Lecture 37

November 30th, 2011

Today's Outline

I Last Lecture

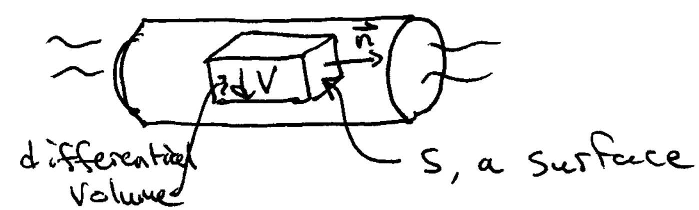
II Isotropy of Pressure

III Stress & Strain: The Elastic Moduli

I Last Lecture

We derived the E.O.M. for a taut string, the wave equation.

We began our study of continua:

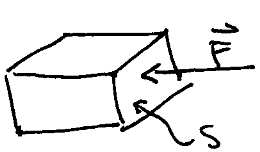


Volume Force = force proportional to dV

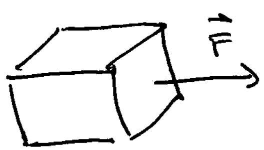
Surface Force = force proportional to dA

~~II Isotropy of pressure~~

More Examples of surface forces:

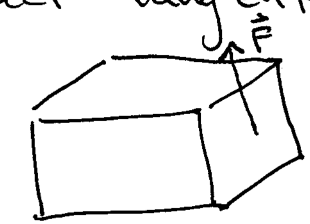


pressure



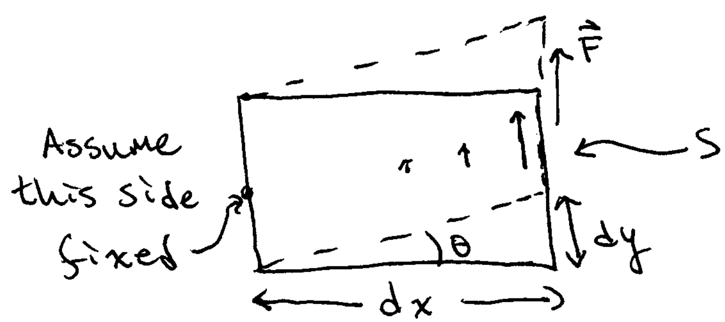
tension

Pressure and tension forces act normally to S, while shear forces act tangentially:



Shear

Zoom in on shear



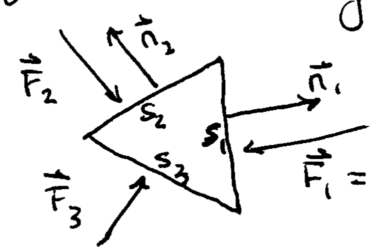
dy, dx, θ quantify how strong a shear force is ~~ap~~ being applied.

II Isotropy of pressure

Already we can prove a neat result: pressure pushes in all directions equally.

Assume we are considering a non-viscous medium (i.e. no shearing).

Choose two directions, $\vec{n}_1$ and $\vec{n}_2$ and build an ~~equilateral~~ isosceles triangle ~~pism~~ prism around them with $dA_1 = dA_2$



$\vec{F}_1 = -p_1 \vec{n}_1 dA_1$

with similar expressions for $\vec{F}_2$ and $\vec{F}_3$. Our aim is to show that $p_1 = p_2 = p_3$. The force in the plane of the figure's cross-section is,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \underbrace{\vec{F}_{vol}}_{\text{net volume force}} = m\vec{a}$$

Now, considering shrinking our prism by a factor λ then this equation becomes

$$\lambda^2 (\vec{F}_1 + \vec{F}_2 + \vec{F}_3) = \lambda^3 (m\vec{a} - \vec{F}_{vol})$$

$$\Rightarrow (\vec{F}_1 + \vec{F}_2 + \vec{F}_3) = \lambda \, (m\vec{a} - \vec{F}_{vol})$$

As $\lambda \to 0$ we find,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

Crossing in $\vec{F}_3$ we find

$$(F_1 F_3 \sin\theta - F_2 F_3 \sin\theta| = 0$$

$$\Rightarrow F_1 = F_2$$

And so,

$$p_1 dA_1 = p_2 dA_2 \Rightarrow p_1 = p_2$$

but $\vec{n}_1$ and $\vec{n}_2$ were arbitrary: this implies pressure is isotropic!

III Stress & Strain: The Elastic Moduli

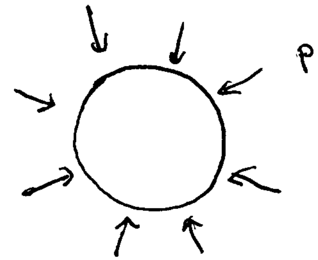
The interesting part of surface forces is the ratio

$$\frac{F}{A} \equiv \text{Stress}$$

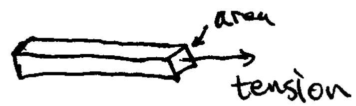
Examples

Stress

Static fluid $\frac{F}{A}$ = pressure, p



wire under tension $\frac{\text{tension}}{\text{area}}$



Shearing ~~forces~~ $\frac{\text{shearing force}}{\text{area}}$

If we don't push too hard then the response should be proportional to how hard we push, that is,

$$\text{stress} = \begin{pmatrix}\text{appropriate} \\ \text{modulus}\end{pmatrix} \times \text{Strain}$$

It's not surprising that a given material might respond differently to different kinds of stress (e.g. a book under shear vs. pressure) and so we need different moduli:

Strain is the fractional change in shape or deformation due to a stress

Strain

$\frac{dV}{V}$ — V the volume of bead

$\frac{dl}{l}$ — l the length of the wire

$\frac{dy}{dx}$ — see zoom in from pg 1

For our three examples:

$$dp = -BM \cdot \frac{dV}{V}$$

BM = "bulk modulus"

$$\frac{dF}{A} = YM \cdot \frac{dl}{l}$$

YM = "Young's Modulus"

$$\frac{F}{A} = SM \cdot \frac{dy}{dx}$$

SM = "Shear modulus"