Laboratory 9: Pendulum Stability

In class and lab, we have discussed the pendulum as the prototypical oscillator for a clock. In particular, in the early labs you established that

- The *restoring* term for the pendulum is **g**, the strength of gravity,
- The *inertial* term for the pendulum is **L**, its length,
- The timing of the oscillation should be nearly independent of its amplitude,
- The frequency and period should scale as:

$$f_0 \propto \sqrt{\frac{\text{Restoring}}{\text{Inertia}}} = \frac{1}{2\pi}\sqrt{\frac{gravity}{Length}} \quad \& \quad T_0 \propto \sqrt{\frac{\text{Inertia}}{\text{Restoring}}} = 2\pi\sqrt{\frac{Length}{gravity}} \; .$$

Today we will begin to evaluate the stability of a pendulum clock.

A. Set up your Pendulum

Set up your pendulum, checking that the hanging mass does not hit the photogate, but that the flag does cross the photogate beam as it passes the photogate. Measure and record the length of your pendulum. Note: *measure length to the center of the mass*.

B. Set up your photogate and the timer

Inspect the box connected to the photogate. Set up the box in the "Timer" mode and select the "Pendulum" Feature. Explore this feature by moving the mass across the laser beam. Notice that the box records the time between when the mass has crossed once and the *second crossing* after that (it skips one crossing). Think about why it is programmed to do that, and include your explanation in your lab report.

C. Measuring multiple pendulum periods

Carefully release the pendulum from a moderate height so that it is not twisting and swings freely through the photogate. Recording the data requires some coordination individually and with your lab partner(s).

One person will want to control the timer box, and read out the period measurement as soon as it is complete, then press the start button again before the pendulum swings through the photogate on its next passage.

This will allow nearly identical repeated measurements of the period. Now that you are more experienced with this setup, try to get **100** pendulum period measurements (more is fine). This will require some practice and you will only want to keep the data from one of your `good' runs.

A lab partner will want to record the reported periods efficiently as they will be coming in rapidly. The partner controlling the timer may only want to read off the digits that have changed, as this can be easier to record. For some it will be simplest to enter these data directly into Excel (it depends on how fast you type).

Analysis advice: Label each of the columns that you use in Excel with a clear title and with the units of the quantity contained in that column.

D. Analyzing the Data, Predicting the Uncertainty in a Time Measurement

Enter your period data in Excel. Remembering that f = 1/T, use the formula function in Excel to compute the frequency for each of your period measurements. Recall that to compute the fractional frequency difference

$$y = \frac{f - f_0}{f_0} = \frac{\Delta f}{f_0}\ , \tag{1}$$

you need to have an expectation for, or measurement of, the ideal frequency of the oscillator, f_0. Interesting subtleties enter into the choice of what to use for f_0. This week we will stick with our theoretical model, since it is the nominal frequency you would expect before making any measurements of your pendulum's oscillations.

1. Theoretical model of a pendulum

You will use your previous experiments and a mechanical model of the pendulum to fix f_0. In the Labs 1 & 2 you established that the squared period of the pendulum was proportional to its length. A careful analysis, neglecting friction and assuming small amplitudes for the pendulum, shows that

$$T_0 \propto \sqrt{\frac{\text{Inertia}}{\text{Restoring}}} = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad f_0 \propto \sqrt{\frac{\text{Restoring}}{\text{Inertia}}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}\ , \tag{2}$$

Recall that the strength of gravity is $g = 9.81\ \text{m/s}^2$. Use this formula and your pendulum length to compute f_0. Then use this as your ideal frequency in computing the fractional frequency difference for each of your data.

2. Allan deviation for three averaging times

(i) Compute the fractional frequency differences for your data. Use these values of y to compute the Allan deviation for your clock averaged over a single period.

(ii) Return to your initial frequency data. Compute the average of these data over the first ten data, e.g. AVERAGE(B2:B11), then over the next ten data, e.g. AVERAGE(B3:B12), and so on until you have a set of measurements averaged over 10 periods. Compute the fractional frequency differences for these data. Use these new values of y to compute the Allan deviation for your clock averaged over ten periods.

(iii) Return to your initial frequency data. Compute the average of these data over the first 50 data, e.g. AVERAGE(B2:B51), then over the next 50 data, e.g. AVERAGE(B3:B52), and so on until you have a set of measurements averaged over 50 periods. Compute the fractional frequency differences for these data. Use these new values of y to compute the Allan deviation for your clock averaged over 50 periods.

You should now have three Allan deviations computed for three different averaging times: 1 period, 10 periods, and 50 periods. Make two new columns in Excel or Sheets.

In the first put the number of seconds that corresponds to 1 period, 10 periods, 50 periods. In the second put your three Allan deviations. Make a scatter plot of Allan deviation vs. time.

3. Timing 100 boings and predicting the quality of your measurement

Use your pendulum to time 100 boings of the spring in the back of the room. Recall that a boing is the time it takes the spring to go up and come back down (a full period of the spring). It is unlikely that 100 boings will be an integer number of periods of your pendulum. Do your best to estimate how much of the last pendulum swing should be included in your timing.

Use f_0 from above to find T_0 and combine this with the number of swings of your pendulum to find the total time T for 100 boings. Use your Allan deviation measurements to estimate your uncertainty in this time.

We will collect the times for 100 boings of all of the groups on the board at the front of the room. Write down these times.

Lab Write-up

Explain in your own words why we cannot say anything about noise in our system for times shorter than the period of your pendulum.

Explain what you did, what conclusions you can draw from it. In particular, explain what the plot of your pendulum's Allan deviation vs. averaging time tells you about using this pendulum as a clock. For what lengths of time will your pendulum give the most accurate measurement?

Find the standard deviation of the 6 times for 100 boings that all the groups found. Does this standard deviation come close to your own prediction for the uncertainty of your time measurement using the Allan deviation? If not, which is bigger and can you explain why?