Time Examined Class Notes – March 26, 2019 (md)

1. If you are like me, over Spring Break you found yourself thinking about all we have studied about *time*. We have learned how the work of Einstein and others have shown us that many of our "common sense" notions about time have to be reconsidered.

These reconsiderations are based on considering extreme situations – moving very fast, large gravitational effects, sizes that go down to the quantum level. But don't let that obscure the fact that we are talking about the same *time* that we use every day.

2. In particular, we are considering *time* the way scientists (physicists in particular) use it. In our theories and measurements, we tend to describe everything in terms of a time variable, t. When we consider how something changes, either qualitatively or (especially) quantitatively, we mark those changes as functions of t:

- Motion in terms of position, velocity, and acceleration, $x(t)$, $v(t)$, and $a(t)$.
- Their relations in terms of changes in *time*, $v(t) = \Delta x / \Delta t$.
- The periodic motion of pendula, masses on springs.
- Vibrations of air pressure with a frequency that produces the pitch of a sound.
- Changes in electric circuits that give oscillations we measured.

Time is our basic descriptor for change, which is why Rovelli's Chapter 8 makes such a big deal about the Wheeler-De Witt equation describing quantum gravity without using a time variable of any kind.

3. I want to concentrate on two phenomena that change in time, hoping to highlight similarities between them. The first is a case of damped motion.

Imagine a block is sliding along a table, being slowed by friction. The faster the block moves, the more friction force there is. This is because friction slows the block due to the interaction between the molecules in the block and the molecules in the table – the faster the motion, the more interactions there are in any instant of time. From careful measurements, we know that the precise form of this velocity dependence can be different for different types of motion. We are going to use one of the simpler forms, namely that the force is directly proportional to the velocity, $f = -b\,v$. The constant b indicates the strength of the interaction between the block and the table.

4. We now apply Newton's Laws to characterize the velocity's behavior in time.

$$F = ma \;\Rightarrow\; m\frac{\Delta v}{\Delta t} = -bv \;\Rightarrow\; \frac{\Delta v}{v} = -\frac{b}{m}\cdot\Delta t \;\Rightarrow\; \frac{\Delta v}{v} = -\frac{\Delta t}{\tau}. \qquad (1)$$

Let me describe this in words: It says that the *fractional* or *percentage change* in velocity is proportional to time.

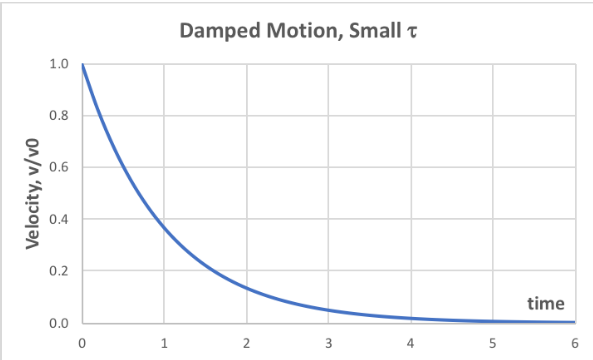
5. I call the proportionality constant the *characteristic time constant*, $\tau = m/b$.

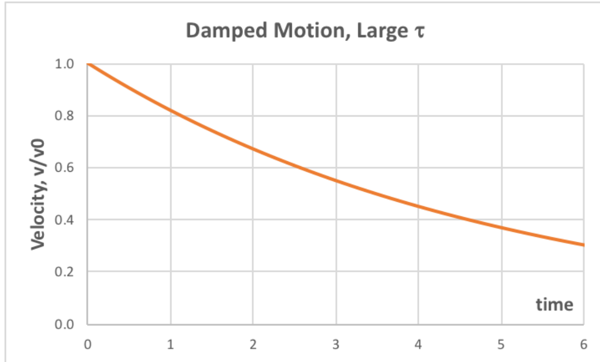
τ describes how quickly the system evolves or changes. Any time-dependent description of the system will involve some multiple of τ. We will develop a better sense of this as we go, but here is a start on the physical sense of it.

- Consideration of b and m shows that the units of $\tau = m/b$ are indeed time.
- If b is large, τ is small. The high friction slows the block down more quickly.
- If m is large, τ is large. The block's large inertia makes it harder to slow down.

6. Let’s get a sense of the qualitative behavior determined by this motion equation.
   - The minus sign means that the velocity is always decreasing (negative slope).
   - The proportionality means that the larger the speed, the steeper the slope.
   - The larger the time constant τ, the less steep the slopes.

Graphs of “exponential decays” following these characteristics are shown below.





7. You will be happy to know that we physicists actually have solved this problem mathematically. It involves the *natural logarithm* function, because (as I showed),

$$\Delta\left[\ln(v(t))\right] = \frac{\Delta v}{v(t)} \ . \qquad (2)$$

Combining this with Eqn. (1) gives

$$\Delta\left[\ln(v(t))\right] = -\frac{\Delta t}{\tau} \quad \text{or} \quad \frac{\Delta\left[\ln(v)\right]}{\Delta t} = -\frac{1}{\tau} \ . \qquad (3)$$

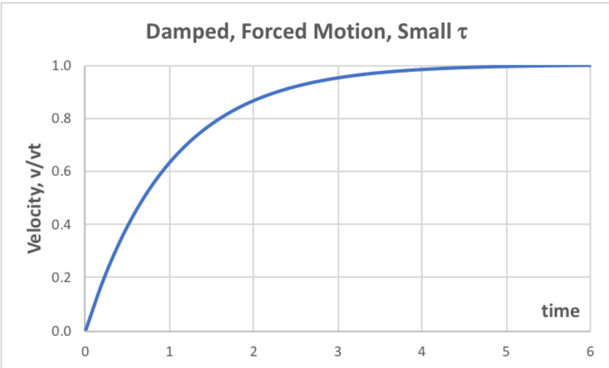
That means that a plot of ln (v) *vs.* t will be a straight line whose slope is $-1/\tau$, which makes the analysis much easier. To find τ, we just have to
   - Measure a series of velocities at different times,
   - Calculate the logs and plot ln [v] *vs.* t,
   - Find the best fit line for this graph and its slope,
   - Take the negative inverse of that slope, yielding τ.

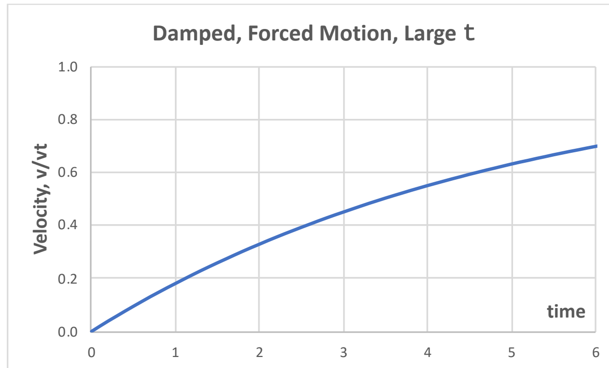
8. What happens if this block is instead subjected to a constant force F_0 pushing it?

$$F = ma \ \Rightarrow \ m\frac{\Delta v}{\Delta t} = F_0 - bv \ . \qquad (4)$$

   - If the friction force is larger than applied force, the body slows down,
   - If the friction force is smaller than the applied force, the body speeds up.
   - If the friction force is equal to the applied force, the body’s speed is constant.
   - The time dependence of the system is still governed by the time constant τ.

Considering the case of a slowly moving block (small friction force), as the force continues working, the body speeds up, but by less and less over time. Eventually, it is moving at a constant “terminal” speed, $v_t = F_0/b$. This is graphed below.





9. I now perform one of my favorite conceptual maneuvers:

   set up a system that has the same mathematical form as one we have analyzed, and use the similarities to help us understand the new situation.

We build an electric circuit with two elements, a capacitor C which stores charge q(t), and a resistor R which impedes the motion of the charge through the system $I = \Delta q/\Delta t$. If the capacitor is initially charged up to a voltage V_0, and then connected to discharge through the resistor, the circuit equation is

$$R\,\frac{\Delta q(t)}{\Delta t} = -\frac{1}{C}\,q(t) \tag{4}$$

A comparison to Eqn. (1) shows the same form, just with different variable names:

$$v \rightarrow q\,,\quad m \rightarrow R\,,\quad b \rightarrow 1/C\,. \tag{5}$$

The mathematical solutions for q(t) are then the same as what we had found before for v(t), with the only change being that the time constant is now $\tau = RC$.

10. So, τ = RC dictates the changes over time in the system.

- Consideration of *R* and *C* shows that the units of $\tau = RC$ are indeed time.
- If *C* is large, τ is large. The large capacitance slows the impetus to discharge.
- If *R* is large, τ is large. The large resistance gives smaller current, increasing the time needed to discharge.
- A graph of ln[q(t)] *vs.* time will give a line with slope $-1/\tau = -1/RC$.

Since the voltage across the capacitor is $V_C = q/C$, a graph of ln[$V_C(t)$] *vs.* t will give a line with slope $-1/RC$. That is what we will do in lab.

- Measure a series of capacitor voltages at different times,
- Calculate the logs and plot ln [V_C] *vs.* t,
- Find the best fit line for this graph and its slope,
- Take the negative inverse of that slope, yielding τ.
- If we know the resistance, we can find $C = \tau/R$.

This is what we will do in lab tomorrow.