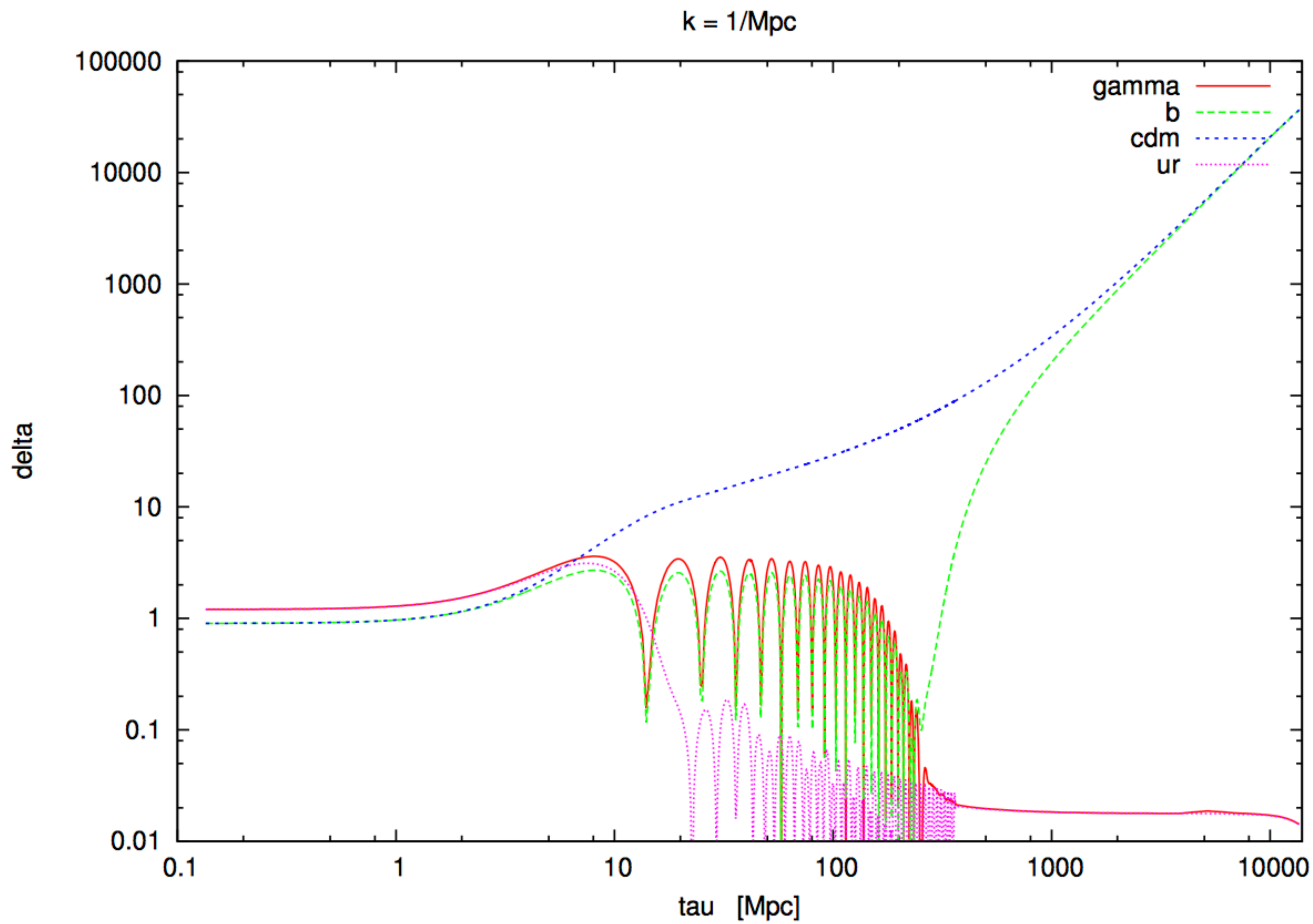
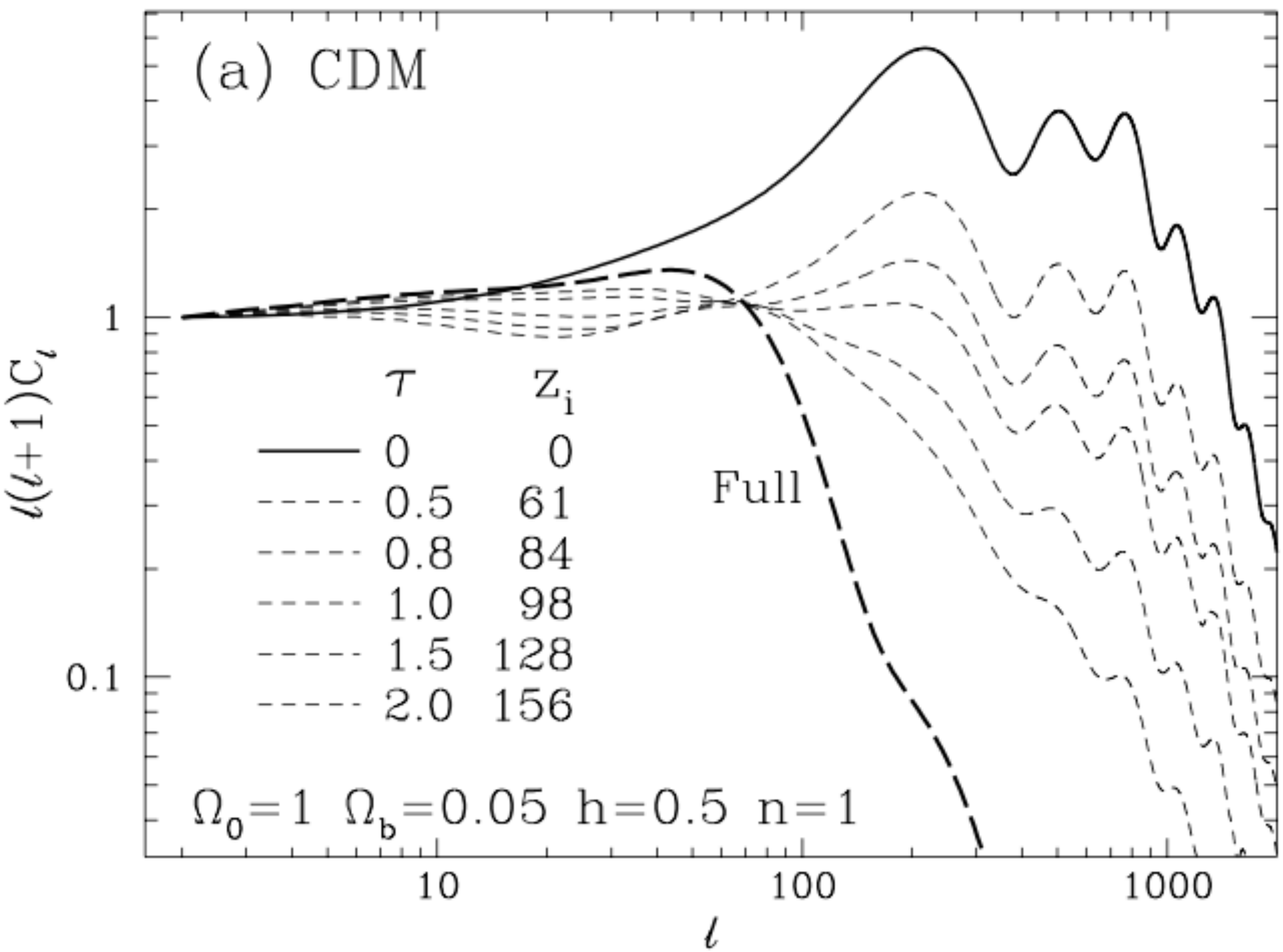


Figure: Lesgourgues

Optical depth

Figure from Hu PhD Thesis



CMB Polarisation

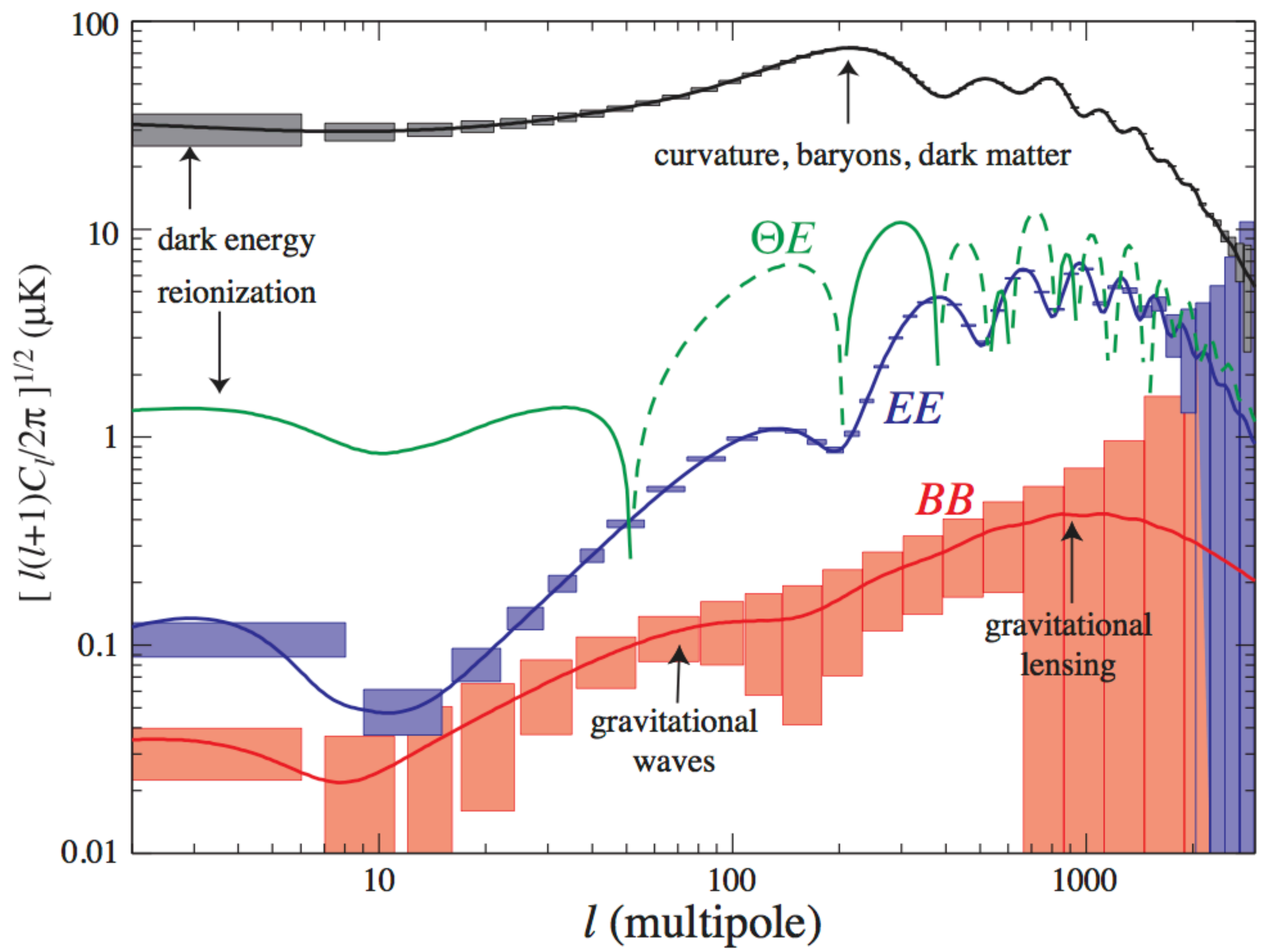


Fig. 19. Polarized landscape. While the E-spectrum and ΘE cross correlation are increasingly well measured, the B-spectrum from inflationary gravitational waves (shown here near the maximal value allowed by the temperature spectrum) and gravitational lensing remains undetected. Shown here are projected error bars associated with Planck sample variance and detector noise. Adapted from Hu and Dodelson (2002).

Figure: Hu

Baryons and matter

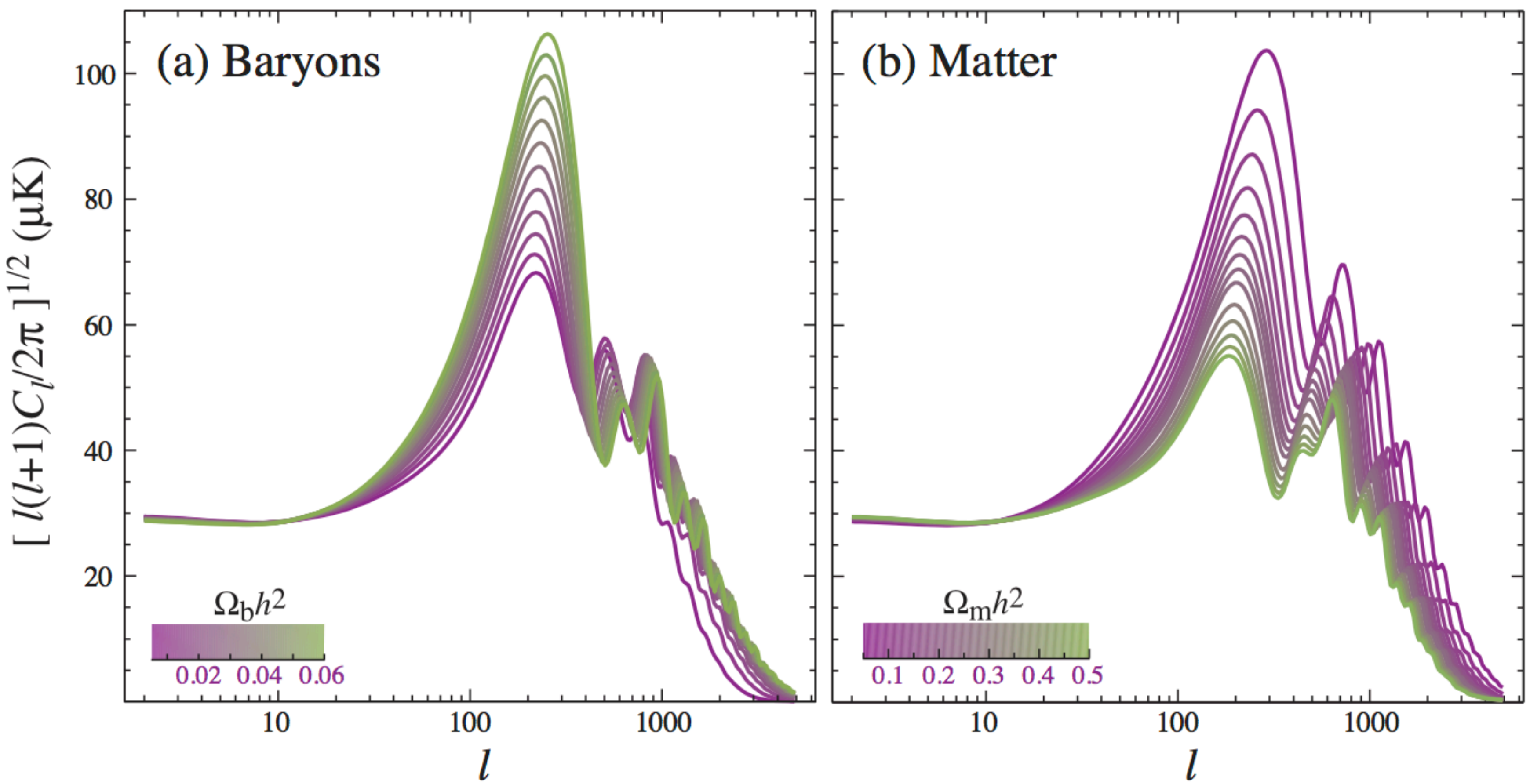


Fig. 15. Baryons and matter. Baryons change the relative heights of the even and odd peaks through their inertia in the plasma. The matter-radiation ratio also changes the overall amplitude of the oscillations from driving effects. Adapted from Hu and Dodelson (2002).

Figure from Hu

Silk Damping

Temperature anisotropy: damped oscillator => dispersion relation

imaginary term of the frequency: damping

$$\mathcal{D}_\ell \approx \exp[-(\ell/\ell_D)^{1.25}]\,,$$

$\ell_D = 2\pi D_*/\lambda_D$ and (Hu 2005)

$$\frac{\lambda_D}{\mathrm{Mpc}} \approx 64.5 \left(\frac{\Omega_m h^2}{0.14}\right)^{-0.278} \left(\frac{\Omega_b h^2}{0.024}\right)^{-0.18}$$

(Hu cmb lecture notes)

Table 1. Independent leading effects controlling the shape of the CMB temperature power spectrum C_l in the minimal ΛCDM model.

	Effect	Relevant quantity	Parameter
(C1)	Peak scale	$\theta_{\rm peak} = \frac{\pi}{l_{\rm peak}} \sim \frac{d_{\rm s}\|_{\rm dec}}{d_{\rm a}\|_{\rm dec}}$	$\leftarrow \omega_{\rm m}, \omega_{\rm b}$ $\leftarrow \Omega_\Lambda, \omega_{\rm m}$
(C2)	Odd/even peak amplitude ratio	$R\|_{\rm dec}$	$\omega_{\rm b}$
(C3)	Overall peak amplitude	$\frac{a_{\rm dec}}{a_0}$	$\omega_{\rm m}$
(C4)	Damping enveloppe	$\theta_{\rm d} = \frac{\pi}{l_{\rm d}} = \frac{a_{\rm dec} r_{\rm d}\|_{\rm dec}}{d_{\rm a}\|_{\rm dec}}$	$\leftarrow \omega_{\rm m}, \omega_{\rm b}$ $\leftarrow \Omega_\Lambda, \omega_{\rm m}$
(C5)	Global amplitude	$\mathcal{P}_{\mathcal{R}}(k_*)$	$A_{\rm s}$
(C6)	Global tilt	$\frac{d \log \mathcal{P}_{\mathcal{R}}}{d \log k}$	$n_{\rm s}$
(C7)	Additional plateau tilting (LISW)	$\frac{a_\Lambda}{a_0}$	Ω_Λ
(C8)	Amplitude for $l \geq 40$ only	$\tau_{\rm reio}$	$\tau_{\rm reio}$

Lesgourgues TASI