Lecture 21

October 17^{th}, 2011

Today's Outline:

I Survey

II Last lecture/Where are we?

III Collections of particles

~~IV Why must we distinguish $\vec{\omega}$ and $\vec{L}$?~~

V Announce Midterm 2: Oct 28^{th} (Fri.)

- Covers Ch.s 7, 8, 9
- Bring blue book
- One pg. single sided of notes

1. What percentage of the time do you feel you've understood the main point of a lecture when its ended?

2. Is the lecture moving too fast, too slow or at the right pace?

3. What would you change were you teaching the course? What's going well?

I Survey

We recently passed the half way mark for the course and I would like your feedback. I will use this, as best I can, to improve this course ~~and~~ to improve my teaching going forward.

I appreciate both constructive critique and positive feedback.

4. In a few weeks we will have completed the core foundation of the course. I currently plan to cover: Hamiltonian Mechanics, Chaos and Collision theory.

Pick your top 3 of the following topics (or pick 2 and say which you would like to spend more time with): Hamiltonian Theory, chaos, collision theory, continuum mechanics, Hamilton-Jacobi theory, or Computational techniques: symplectic integrators

Any additional comments:

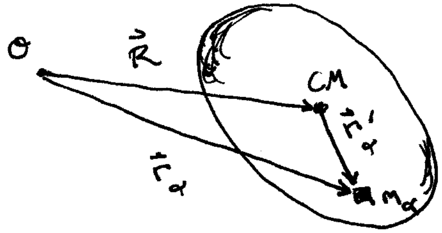
II Last Lecture / Where are we?

We investigated rotating frames for two reasons:

1. To understand motion in noninertial frames: modify Newton's 2nd law to ~~take~~ include fictitious forces.
2. To setup context for investigating rotating <u>bodies</u> (as opposed to frames).

This formula is <u>amazing</u> (!): We can treat collections of particles as a single pt. particle with mass M subject to $\vec{F}^{ext}$.

Consider



a rigid body and define $\vec{r}_\alpha'$ as the position of m_α w.r.t. the C.M.

III Collections of particles

CM: $\vec{R} = \frac{1}{M} \sum_\alpha m_\alpha \vec{r}_\alpha = \frac{1}{M} m_\alpha \vec{r}_\alpha$ $(\alpha = 1, \dots, N)$

Einstein summation convention: repeated index <u>means</u> sum over that index.

Total Momentum: $\vec{P} = \sum_\alpha p_\alpha = m_\alpha \dot{\vec{r}}_\alpha = M\dot{\vec{R}}$.

$$\left(\Sigma \frac{d}{dt}(m_\alpha \vec{r}_\alpha) = \frac{d}{dt}(M\vec{R}) \right)$$

Total external Force: $\dot{\vec{P}} = \vec{F}^{ext} = M\ddot{\vec{R}}$

[Aside: Throughout this chapter Taylor forgoes integration unless it is absolutely necessary. A nice convention but do be careful.]

$$\vec{r}_\alpha = \vec{R} + \vec{r}_\alpha'$$

Ang. mom. of m_α: $\vec{l}_\alpha = \vec{r}_\alpha \times \vec{p}_\alpha = \vec{r}_\alpha \times m_\alpha \dot{\vec{r}}_\alpha$ (no sum)

Tot. Ang. mom.: $\vec{L} = \sum_\alpha \vec{l}_\alpha = \sum_\alpha \vec{r}_\alpha \times m_\alpha \dot{\vec{r}}_\alpha$

this simplifies

$$= \sum_\alpha (\vec{R} + \vec{r}_\alpha') \times m_\alpha (\dot{\vec{R}} + \dot{\vec{r}}_\alpha')$$

So,

$$\vec{L} = \sum \vec{R} \times m_\alpha \dot{\vec{R}} + \sum \vec{R} \times m_\alpha \dot{\vec{r}}'_\alpha + \sum \vec{r}'_\alpha \times m_\alpha \dot{\vec{R}} + \sum \vec{r}'_\alpha \times m_\alpha \dot{\vec{r}}'_\alpha$$

$$= \vec{R} \times M\dot{\vec{R}} + \vec{R} \times \Big(\sum m_\alpha \dot{\vec{r}}'_\alpha\Big) + \Big(\sum \vec{r}'_\alpha m_\alpha\Big) \times \dot{\vec{R}} + \sum \vec{r}'_\alpha \times m_\alpha \dot{\vec{r}}'_\alpha$$

0, CM position relative to CM

0, time derivative of zero

$$= \vec{R} \times \vec{P} + \sum \vec{r}'_\alpha \times m_\alpha \dot{\vec{r}}'_\alpha$$

$$\Rightarrow \vec{L} = \vec{L}(\text{motion of CM}) + \vec{L}\left(\begin{matrix}\text{motion}\\ \text{relative to CM}\end{matrix}\right)$$

Also simplifies:

$$\dot{\vec{r}}_\alpha^{\,2} = (\dot{\vec{R}} + \dot{\vec{r}}'_\alpha)^2 = \dot{\vec{R}}^2 + \dot{\vec{r}}'^2_\alpha + 2\dot{\vec{R}} \cdot \dot{\vec{r}}'_\alpha$$

$$\Rightarrow T = \frac{1}{2}\sum m_\alpha \dot{\vec{R}}^2 + \frac{1}{2}\sum m_\alpha \dot{\vec{r}}'^2_\alpha + \dot{\vec{R}}\, m_\alpha \dot{\vec{r}}'_\alpha$$

zero as before

$$= \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} m_\alpha \dot{\vec{r}}'^2_\alpha$$

In words

$$T = T(\text{motion of CM}) + T\left(\begin{matrix}\text{motion relative}\\ \text{to CM}\end{matrix}\right)$$

(~~rotation~~)

For a rigid body any motion relative to the CM is a rotation (Euler's theorem)

For example a planet orbiting the sun (~~or roughly at sun~~ assume sun fixed):

$$\vec{L} = \vec{L}_{orb} + \vec{L}_{spin}$$

orbital angular momentum due to motion around the sun

spin ang. mom. due to motion relative to CM.

Total kinetic Energy: $T = \sum_\alpha \frac{1}{2} m_\alpha \dot{\vec{r}}_\alpha^{\,2}$

and so

$$T = T\left(\begin{matrix}\text{motion of}\\ \text{CM}\end{matrix}\right) + T\left(\begin{matrix}\text{rotation}\\ \text{about CM}\end{matrix}\right)$$

Finally, if the forces on and within a body are conservative,

$$U = U^{ext} + U^{int}$$

total external potential energy

gives rise to forces that hold body together

$$U^{int} = \sum_{\alpha<\beta} U_{\alpha\beta}(r_{\alpha\beta}) = \text{constant we can drop.}$$

dist. btwn particles α and β