Today's Outline

I Last Lecture & Comments on Strominger's talk

II More on Event Horizons

III Kruskal-Szekeres Coords

Lecture 16

Mar 13^{th}, 2012

I Last Lecture

- Using Eddington-Finkelstein coords we showed that $r=2M$ is a coord. singularity of the Schw. metric.
- Also showed that ~~this~~ $r=2M$ is a black hole event horizon.
- Interpretted the Schw. radius r_s as an areal radius: sphere of radius r_0 has an area $4\pi r_0^2$.

Strominger's colloquium gave a very nice summary of the history of black holes; I've asked for his slides to post on our site and check out the video.

In my <u>opinion</u> his view that string theory is the only game in town is biased.

I believe that ~~both~~ string theory, loop gravity and all other approaches to quantum gravity receive an overall grade of:

$$\boxed{F}$$

This is because of their Fs in the category of observational predictions — after all we're doing <u>physics</u>.

You, the students in this room have to improve this grade.

II More on Event Horizons

Area

Recall, (v, r, θ, ϕ),

$$ds^2 = -\left(1-\frac{2M}{r}\right)dv^2 + 2\,dv\,dr + r^2 d\Omega^2$$

The area of the event horizon is given by integrating over a $v=$ const. slice of it,

$$d\Sigma^2 = (2M)^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$\Rightarrow \quad A = 16\pi M^2$$

Because Schw. is static this is independent of v (and t).

Inside the horizon r acts like a time coordinate and the $r=0$ singularity is not a place in space but rather a moment in time!

This is a dramatic way of saying that motion towards the singularity is inevitable.

The sign switch

Inside $r=2M$ note that

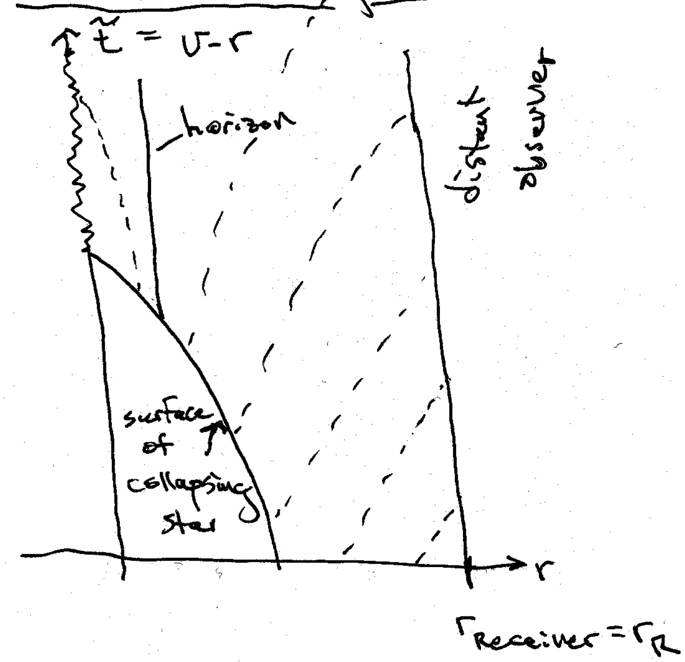
$$ds^2 = \underbrace{-\left(1-\frac{2M}{r}\right)}_{\text{is positive}} dv^2 + 2\,dv\,dr + r^2 d\Omega^2$$

This means that a 3-surface of constant r has line elem.

$$dS^2 = \underbrace{-\left(1-\frac{2M}{r}\right)}_{\text{positive}} dv^2 + r^2 d\Omega^2$$

which is a spacelike surface!

Spherical Collapse



3 stories:

(i) An observer who sits on the collapsing star at τ-dependent radius $r_{Emitter} \equiv r_E$.

- Emits light signals at regular intervals of proper time.
- Sees nothing special at $r = 2M$ and can leave surface of star in rocket up to $r=2M$.
- Can neither communicate w/ distant observer or avoid singularity with rocket for $r \leq 2M$

(ii) An observer who sits at $r = r_{Receiver} \equiv r_R$.

- Receives irregularly spaced light signals as measured by her proper time $\tau_R \simeq \tilde{t} \simeq t$, w/ spacing going to ∞ as emitter approaches $r=2M$.

- Light signals are of lower and lower energy, i.e. red shifted.
- Quickly the geometry is indistinguishable from Schwarzschild, and the presence of the black hole can only be inferred indirectly (see next lecture).

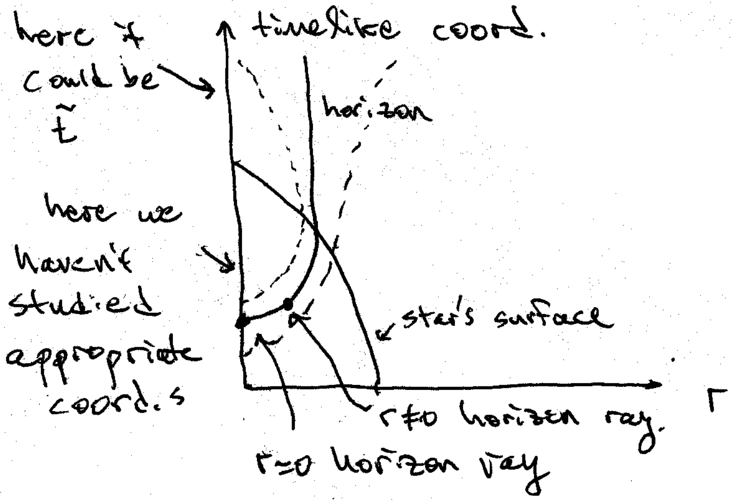
(iii) Light rays

- They escape if emitted before $r_E = 2M$ and don't otherwise.
- If a light ray is emitted at $r=2M$ it remains there.

<u>Increase of horizon area</u>

We have not (and unfortunately will not) study the interior metric but one aspect we can discuss qualitatively: the area of the horizon increases as mass falls in.

Hartle's discussion is a little confusing. He speaks of rays that leave $r=0$ and form the horizon, I would speak of a ray:



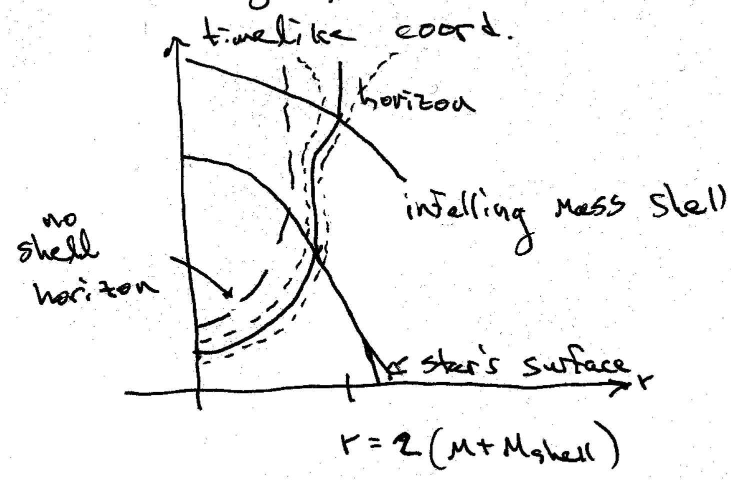
depends not just on the present mass within it but any future mass that might fall in. Horizons are a global feature of spacetimes.

- Partially motivates study of Penrose diagrams and of more systems of coord.s (Kruskal-Szekeres coord.s)

II Kruskal-Szekeres Coord.s

New coordinates (V, U, θ, ϕ):

Fascinating phenomenon:



- Area of the horizon grows with infalling mass
- The horizon's location at each moment

$$\left.\begin{aligned} U &= \left(\frac{r}{2M}-1\right)^{1/2} e^{r/4M} \cosh\left(\frac{t}{4M}\right) \\ V &= \left(\frac{r}{2M}-1\right)^{1/2} e^{r/4M} \sinh\left(\frac{t}{4M}\right) \end{aligned}\right\} r > 2M$$

$$\left.\begin{aligned} U &= \left(1-\frac{r}{2M}\right)^{1/2} e^{r/4M} \sinh\left(\frac{t}{4M}\right) \\ V &= \left(1-\frac{r}{2M}\right)^{1/2} e^{r/4M} \cosh\left(\frac{t}{4M}\right) \end{aligned}\right\} r < 2M$$

leads to,

$$ds^2 = \frac{32M^3}{r} e^{-r/2M}\left(-dV^2 + dU^2\right) + r^2 d\Omega^2$$

... continued next lecture