Lecture 16
Oct. 5^{th}, 2011

Today's outline

I Finish up orbit transfer

II Noninertial Frames:

Acceleration w/out rotation

III ~~Major~~ Great Example: Tides

Putting these together we find

$$\frac{c_1}{1+\epsilon_1} = \frac{\lambda^2 c_1}{(1+\epsilon_2)}$$

$$\Rightarrow \quad 1+\epsilon_2 = \lambda^2 + \lambda^2 \epsilon_1$$

$$\Rightarrow \quad \epsilon_2 = \lambda^2 \epsilon_1 + (\lambda^2 - 1)$$

$\lambda > 1 \Rightarrow \epsilon_2 > \epsilon_1$, orbit is more eccentric until escape.

$\lambda < 1 \Rightarrow \epsilon_2 < \epsilon_1$, orbit is less eccentric $\rightarrow$ circular $\rightarrow$ perigee and apogee switch.

I Orbit transfer

Special case: tangential thrust at perigee.

Choose thrust s.t. $\phi = 0 \Rightarrow \phi_0 = 0$, $\delta_1 = 0$, $\delta_2 = 0$. In this case the orbit equation becomes

$$r_1 = \frac{c_1}{1+\epsilon_1} = r_2 = \frac{c_2}{1+\epsilon_2}.$$

Thrust factor: $v_2 = \lambda v_1$ $\begin{pmatrix} \lambda > 1 & \text{speed up} \\ \lambda < 1 & \text{slow down} \end{pmatrix}$

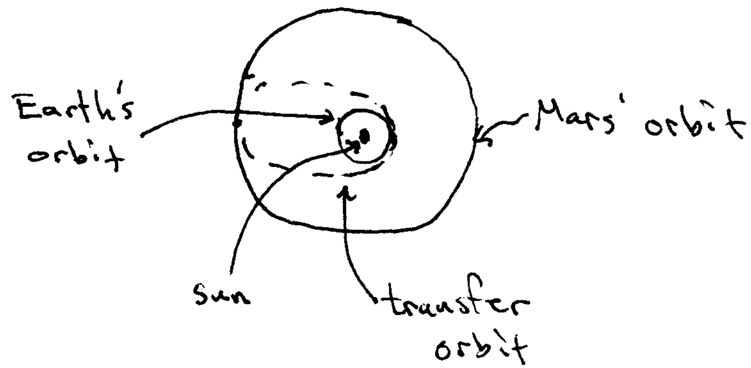
Perigee: $v = r\dot{\phi}$ and $\ell = \mu r v$

$$\Rightarrow \ell_2 = \lambda \ell_1 \quad \text{and} \quad c_2 = \lambda^2 c_1 \quad \left(c = \frac{\ell^2}{\gamma \mu}\right)$$

Example: Most efficient transfer to Mars: "Hohmann transfer". Both the earth and mars have roughly circular orbits:

	Eccen.	Orbit radius
Earth	$\epsilon_e = 0.0167$	$R_e = 1.5 \times 10^{11}$ m
Mars	$\epsilon_M = 0.0933$	$R_M = 2.3 \times 10^{11}$ m

A Hohmann transfer involves two thrusts, the first takes you from the circular (Earth) orbit to an elliptical orbit that intersects the desired ~~orbit~~ circular orbit (Mars in our case) and the second that brings you onto the target orbit. (Note: I'm speaking about orbits around the Sun now.)



Take $\epsilon_e \approx \epsilon_M \approx 0$. Denote the transfer orbit quantities w/ t subscripts. Then

$$c_1 = c_e = R_e \quad \left(\text{recall geometrical interp. of } c\right)$$

and

$$c_t = \lambda^2 R_e$$

$$\epsilon_t = \lambda^2(0) + (\lambda^2 - 1) = \lambda^2 - 1$$

Now R_M must be ~~apogee~~ aphelion of transfer so,

$$R_M = \frac{c_t}{1-\epsilon_t} = \frac{\lambda^2 R_e}{1-(\lambda^2-1)} = \frac{\lambda^2 R_e}{2-\lambda}$$

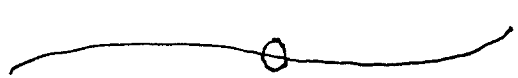
$$\Rightarrow \quad 2R_M - \lambda^2 R_M = \lambda^2 R_e$$

$$\Rightarrow \quad \lambda^2 (R_e + R_M) = 2R_M$$

$$\Rightarrow \quad \lambda = \sqrt{\frac{2R_M}{R_e+R_M}} = \sqrt{\frac{4.6}{3.8}} \approx 1.10$$

Need a 10% increase in speed!

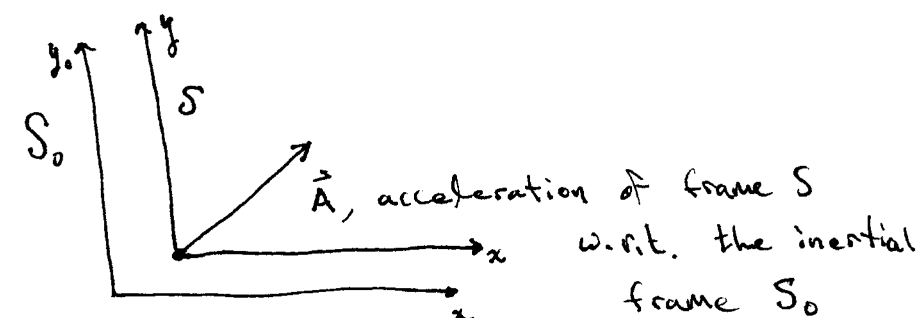
I leave it to you to find λ_2, the thrust factor to go into Mars' orbit



II Noninertial Frames

- Newton's laws only hold in inertial frames
- We are careful to write down the Lagrangian in an inertial frame.

For the next few lectures we explore non-inertial frames.



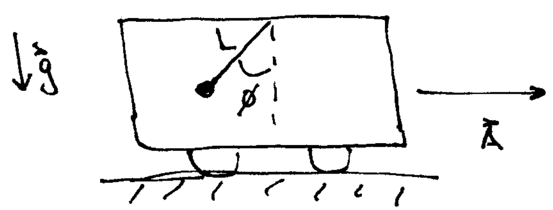
Consider the position of a particle. In the frame S_0 this is measured to be $\vec{r}_0$ and it obeys Newton's 2nd law

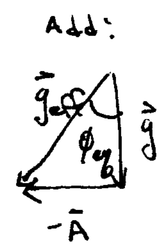
$$m\ddot{\vec{r}}_0 = \vec{F}$$

In S they measure $\vec{r}$ and

$$\dot{\vec{r}}_0 = \dot{\vec{r}} + \vec{V}$$

particle's vel. w.r.t. ground = particles vel. w.r.t. moving frame + vel. of moving frame w.r.t. ground





Noninertial (boxcar) frame!

$$m\ddot{\vec{r}} = \vec{T} + m\vec{g} - m\vec{A}$$

$$= \vec{T} + m(\vec{g} - \vec{A})$$

$$= \vec{T} + m\vec{g}_{eff}\,, \quad \vec{g}_{eff} \equiv \vec{g} - \vec{A}$$

Then

$$\phi_{eq} = \arctan\left(\frac{A}{g}\right)$$

and

$$\omega = \sqrt{\frac{g_{eff}}{L}} = \sqrt{\frac{\sqrt{g^2 + A^2}}{L}}$$

This then implies,

$$\ddot{\vec{r}}_0 = \ddot{\vec{r}} + \vec{A} \quad \text{or} \quad \ddot{\vec{r}} = \ddot{\vec{r}}_0 - \vec{A}$$

So that

$$m\ddot{\vec{r}} = m\ddot{\vec{r}}_0 - m\vec{A}$$

$$= \vec{F} - m\vec{A}$$

This looks a lot like Newton's 2nd law. In fact it is if we agree to introduce an "inertial force"

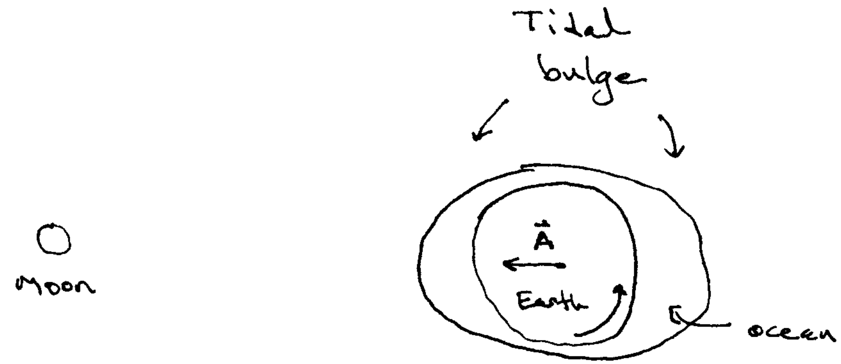
$$\vec{F}_{inertial} = -m\vec{A}$$

<u>Examples</u>: Airplane take off, elevator, car, etc.

This is computationally fast, conceptually subtle route!

III ~~Major~~ Great Example: Tides

The Earth is a noninertial frame because it is rotating. We will discuss this more next lecture. However it is also a noninertial frame because it is accelerating towards the moon (sun also). The tides are a combined effect of the moon's gravitational attraction of the ocean and of this acceleration.



The acceleration $\vec{A}$'s magnitude and direction are determined by treating the Earth and moon as point masses concentrated at their respective centers.

Forces on m: (1) Earth's gravity: $m\vec{g}$

(2) Moon's gravity: $-GM_m m\,\hat{d}/d^2$ (M_m = moon's mass)

(3) Net non-gravitational force: $\vec{F}_{ng}$

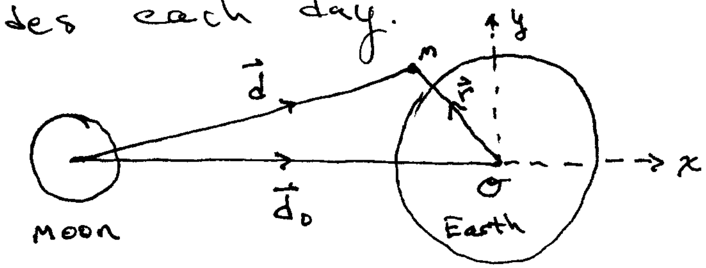
And the earth is a noninertial frame with acceleration,

$$\vec{A} = -\frac{GM_m}{d_o^2}\hat{d}_o$$

We can use our noninertial frame law

$$m\ddot{\vec{r}} = \vec{F} - m\vec{A}$$

On the other hand the gravitational pull of the moon on the oceans is <u>greater</u> on the side closer to the moon and <u>weaker</u> on the opposite side. This is why there are two high tides each day.



$$\Rightarrow m\ddot{\vec{r}} = \left(m\vec{g} - GM_m m\frac{\hat{d}}{d^2} + \vec{F}_{ng}\right) + GM_m m\frac{\hat{d}_o}{d_o^2}$$

$$\Rightarrow m\ddot{\vec{r}} = m\vec{g} \underbrace{- GM_m m\left(\frac{\hat{d}}{d^2} - \frac{\hat{d}_o}{d_o^2}\right)}_{\equiv \vec{F}_{tid}} + \vec{F}_{ng}$$

Illustrate direction of tidal force on figure.

Next time: Quantitative analysis of tides, Begin rotating frames