Time Examined
Day 7

<u>Today</u>

I Last time

II Spacetime diagrams

I. We collected all of the consequences of Einstein's two postulates:

(1) Relativity of simultaneity

(2) Time dilation

$$\Delta t = \gamma \Delta t'$$

(3) Length contraction

$$L = \frac{1}{\gamma} L'$$

or

$$\Delta x = \frac{1}{\gamma} \Delta x'.$$

and finally

(4) Velocity addition

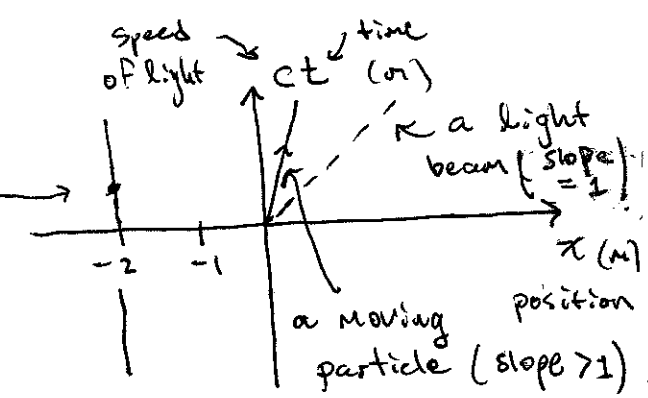
$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB} v_{BC}}{c^2}}.$$

We've shown that when you move your notions of time and length change — and the ways in which they change are related!

This is a hint that it might be useful to consider them together and to think of <u>spacetime</u>. This will be our goal today.

II We draw

a particle at rest at $x = -2$ m.



On these spacetime diagrams

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{c\,\Delta t}{\Delta x}$$

$$= \frac{c}{\left(\frac{\Delta x}{\Delta t}\right)} = \frac{c}{v}$$

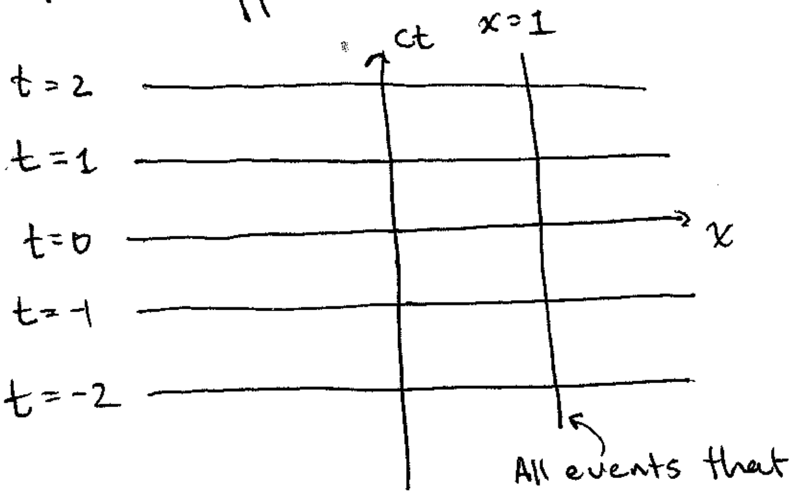
or

$$\frac{v}{c} = \frac{1}{\text{slope}}$$

or

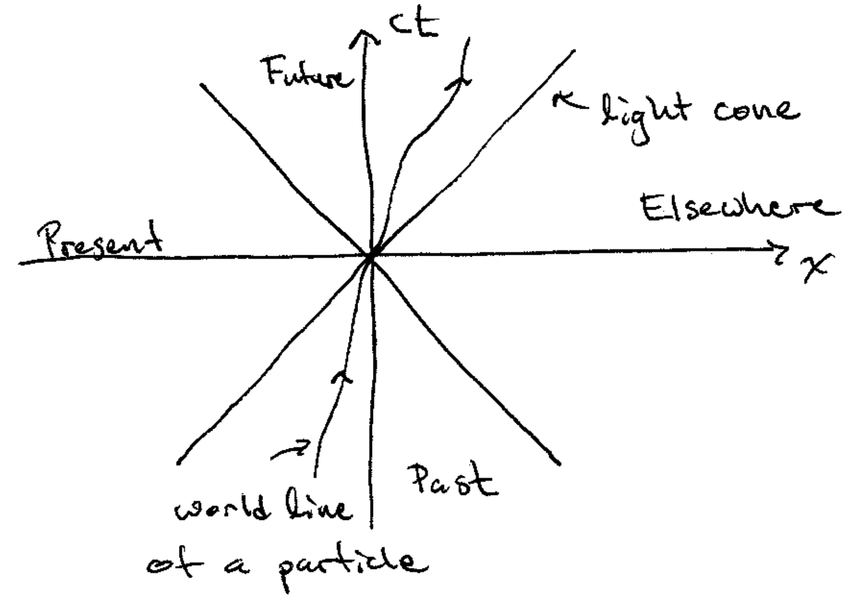
$$\boxed{v = \frac{1}{\text{slope}}\, c}$$

Horizontal lines, like the x-axis represent all events that happen at the same time

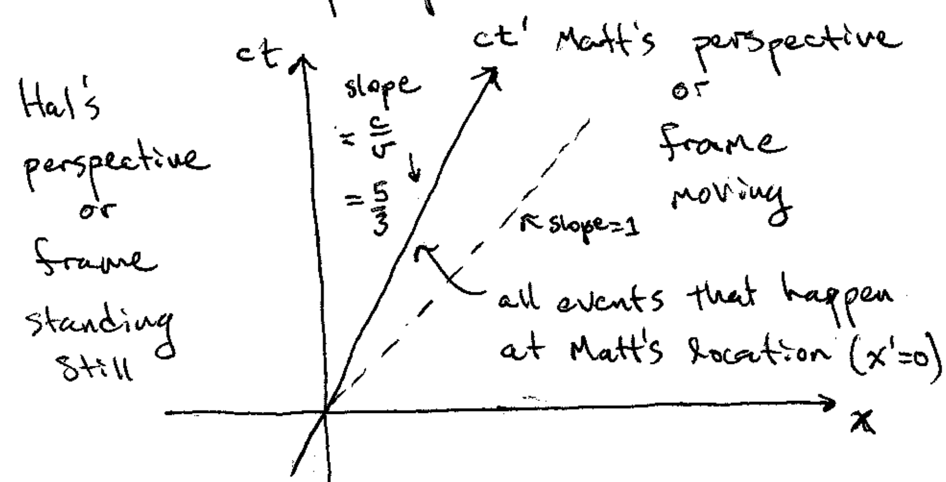


happen at the same location

Because massive particles always have $v < c \Rightarrow$ slope > 1



These diagrams become truly useful when they compare two different observers' perspectives on events



Matt moves through at $\frac{3}{5}c$

But, which events does Matt think are simultaneous? E.g. where is Matt's x'-axis? What is its slope?

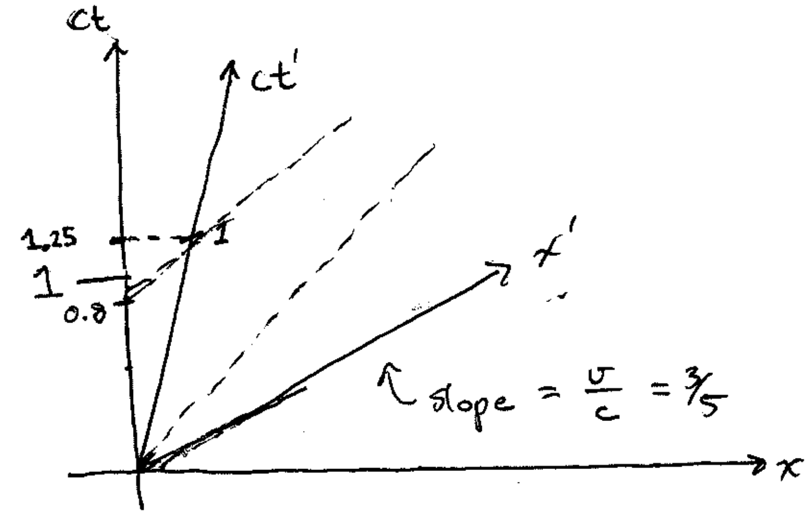
To answer these questions, let's compare how Matt and Hal see time pass. Suppose Matt measures the interval

$$c\Delta t' = 1\,\text{m} \text{ from the origin}$$

But, notice from Matt's perspective it's Hal that's moving! This means that according to Matt Hal's clock is running slow and when Matt measures $c\Delta t' = 1\,\text{m}$ Hal has only measured

$$c\Delta t' = \gamma c \Delta t$$

$$\Rightarrow c\Delta t = \frac{1}{\gamma} c\Delta t'$$
$$= \frac{4}{5}(1\,\text{m}) = 0.8\,\text{m}$$



In this same time Hal measures

$$c\Delta t = \gamma c\Delta t'$$
$$= \frac{1}{\sqrt{1-\frac{9}{25}}} c\Delta t' = \frac{5}{4}\,\text{m} = 1.25\,\text{m}$$

So, according to Matt, Hal's event $(0, 0.8)$ and Matt's event $x'=0$ $ct'=1$ are simultaneous. In Hal's frame the second event occurs at

$$x = \frac{v}{c} c\Delta t = \frac{3}{5}\cdot\frac{5}{4} = \frac{3}{4}\,\text{m}$$

and

$$ct = 1.25 = \frac{5}{4}\,\text{m}$$

So, the slope of the line that connects events simultaneous for Matt is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\frac{5}{4} - \frac{4}{5}}{\frac{3}{4} - 0}$$

$$= \frac{4}{3}\left(\frac{25}{20} - \frac{16}{20}\right)$$

$$= \frac{4}{3}\left(\frac{9}{20}\right) = \frac{3}{5} = \frac{v}{c}\,!$$

The events that Matt sees as simultaneous are connected by lines that have slope $= \frac{v_{matt}}{c}$!

According to Hal A and B are simultaneous, but according to Matt B happens before A!

This allows for immediate comparisons: which event, A or B, is first according to Hal? To Matt?