Homework 4

Due Tuesday, February 12th in class

Read Chapter 4 of Jespersen and Fitz-Randolph and Chapter 3 of Rovelli.

Below are three questions to think about and answer. The first two are not realizable as experiments; we can't get trains and meter sticks to move at speeds near the speed of light. However, these effects do play an important role for the small particles out of which everything around us is made. The third problem explores this case.

1. A clock on a train runs at a rate that is 3/4 the rate of a clock in Penn station. How fast must the train be moving? [For both this problem and the next it is simplest to write your answer as a fraction of the speed of light.]

2. How fast would a meter stick have to be moving through a laboratory for its length to be observed to shrink to 0.5 m in the laboratory?

3. Cosmic ray showers are created in the Earth's atmosphere when protons in outer space crash into our atmosphere and interact with atmospheric protons to create showers of particles. There are many kinds of particles in nature and physicists have named many of them using the greek alphabet, for example there are pions (after the greek letter pi or π) and muons (after the greek letter mu or μ) and neutrinos, an Italianism for "little neutral ones", and denoted by the greek letter nu or ν.

   You don't need many of the details of these particles for this problem, but a muon is much like the electron that you are more familiar with except that it is about 200 times heavier. The one thing that you will need to know is that heavy particles often decay into lighter particles and that when this happens the heavy particle has a characteristic lifetime, a period of time you have to wait before the decay happens. The charge of a particle is often shown by decorating its letter with a + sign for positive charge and a − for negative charge. [In detail then, the cosmic ray sequence is proton (from outer space) hits proton (in atmosphere) $\rightarrow p + p +$ pions. The pions then decay into muons: $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$; $\pi^+ \rightarrow \mu^+ + \nu_\mu$, and the muons decay into electrons and positrons: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$; $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$] After that setup, here are some questions:

   Muons are produced high in the atmosphere (at 8000 m, say) and travel toward the earth at very nearly the speed of light ($0.998c$, say).
   (a) Given the lifetime of the muon (2.2×10^{-6} sec), how far would it go before disintegrating, according to pre-relativistic physics? Would the muons make it to ground level?
   (b) Now answer the same question using relativistic physics. (Because of time dilation, the muons last longer, so they travel farther.)
   (c) Now analyze the same process from the perspective of the muon. (In its reference frame it only lasts 2.2×10^{-6} sec; how, then, does it make it to ground?)
   (d) Pions are also produced in the upper atmosphere. But the lifetime of the pion is much shorter, a hundredth that of the muon. Should the pions reach ground level? (Assume that the pions also have a speed of $0.998c$.)