Homework 17

Due Wednesday, April 17$^{\text{th}}$ in lab

In Hafele and Keating's paper on the results of their flying clocks experiment (which you can download here from our course website) they report the following results:

Table 1. Observed relativistic time differences from application of the correlated rate-change method to the time intercomparison data for the flying ensemble. Predicted values are listed for comparison with the mean of the observed values; S.D., standard deviation.

Clock serial No.	$\Delta\tau$ (nsec)	
	Eastward*	Westward
120	− 57	277
361	− 74	284
408	− 55	266
447	− 51	266
Mean ± S.D.	− 59 ± 10	273 ± 7
Predicted ± Error est.	− 40 ± 23	275 ± 21

* Negative signs indicate that upon return the time indicated on the flying clocks was less than the time indicated on the MEAN(USNO) clock of the U.S. Naval Observatory.

1. To gear you up for doing these calculations in general, compute the standard deviation of each of the Eastward and Westward clock measurements by hand. Do your results agree with what Hafele and Keating report?

2. (a) Use excel's `RAND()` function to generate 1000 random numbers. Elsewhere in your sheet compute the variance and standard deviation for the first 20 of these numbers. Repeat for 100 of the numbers and for all 1000 of them.
   (b) Make a new column that contains the squared difference of neighboring data points, e.g. `(A2-A1)`^{2}. Use the `COUNT()` and `SUM()` functions to write your own functions that compute the Allan variance and Allan deviation. Compute the Allan variance and Allan deviation of collections of the first 20, first 100, and all of your 1000 data points. As you increase the number of data, does the Allan deviation get closer and closer to the standard deviation? (It should, and if it doesn't you should explore if you have made an error in code.)

Reminder: The Allan variance and Allan deviation are given by

$$\text{avar} = \sigma_y^2 = \frac{1}{2}\frac{1}{N-1}\sum_{i=1}^{N-1}(y_{i+1}-y_i)^2 \qquad \text{and} \qquad \text{adev} = \sqrt{\sigma_y^2}.$$