Homework 12

Due Wednesday, April 25th at 5pm

Our last problem set! We'll use this opportunity to review each of the big metrics we've explored. Read chapter 21 of Hartle's *Gravity*, which we started yesterday in lecture. We will continue treating this material through tomorrow and the rest of next week.

1. Tomorrow in lecture we will introduce the Riemann curvature tensor,

$$R^{\alpha}{}_{\beta\gamma\delta} = \frac{\partial \Gamma^{\alpha}_{\beta\delta}}{\partial x^{\gamma}} - \frac{\partial \Gamma^{\alpha}_{\beta\gamma}}{\partial x^{\delta}} + \Gamma^{\alpha}_{\gamma\epsilon}\Gamma^{\epsilon}_{\beta\delta} - \Gamma^{\alpha}_{\delta\epsilon}\Gamma^{\epsilon}_{\beta\gamma}.$$

   Calculate the full Riemann tensor for the static weak field metric,

$$ds^2 = -(1 + 2\Phi(x^i))dt^2 + (1 - 2\Phi(x^i))(dx^2 + dy^2 + dz^2), \qquad (\Phi(x^i) << 1),$$

   as always you should only work to lowest order in Φ, i.e. you can take $\Phi^2 \approx 0$.

2. Find the orthonormal frame of a stationary observer at a point (t, r, θ, ϕ) outside of a Schwarzschild black hole. Express the components of these frame vectors in a coordinate basis, e.g. find $(\mathbf{e}_{\hat{0}})^{\alpha}$ along with the components of the other frame vectors. In addition to writing these vectors in our usual component form, i.e. (*,*,*,*), write them in a derivative notation, i.e. $(*)\partial/\partial x^0 + (*)\partial/\partial x^1 + (*)\partial/\partial x^2 + (*)\partial/\partial x^3$. Also find the coordinate components of the dual orthonormal frame, e.g. $(\mathbf{e}^{\hat{0}})_{\alpha}$ along with the components of the other dual frame vectors.

3. Hartle 14.9, p309. Elaboration of the hint: The key to this problem is to note that the leading order $1/c$ answer will be proportional to the rotation term in the metric (14.22) and GM/rc^2 terms will not enter at all. (Explain this!) The leading order deflection can therefore be calculated in the metric (G=c=1),

$$ds^2 = -dt^2 + dr^2 + r^2 d\phi^2 - \frac{4J}{r} d\phi dt,$$

   assuming the orbit is in the equatorial plane $\theta = \pi/2$. With this setup the calculation is very similar to our previous calculation of the deflection of starlight.

4. Hartle 15.3, p328

5. Look at the document ListOfEquations.tex on our bspace site in the resources folder. Write down the latex code for two general relativity equations you would like to be on this list for the final exam. Take the opportunity to talk to your peers and figure out which ones they are doing and make yours distinct. This problem is a way for you to prepare for when it's your turn to add to the actual list but it's fine if the equations you end up adding are distinct from the ones you submit for this problem.