Laboratory 8: The Changing Frequency of a Pendulum

In class and lab, we have discussed the pendulum as the prototypical oscillator for a clock. In particular, in the early labs you established that

- The *restoring* term for the pendulum is **g**, the strength of gravity,
- The *inertial* term for the pendulum is **L**, its length,
- The timing of the oscillation should be nearly independent of its amplitude,
- The frequency and period should scale as:

$$f_0 \propto \sqrt{\frac{\text{Restoring}}{\text{Inertia}}} = \frac{1}{2\pi}\sqrt{\frac{gravity}{Length}} \quad \& \quad T_0 \propto \sqrt{\frac{\text{Inertia}}{\text{Restoring}}} = 2\pi\sqrt{\frac{Length}{gravity}} \; .$$

We will take a closer look at the latter two results experimentally today.

Experimental advice: carefully check the alignment of your pendulum and your photogate. We want to be sure not to hit the photogate with the mass.

A. Set up your Pendulum

Set up your pendulum, checking that the hanging mass does not hit the photogate, but that the flag does cross the photogate beam as it passes the photogate. Measure and record the length of your pendulum. Note: *measure length to the center of the mass*.

B. Set up your photogate and the timer

Inspect the box connected to the photogate. Set up the box in the "Timer" mode and select the "Pendulum" Feature. Explore this feature by moving the mass across the laser beam. Notice that the box record the time between when the mass has crossed once and the *second crossing* after that (it skips one crossing). Think about why it is programmed to do that, and include your explanation in your lab report.

C. Measuring multiple pendulum periods

Carefully release the pendulum from a moderate height so that it is not twisting and swings freely through the photogate. Recording the data requires some coordination individually and with your lab partner(s).

One person will want to control the timer box, and read out the period measurement as soon as it is complete, then press the start button again before the pendulum swings through the photogate on its next passage.

This will allow nearly identical repeated measurements of the period. Try to get at least 40 pendulum period measurements (more is fine). This will require some practice and you will only want to keep the data from one of your `good' runs.

A lab partner will want to record the reported periods efficiently as they will be coming in rapidly. The partner controlling the timer may only want to read off the digits that have changed, as this can be easier to record. For some it will be simplest to enter these data directly into Excel (it depends on how fast you type).

D. Analyzing the Data, Building an Explanatory Model

Enter your period data in Excel. Remembering that $f = 1/T$, use the formula function in Excel to compute the frequency for each of your period measurements. Recall that to compute the fractional frequency difference

$$y = \frac{f - f_0}{f_0} = \frac{\Delta f}{f_0}, \qquad (1)$$

you need to have an expectation for, or measurement of, the ideal frequency of the oscillator, f_0. Interesting subtleties enter into the choice of what to use for f_0. We will use two different choices for f_0 to analyze our data:

1. Theoretical model of a pendulum

In the first approach you will use your previous experiments and a mechanical model of the pendulum to fix f_0. In the Labs 1 & 2 you established that the squared period of the pendulum was proportional to its length. A careful analysis, neglecting friction and assuming small amplitudes for the pendulum, shows that

$$T_0 \propto \sqrt{\frac{\text{Inertia}}{\text{Restoring}}} = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad f_0 \propto \sqrt{\frac{\text{Restoring}}{\text{Inertia}}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}, \qquad (2)$$

And we will explain why the factors of 2π and the strength of gravity, $g=9.81\ \text{m/s}^2$, come into the final formula. Use this formula and your pendulum length to compute f_0. Then use this as your ideal frequency in computing the fractional frequency difference for each of your data.

Once you have your fractional frequency differences compute the standard deviation for six different sets of 5 of your data. Repeat this process for 4 sets of 10 data points each and for 2 sets of 20 data points each. Is the standard deviation a good way to measure the spread of your data? For example, is the s.d. converging to a clearer value as you include more data? Make a scatter plot of your fractional frequency differences. Use this plot to help in explaining why the standard deviation is not the best measure of spread for these data.

2. Empirical trend of the data

There is a second, and often more useful, way to compute the fractional frequency differences. If instead of thinking of f_0 as a prediction fixed from the outset, we could think of using the measurements themselves to predict what the frequency of the pendulum on its next swing will be. For example, if in the first swing we measured a frequency f_1 we might predict that the second swing will have the same frequency, and so compute the fractional frequency difference as

$$y_1 = \frac{f_2 - f_1}{f_1}.$$

Continuing in this way you would compute

$$y_2 = \frac{f_3 - f_2}{f_2} \quad \text{and} \quad y_3 = \frac{f_3 - f_2}{f_2}, \text{ etc.}$$

Use this second method to compute the fractional frequency differences of your data. Plot the result. How does this plot of fractional frequency difference differ from the first one you made? Explain why the two plots behave as they do.