Time Examined

Day 16

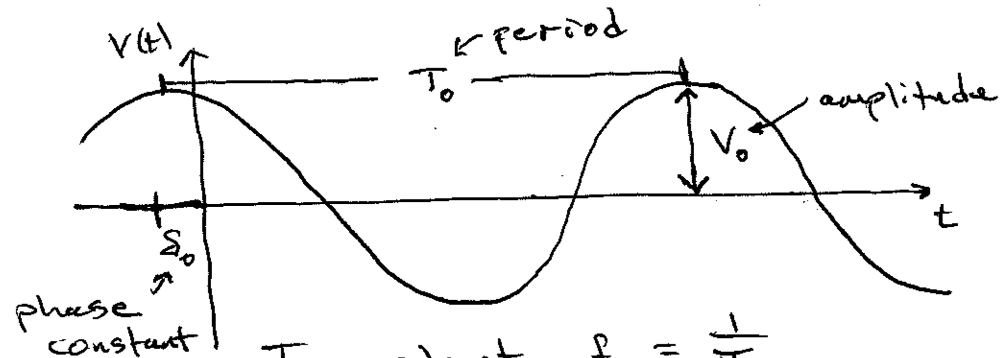
Today

I Lab Analysis

II Recollection of Day 14 (before Spring break)

I Reviewed how to do the lab analysis.

II We studied how to describe an oscillator mathematically



Important $f_0 = \frac{1}{T_0}$

Ideal oscillator:

$$V(t) = V_0 \cos(\underbrace{2\pi f_0 t + \delta_0}_{\Phi(t) \text{ the total phase}})$$

A physical oscillator has noise. Remarkably all the important sources of noise can be summarized in a single correction

$$V(t) = V_0 \cos(2\pi f_0 t + \delta_0 + \phi(t))$$

$\phi(t)$: the time variation of the phase

A clock counts the oscillations (say the peaks) of the oscillator to measure how much time has passed. Clearly $\phi(t)$ can mess this up, if it changes the duration between the peaks. One way to capture this is to compare the physical clock's total phase to the ideal clock's phase, that is

Consider

$$\frac{\Phi(t)}{2\pi f_0} = t + \frac{\delta_0}{2\pi f_0} + \frac{\phi(t)}{2\pi f_0}$$

The $\delta_0/2\pi f_0$ is a constant, we can always get rid of it by starting our clock at the right time. But,

$$x(t) = \frac{\phi(t)}{2\pi f_0} \quad \text{"relative phase"}$$

tells us how much our physical clock's time differs from a perfect clock of frequency f_0.

Before Spring break we also saw how to extract the frequency of a clock:

$$\frac{\Delta\Phi}{\Delta t} = \frac{\Phi(t_2)-\Phi(t_1)}{t_2-t_1}$$

$$= \frac{(2\pi f_0 t_2 + \delta_0 + \phi(t_2)) - (2\pi f_0 t_1 + \delta_0 + \phi(t_1))}{t_2-t_1}$$

$$= \frac{2\pi f_0(t_2-t_1) + \phi(t_2) - \phi(t_1)}{t_2-t_1}$$

Dividing both sides by 2π gives,

$$f = \frac{1}{2\pi}\frac{\Delta\Phi}{\Delta t} = f_0 + \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t}$$

Then,

$$\Delta f = f - f_0 = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t}$$

So, we see that, the difference $\Delta\phi$ also characterizes how our physical oscillator's frequency differs from the ideal oscillator's frequency f_0.

In practice there are several reasons that it is even better to work with

$$y = \frac{\Delta f}{f_0} \quad \text{"the fractional frequency difference"}$$

$$= 1 - \frac{f}{f_0} \quad \text{A pure \#}$$

$$= \frac{1}{2\pi f_0}\frac{\Delta\phi}{\Delta t} = \frac{\left(\frac{\Delta\phi}{2\pi f_0}\right)}{\Delta t} = \frac{\Delta x}{\Delta t}$$

The last expression,

$$y = \frac{\Delta x}{\Delta t}$$

will give us a very practical way to measure y in the lab. More on that soon.

Why do we focus on y? First and foremost it is because y is what we can measure. We take

The best oscillator we own and treat it as ideal and having exact frequency f_0. Then we measure the number of oscillations of our non-ideal oscillator in an oscillation of our ideal oscillator. This is precisely

$$T_0 f = \frac{f}{f_0}!$$

There is another reason too: why not just consider $\Delta f = f - f_0$? Suppose we had a pair of oscillators with

$$f = 11\ Hz \quad \text{and} \quad f_0 = 10\ Hz$$

and a second pair with

$$f = 101\ Hz \quad \text{and} \quad f_0 = 100\ Hz.$$

Both pairs have $\Delta f = 1\ Hz$, but if my clock's counter counts one extra cycle of the 1st case, it is off by about 1/10 of a second, while in the second clock it is only off by about 1/100 of a second. This difference is perfectly captured by y:

case 1: $y = \Delta f/f_0 = 1/10 = 0.1$

case 2: $y = \Delta f/f_0 = 1/100 = 0.01$.