Lecture Notes

September 1st, 2011

Dear Class,

These lecture notes are intended to help you review the material covered in the course. They are not an ideal replacement for attending lecture. The act of writing your own notes, thinking through what is being said and actively questioning it will, I believe, be a huge part of your learning experience. You can do this with lecture notes but it is generally aided by interaction with other people. Come to lecture!

If you have comments on these notes or suggestions for their improvement, always feel free to contact me: hal “usual at” berkeley.edu.

Enjoy!

Hal

Course Logistics   Lecture 1   5 min   Aug. 26th, 2011

Hal Haggard
hal@berkeley.edu
(510) 435-1747
Office: 443 Birge

Nadir Jeevanjee
jeevanje@berkeley.edu

Office: 443 Birge

W 4-5pm 75 Evans

Sections Tu. 1-2pm 71 Evans

Everything on course websites:
http://bohr.physics.berkeley.edu/hal/teaching/phys105

bspace site

HW 1 posted

Come to Lecture!

Come to sections!

Sections will be co-taught.

Ask questions

Names

Baby

Today's Outline:

0. ~~Review Newtonian~~ mechanics (5 min)

I. Why are simple harmonic oscillations everywhere? (15 min)

II The Standard guess (10 min)

III 2D oscillations (10 min)

-I. Course Overview (5 min)

0. Review Newtonian Mechanics

$$\vec{F} = m\vec{a} = m\ddot{x}$$

↑ frequently in this course

$$\dot{x} \equiv \frac{dx}{dt} = v \qquad \ddot{x} = \frac{d^2x}{dt^2} = a$$

$$\overset{n \text{ times}}{\overbrace{\dot{\cdots}}}\, x = \frac{d^n x}{dt^n}$$

(That's our review of Newtonian mechanics)

That's it!

Why begin with oscillations?

Our world is permeated by oscillations, from the yielding sway of tree branches to the bumpy ride of the 52 bus. The mathematics of oscillations runs ~~much~~ even deeper than the mechanical examples that come immediately to mind.

Physical examples surround us.

Mathematical machinery is invaluable throughout physics: Electronics to Quantum Field Theory.

Why are (1D) oscillations so generic?

Physical systems are often well described by a potential energy:

$$U(x).$$

This potential is closely related conceptually to forces because...

we can see a force as arising from a "desire" to minimize potential energy

Mathematically the force is captured by

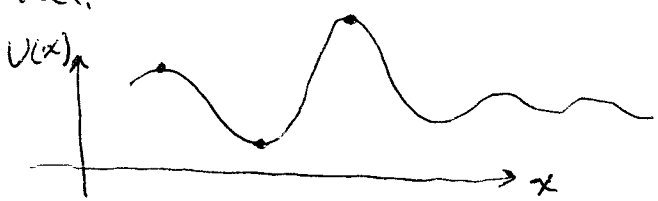
towards decreasing potential

$$F = -\frac{dU}{dx}$$

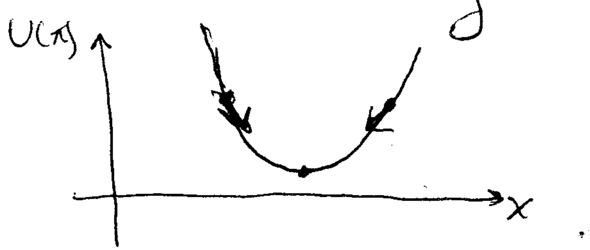
Equilibrium is when there is no net force or when

$$F = 0 = \frac{dU}{dx}.$$

This is also the condition for any extremum (maximum, minimum or inflection) of the potential.



Oscillations arise around stable equilibria! A small displacement (in either direction) pushes the system back towards the equilibrium position — there's a restoring force.



A maximum of the potential energy is an <u>unstable</u> equilibrium. Find 'em by

$$\frac{d^2U}{dx^2} < 0 \quad \left(\text{and } \frac{dU}{dx} = 0\right)$$

A minimum is a <u>stable</u> equilibrium. Find 'em by

$$\frac{d^2U}{dx^2} > 0$$

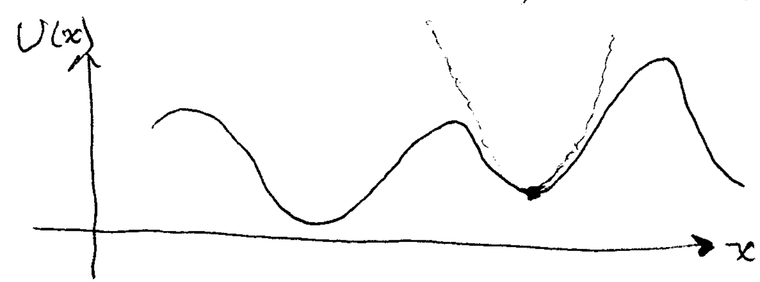
~~Inflections are metastable~~

$d^2U/dx^2 = 0 \rightarrow$ second deriv. test fails.

We can say more, Taylor expand the potential near an equilibrium at $x=0$

$$\cancel{U(x) = U(x_0) + U'(x_0)(x-x_0) + \tfrac{1}{2}U''(x_0)(x-x_0)^2}$$

$$U(x) = U(0) + \overset{0 \leftarrow \text{equilibrium}}{\cancel{U'(0)}}(x) + \frac{1}{2}U''(0)\,x^2 + \dots$$

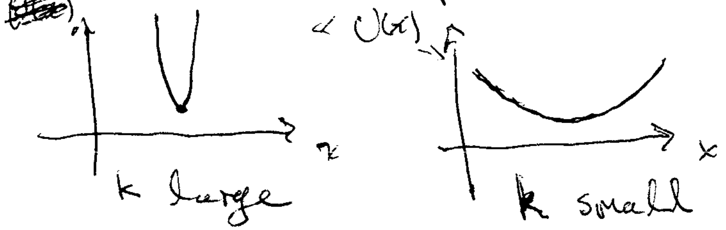


Compare this to the potential energy of a spring

that's my "kay"

$$U(x) = \tfrac{1}{2} k x^2$$

What does the spring constant tell us geometrically? The width of the potential



We've shown that

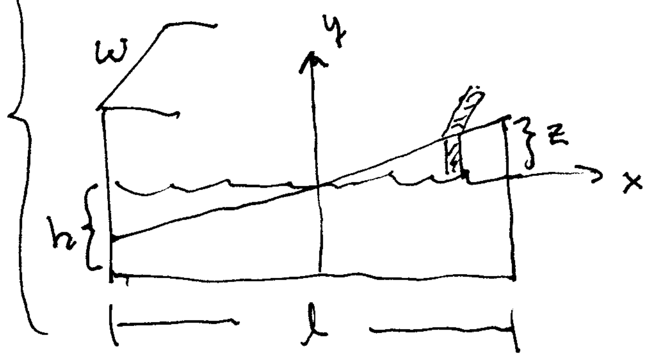
$$k = U''(x_{equil.})$$

for positive $U''(x_{equil.})$.

No matter what shape your potential has it looks like a harmonic oscillator near its minima (as long as $U'' > 0$)

skipped this example due to time constraints.

Example:



$$y = mx$$

$$m = z/(l/2)$$

$$y = \frac{2z}{l} x$$

$$U(x) = \int_{-l/2}^{l/2} dx \, \rho w y dx \, g \tfrac{1}{2} y$$

$$= \underbrace{\int_{-l/2}^{l/2} x^2}\left(\tfrac{1}{2} \rho w \left(\frac{2z}{l}\right)^2\right)$$

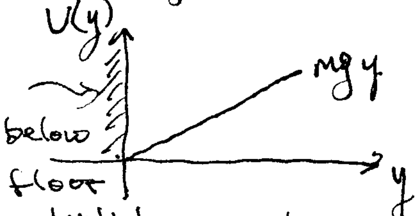
$$= \frac{1}{3}\left(\left(\frac{l}{2}\right)^3 - \left(-\frac{l}{2}\right)^3\right) = \frac{1}{3} \cdot \frac{2}{8} l^3$$

$$= \frac{1}{6} \rho w l g z^2$$

Did this;

Counter Example:

Bouncy Ball



below floor potential is ∞

Can't do Taylor expansion because it's not differentiable

I hope to have convinced you that oscillations are everywhere.

Change of perspective in this course

$$F = ma = -kx$$

$$\downarrow \quad m\frac{d^2x}{dt^2} = -kx$$

$$\downarrow \quad m\ddot{x} = -kx$$

We will learn many techniques for writing down eq.s of motion but also for solving them.

Ordinary diff. eq. (ODE): only ordinary derivatives

Partial diff. eq. (PDE): partial derivatives

Order of diff. eq.: The highest derivative.

Most important method of solutions is the "standard guess"

$$x(t) = e^{\lambda t}$$

$$\dot{x} = \lambda e^{\lambda t}$$

$$\ddot{x} = \lambda^2 e^{\lambda t}$$

$$m\ddot{x} = m\lambda^2 e^{\lambda t} = -kx = -ke^{\lambda t}$$

turned a differential equation into an algebraic one.

$$\Rightarrow \quad \lambda^2 = -k/m$$

$$\Rightarrow \quad \lambda = \pm\sqrt{-1}\sqrt{\frac{k}{m}} = i\sqrt{\frac{k}{m}} \equiv i\omega$$

Solution $x(t) = $ ~~$e^{i\omega t}$~~ $c_1 e^{i\omega t} + c_2 e^{-i\omega t}$ superposition

This is generally true — 2nd order diff. eq. has a general solution depending on two constants.

Physically it means that you need boundary conditions:

$$x(0) = c_1 + c_2$$

$$\dot{x}(0) = i\omega c_1 - i\omega c_2.$$