Lecture 38

Final Lecture

December 2nd, 2011

Today's Outline

I Last Lecture

II The stress Tensor

I Last Lecture

- We explored some examples of surface forces
- We showed that for a non-viscous medium pressure is isotropic
- We introduced stress, strain and elastic moduli:

$$\text{stress} = \frac{\text{force}}{\text{area}}$$

strain = fractional deformation

$$\text{elastic modulus} = \frac{\text{stress}}{\text{corresponding strain}}$$

Before taking a bird's eye view of the course and its applications we introduce one more object from continuum mechanics, the strain tensor. This gives us one more opportunity to explore a practical tensor.

II The stress Tensor

Let's derive the general expression for surface force on a small area dA of a closed surface S in a continuous medium.

First let $d\vec{A} \equiv \hat{n}\, dA$. We'll proceed in two steps

(i) show that $\vec{F}(\alpha_1 d\vec{A}_1 + \alpha_2 d\vec{A}_2)$

$$= \alpha_1 \vec{F}(d\vec{A}_1) + \alpha_2 \vec{F}(d\vec{A}_2)$$

(ii) From (i) we argue that it follows that $\vec{F}$ and $d\vec{A}$ are related by a tensor $\overleftrightarrow{\Sigma}$, the "stress tensor."

Let's start with (i). By the definition of a surface force

$$\vec{F}(\alpha\, d\vec{A}) = \alpha\, \vec{F}(d\vec{A})$$

for α positive and not too large. Now

$d\vec{A} \to -d\vec{A}$ switches inside and outside

It's not too hard to prove that

$$d\vec{A}_1 + d\vec{A}_2 + d\vec{A}_3 = 0$$

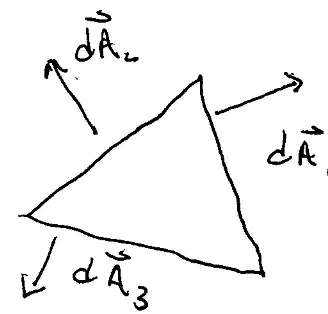
in general but here's a nice intuitive argument. ~~that's not totally general~~ Imagine the prism is immersed in a fluid that is non-viscous then because it's in equilibrium the surface forces satisfy

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

and by Newton's 3rd law this also switches $\vec{F}$ and $-\vec{F}$ so,

$$\vec{F}(-d\vec{A}) = -\vec{F}(d\vec{A}).$$

Now, what about adding $d\vec{A}$s? Consider again a triangular prism



but they are just pressures so

$$p\, d\vec{A}_1 + p\, d\vec{A}_2 + p\, d\vec{A}_3 = 0$$

$$\Rightarrow \quad d\vec{A}_1 + d\vec{A}_2 + d\vec{A}_3 = 0$$

This last result is geometrical and can't depend on how we arrived at, so it's totally general.

Now, immerse the prism in

in any medium (or consider a prism shaped piece of any material) then

$$\vec{F}(d\vec{A}_1 + d\vec{A}_2) = \vec{F}(-dA_3)$$

$$= -\vec{F}(d\vec{A}_3)$$

$$= \vec{F}(d\vec{A}_1) + \vec{F}(d\vec{A}_2)$$

where the last equality follows from the equilibrium condition on the forces. That does it, the surface force depends <u>linearly</u> on $d\vec{A}$.

(ii) But linearity <u>means</u> that we can write

$$F_i = \sum_j \sigma_{ij}\, dA_j$$

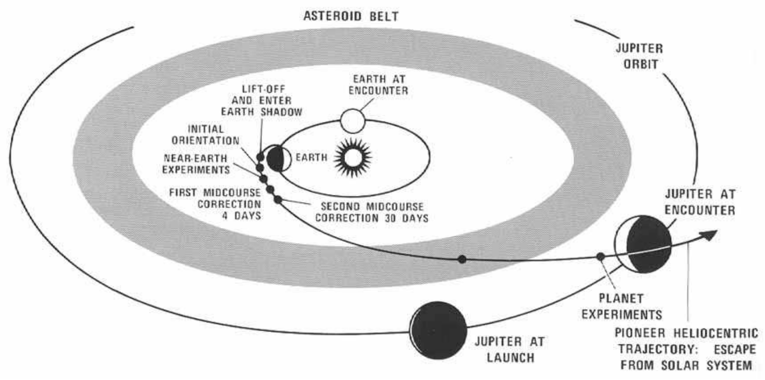
which is precisely what it means to say there is a tensor connecting $\vec{F}$ and $d\vec{A}$:

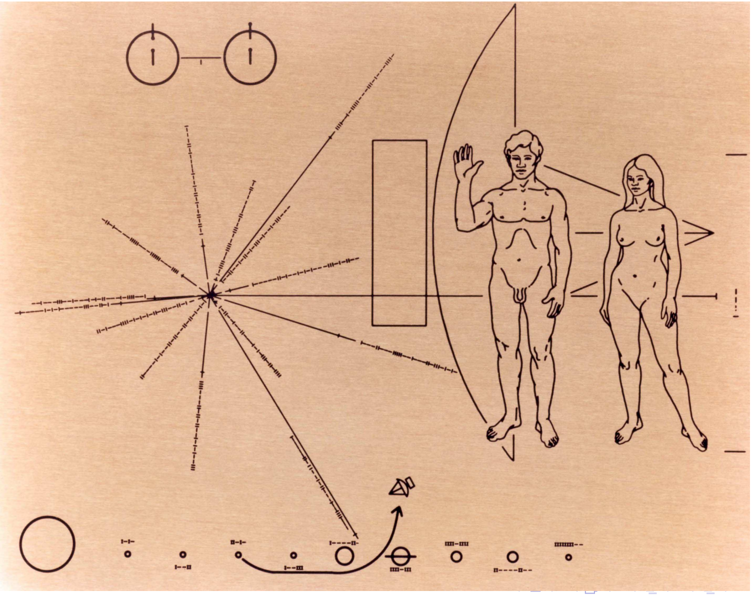
$$\boxed{\vec{F} = \overleftrightarrow{\Sigma}\, d\vec{A}}$$

with

$$[\overleftrightarrow{\Sigma}]_{ij} = \sigma_{ij}$$

the "stress tensor". If we were to go on from here we would derive a strain tensor and the E.O.M. would follow from Newton's laws and the relationship between these tensors. Instead, we will briefly comment on the course as a whole. See attached slides.


ASTEROID BELT
JUPITER ORBIT
EARTH AT ENCOUNTER
LIFT-OFF AND ENTER EARTH SHADOW
INITIAL ORIENTATION
NEAR-EARTH EXPERIMENTS
EARTH
FIRST MIDCOURSE CORRECTION 4 DAYS
SECOND MIDCOURSE CORRECTION 30 DAYS
JUPITER AT ENCOUNTER
PLANET EXPERIMENTS
PIONEER HELIOCENTRIC TRAJECTORY: ESCAPE FROM SOLAR SYSTEM
JUPITER AT LAUNCH



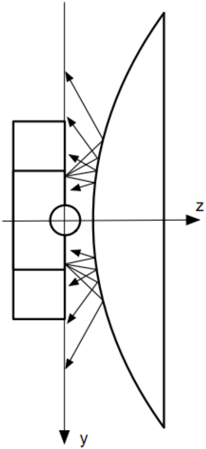
The Pioneer anomaly is an unexplained acceleration towards the Sun with magnitude,

$$8.74 \pm 1.33 \times 10^{-10} m/s^2.$$

The current best explanation...

...is completely *classical*.

In fact, it uses ray tracing techniques developed in the '70s and mostly used for video games!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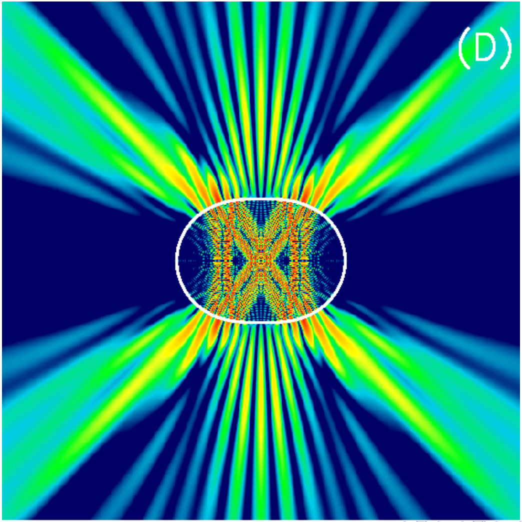
y

A little quantum chaos...



... and a meditation on quantum gravity.

$\hbar$ is crazy small 6.626×10^{-34}. In fact, we are closer to the size of the observable universe, 45.7 billion light years or 4×10^{26} meters than we are...

... to the Planck length, $\ell_{Pl} = \sqrt{\hbar G/c^3} = 1.6 \times 10^{-35}$m.