Time Examined
Day 4

Today

I Last time

II More consequences of Einstein's two postulates:

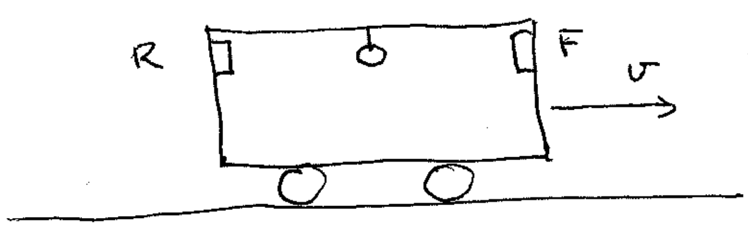
[(1) Relativity of simultaneity]

(2) Time dilation

(3) Lorentz contraction

These two postulates had a remarkable consequence:

(1) the relativity of simultaneity.



(A) Observer on train: R & F fire simultaneously

(B) observer on ground: R before F

I We discussed two postulates:

- Principle of relativity:

No (physical) phenomena have properties corresponding to the concept of absolute rest.

- The speed of light (3×10^8 m/s) is the same regardless!

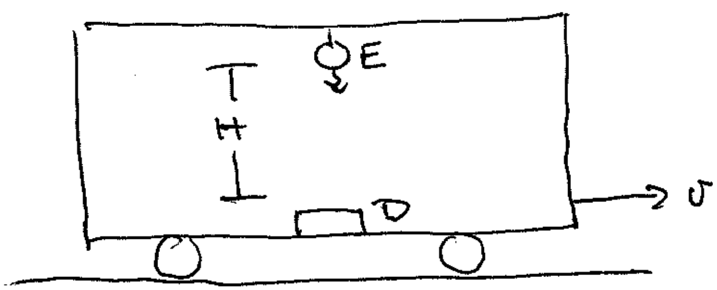
Two events simultaneous to one (uniformly moving) observer, may not be to another!

II (2) Time dilation:

Another train experiment.

In lab we have begun the process of understanding what a clock is: something that oscillates, an energy source that keeps it oscillating, and something that counts those oscillations.

Consider the following light clock



Last time we saw that events simultaneous in one frame may not be in another. Let's quantify this.

So, how long between emission (E) and detection (D)?

(A) According to an observer on the train: recall

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

or $$\Delta t' = \frac{\text{distance}}{\text{speed}},$$

Then

$$\Delta t' = \frac{H}{c}$$

Call the ground frame S and the train frame S' — we'll use a prime ' to refer to all quantities in the train frame.

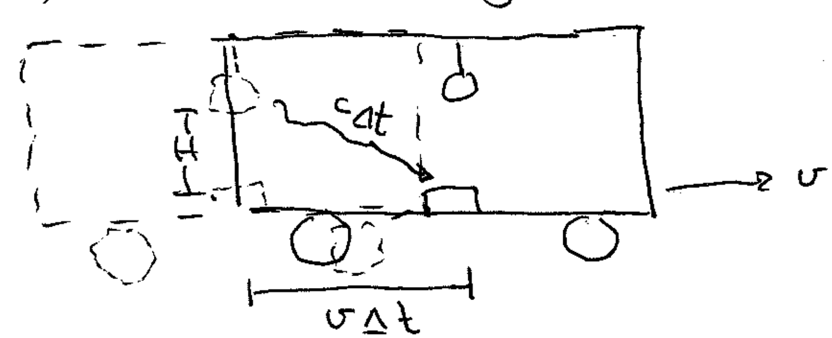
To quantify the differences between frames we want to relate

$$\Delta t = t_D - t_E$$ time of detection & time of emission both in ground frame

and

$$\Delta t' = t_D' - t_E'$$ Both in train frame

(B) Observer on ground:



Call it Δt, then the distance the train travels $v\Delta t$, the height of the bulb H, and the distance the light travels make a triangle. According to the Pythagorean formula

$$c^2 \Delta t^2 = v^2 \Delta t^2 + H^2.$$

Let's solve for Δt in steps, subtract $v^2 \Delta t^2$ from each side,

$$c^2 \Delta t^2 - v^2 \Delta t^2 = H^2$$

and

$$(c^2 - v^2)\Delta t^2 = H^2$$

Dividing through by $(c^2 - v^2)$

$$\Delta t^2 = \frac{H^2}{(c^2 - v^2)}$$

Dividing top and bottom of right hand side by c^2 gives

$$\Delta t^2 = \frac{H^2/c^2}{(1 - \frac{v^2}{c^2})}$$

Finally, taking a square root

$$\Delta t = \frac{H/c}{\sqrt{1 - v^2/c^2}}.$$

H/c ← This is $\Delta t'$

Introducing $\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$ we have

$$\boxed{\Delta t = \gamma \Delta t'.}$$

A few example γ's

v	$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$	$\Delta t = \gamma \Delta t'$
0	$\frac{1}{\sqrt{1-0}} = \frac{1}{\sqrt{1}} = 1$	$\Delta t = \Delta t'$ makes sense train not moving
$\frac{1}{4}c$ (1.7×10^8 mph)	1.033	$\Delta t = 1.033 \Delta t'$
$\frac{1}{2}c$	1.155	$\Delta t = 1.155 \Delta t'$
$\frac{3}{5}c$	$\frac{1}{\sqrt{1 - \frac{9}{25}\frac{c^2}{c^2}}} = \frac{1}{\sqrt{\frac{16}{25}}} = \frac{5}{4}$	$\Delta t = 1.25 \Delta t'$
0.866c	2	$\Delta t = 2 \Delta t'$
0.99c	7.09	$\Delta t = 7.09 \Delta t'$

Notice that because v is always less than c, $\gamma \geq 1$ with equality for $v = 0$ only. Looking at $\Delta t = \gamma \Delta t'$, this means that the number of ticks of a moving clock is always smaller than the number of ticks of a stationary clock (you have to multiply by $\gamma > 1$ to get to the number of ticks $\Delta t'$ up to the number of ticks Δt).

We call a clock that ticks off fewer tick then a standard, a "slow clock".

Hence, we conclude: "Moving clocks run slow." They run slow by a factor γ.

<u>Example</u>: The pilot of a futuristic rocket speeds away from Earth at $3/5\,c$. If 10 seconds pass on Earth how many seconds will the pilot's watch register having passed?

From our table $\gamma = 5/4$ and so

$$\boxed{\Delta t' = \frac{1}{\gamma}\Delta t = \frac{4}{5}(10\text{ secs}) = 8\text{ secs.}}$$

Then observers on the ground will measure it to have a smaller length L. These two lengths are related by

$$L = \frac{1}{\gamma} L'$$

where again $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

just as before. If the train moved at $0.866\,c$ and the train observers measured it to be a meter stick then the ground observers would say it was

$$L = \frac{1}{2}(1\text{ meter}) = 0.5\text{ meters}$$

half a meter long.

Next time we will show another result that you will need for this week's homework:

Say you have a stick measured to have length L' on a train,