1. In lab last week, we looked at reasonable candidates for a regularly repeating motion. It was suggested that the pendulum might be more regular than the pulse in part because it is simpler – fewer possible unknown variables.

To really understand a system, we need to imbed it in a more extensive explanation of how we think things behave. That is, we need a theory of how the oscillation happens. We need to first start with a more general theory of motion.

2. Historically, the dominant theory of motion in Europe was that of Aristotle.
Its fundamental ideas were that,

- Objects have natural motions (earth & water go *down*, air & fire go *up*),
- To maintain any other motion, you need to exert a force on it.

In a lot of ways, this matches our everyday experience. You push on something, it moves; you stop pushing, it comes to a stop.

3. In the 1600s and 1700s, Galileo & Newton developed a contrasting theory, with

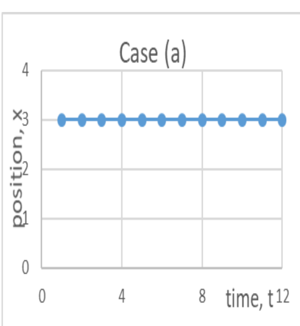
- (Quantification) We need to compare numerical measurements of the motion as over time. Specifically, we talk about *position, velocity,* and *acceleration.*
- (Inertia Law) An object maintains a constant *velocity* unless a force acts on it,
- (Force Law) A force causes an object to undergo a constant *acceleration*, according to F = m a, with mass m indicating how hard the object is to move.

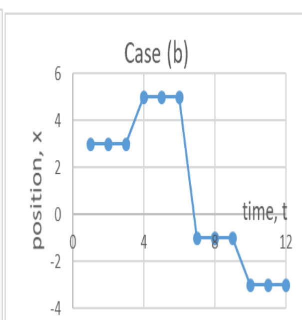
To make sense of this, we need to develop our *descriptions* of motion.
I will have you sketch graphs of various motions that I describe.

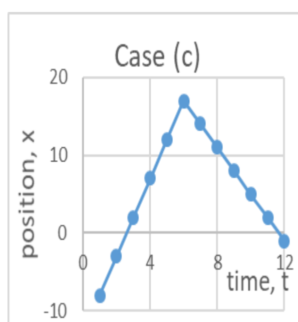
4. By a graph, I mean to draw a curve that indicates the object's position (x) along the vertical axis, plotting it out over time (t), indicated along the horizontal axis.

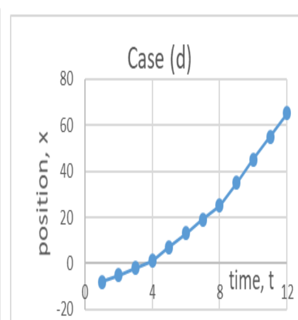
To give numerical values for x, we chose a coordinate system at the front of the room, with 0 in the middle (the origin), positive x to the right, and negative x to the left.

Cases:
a) Me standing still.
b) Me standing at different points at successive times.
c) Me moving in the positive, and then the negative direction.
d) Me moving in the positive direction, first slowly, then more quickly.









5. In describing the motion, we pay attention to both the value of something (*e.g.*, x is positive||zero||negative) *and* how values change (increasing||constant||decreasing). For the graph, we focus both on the value of the plotted point (position) and the slope of the curve or line at that point. Slope indicates *velocity.*

6. This emphasis on change is notated by the Greek letter Delta: Δ .
Δ signifies the change from an older to a newer value, so

$\Delta x = x_{final} - x_{initial}$ $\Delta t = t_{now} - t_{before}$ $\Delta v = v_{later} - v_{now}$.

The individual values are measured at particular times, each is at a certain *when*: the difference between two times is the *time interval* idea we have been using.

7. The behaviors of motion that we want to keep track of are:

Position (x): where an object is. + / 0 / – is the vertical place on the graph.

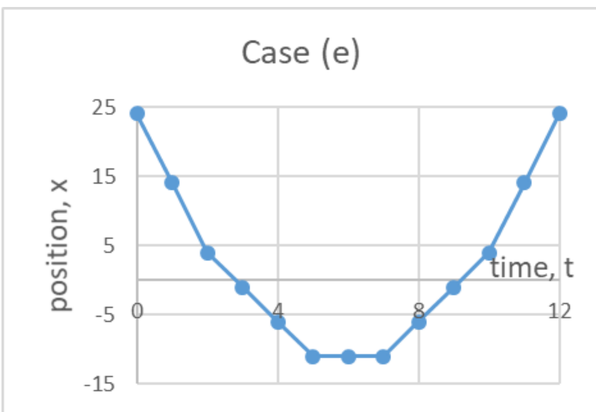
Velocity (v): how an object moves, $v = \Delta x / \Delta t$. + / 0 / – is the slope of the curve.

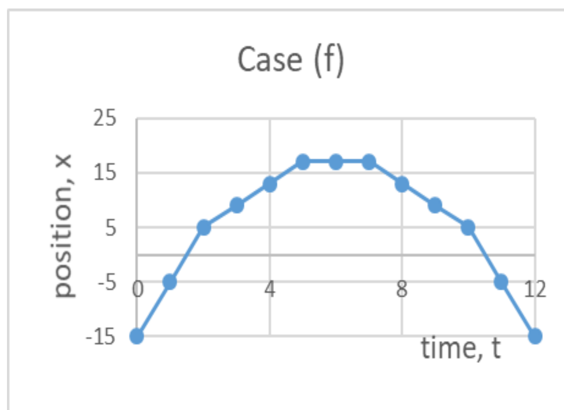
Acceleration (a): speeding or slowing, $a = \Delta v / \Delta t$. + / 0 / – is the change in slope.

We considered these two cases of changing velocities,

e) – – to – to 0 to + to ++ . $a = \Delta v / \Delta t$ is positive in each case.

f) ++ to + to 0 to – to – – . $a = \Delta v / \Delta t$ is negative in each case.





For obvious reasons, I call these *smiling curves* and *frowning curves*.

8. The motion variables then break up into numerical cases, depending on their signs:

Verbal descriptions of my motion along the front board

<table>
<tr><th>Motion Variable</th><th>+</th><th>0</th><th>–</th></tr>
<tr><td>x: position</td><td>On the right</td><td>In the middle</td><td>On the left</td></tr>
<tr><td>v: velocity</td><td>Moving to the right</td><td>Not moving</td><td>Moving to the left</td></tr>
<tr><td rowspan="2">a: acceleration</td><td>Moving left, and slowing down</td><td rowspan="2">Constant velocity</td><td>Moving right and slowing down</td></tr>
<tr><td>Moving right, and speeding up</td><td>Moving left and speeding up</td></tr>
</table>

Indicators on the v *vs.* t graph

Motion Variable	**+**	**0**	**–**
x: position	Above the t-axis	On the t-axis	Below the t-axis
v: velocity	Upward slope	Flat, no slope	Downward slope
a: acceleration	Smiling curve	Constant slope	Frowning curve

9. We apply these ideas of motion to a mass on a spring as it oscillates around its equilibrium position. In this case, x is taken as

   - positive if the mass is above the equilibrium position,
   - 0 if it is at the equilibrium position,
   - negative if it is below the equilibrium position.

   The net force on the mass is the combination of gravity (down) and spring (up).

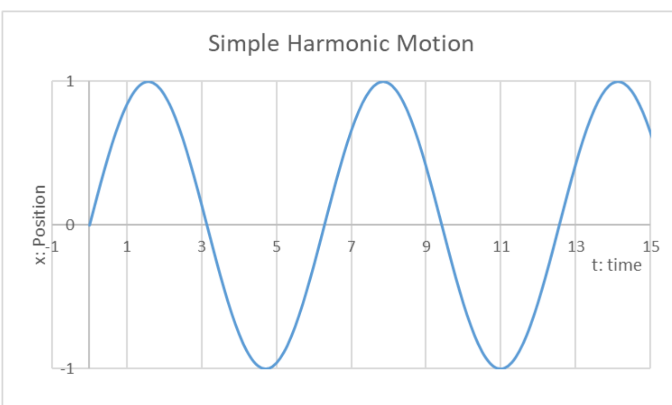
   - Above the equilibrium, gravity > spring, and the net force is downward,
   - At equilibrium, the net force is 0,
   - Below the equilibrium, gravity < spring, and the net force is upward.

   This case when F always opposes x, we call a *restoring force*. In this case, the equation is: $F = -k x$, with k as a measure of the spring strength or tension.

10. Since $F = ma$ *and* $F = -k x$, we get $a = -(k/m) x$.

    In words, if x is positive, a is negative (frowning curve), and
    if x is negative, a is positive (smiling curve).

    Graphing this relationship shows us the oscillation motion, matching what we see the mass on the spring go through when we release it.



11. A more fully developed treatment of this problem within Newtonian physics (which we will not take you through) convinces us that we know why this system oscillates as regularly as it does. I will argue, without full details,

- Stronger springs (larger k) have more restoring force, and higher frequencies,
- More inertia (larger m) is harder to move, and lower frequencies,
- The amplitude of the motion does not affect the frequency,
- The numerical dependence is: $\text{Period}^2 = \text{Inertia} / \text{Restoring}$.