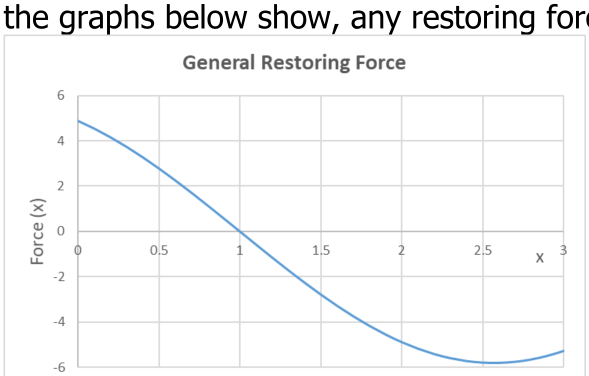
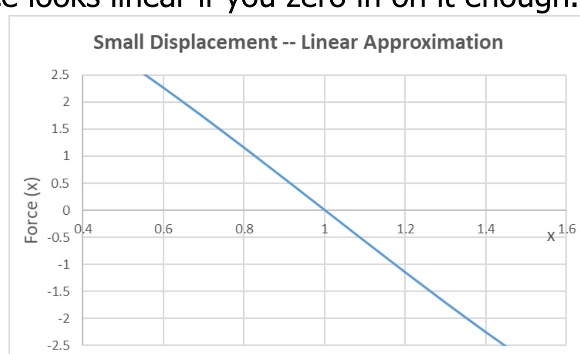
1. Last Tuesday, we systematically studied motion, and applied this to the motion of a mass on a spring. This resulted in what we called *Simple Harmonic Motion* of the mass around its equilibrium. To summarize some points,
   - The spring force and gravity combine to establish an *equilibrium position*, x = 0,
   - The spring exerts a *restoring force*, always pushing the mass back to x = 0,
   - For a spring, the force is linear, **F = – k x**,
   - So, **F = – k x** and **F = m a** combine to give **a = – (k/m) x**,
   - That means that when
     - x > 0 (above equilibrium), the acceleration is negative (frowning curve),
     - x < 0 (below equilibrium), the acceleration is positive (smiling curve),
   - This is then a graph of an oscillation, which matches the motion that we see.
2. This is true qualitatively for any restoring force, they all
   - have an equilibrium position at which F = 0, and
   - give oscillations around that equilibrium.

For instance, release a marble so that it can roll in a bowl, and it will oscillate around the equilibrium position at the bottom of the bowl.

3. But the linear restoring forces, when **F = – k x**, are special. For that case *only*,
   - the oscillation is actually a pure sine/cosine motion,
   - the frequency or period are independent of the amplitude of the motion.
4. It may seem like this is too special a case to be useful, but quite to the contrary, it applies to most oscillations, as long as the displacement from equilibrium is small. As the graphs below show, any restoring force looks linear if you zero in on it enough.





5. So, what *does* determine the timing? The key comes from considering the motion equation $\mathbf{a = -(k/m)\,x}$.

k, is a constant which determines the strength of the spring force, *i.e.*, how much force the spring exerts when it is stretched some distance, which we call the *restoring term.*

m, is the mass on the spring, which quantifies the *inertia*. As $\mathbf{F = m\,a}$ shows, an object with a lot of inertia requires a lot of force to be accelerated. *i.e.*, it takes a lot of force to get it moving, but it also takes a lot of force to slow it down or reverse motion.

6. I will try to make a non-rigorous, but I hope plausible argument that since

$$v = \frac{\Delta x}{\Delta t} \ \& \ a = \frac{\Delta v}{\Delta t} = \frac{\Delta(\Delta x / \Delta t)}{\Delta t} \quad \text{so } a \sim \frac{\text{Distance}}{\text{Time}^2},$$

then acceleration is related to square of the frequency. So, for oscillations,

$$f^2 \sim \frac{\text{Restoring}}{\text{Inertia}} \quad \text{and} \quad T^2 \sim \frac{\text{Inertia}}{\text{Restoring}}.$$

Fortunately, you don't have to just trust me, we will confirm this in lab tomorrow.

7. This timing relationship can be generalized to any oscillation. You just have to figure out what the restoring and inertial terms are in each case. Then,

- More/less restoring force will increase/decrease the frequency, and
- More/less inertia will decrease/increase the frequency.

System	Restoring Term	Inertial Term
Mass on a spring	spring strength, k	mass, m
Pendulum	force of gravity, g	string length, L
Marble in a bowl	force of gravity	curvature of the bowl
Guitar string	string tension	string length and density
Flute	air compressibility	length of the tube
Xylophone bar	stiffness of the metal	length of the bar
Water drop	surface tension	mass of the drop
Diatomic molecule	bond strength	masses of atoms
Star	gravity *vs.* radiation pressure	mass of the star
Moon orbiting Earth	gravity due to mass of Earth	radius of moon orbit
Planet orbiting Sun	gravity due to mass of Sun	radius of planet orbit

It may be a surprise to see the orbital motions on this list, but as I will suggest, SHM and circular are intimately related. And just like the pendulum, the mass of the moving object (the planet) does not factor into the timing, only the size of the motion (the radius of the orbit). Orbital periods of those "clocks" match orbital radii:

Mercury < Venus < Earth < Mars < Jupiter < Saturn < Uranus < Neptune .

8. The general relationship of period or frequency to restoring and inertial factors is perhaps easiest to demonstrate with a guitar string.

Pluck the guitar string, and the note you hear has a pitch which is determined by its frequency: high pitches are high frequencies and low pitches are low frequencies.

- If you tighten the string, increasing the restoring force makes the frequency (pitch) go up. Loosening the string makes the pitch go down.
- If you change the length of the string by fretting, using a longer string makes the frequency (pitch) go down. Using a shorter string makes the pitch go up.
- If you switch to a different string of the same length, choosing a heavier string makes the frequency go down. Using a lighter string makes the pitch go up.

9. If we add in the cases of the pendulum and the mass-on-a-spring that we did in lab, hopefully you will start to intuit the general relationship between timing and the two factors of *restoring* and *inertia*, even when we move on to less tangible cases.