Lecture 17

Mar 15th, 2012

Today's Outline:

I Kruskal-Szekeres coord.s

II Astrophysical Black Holes

I Kruskal-Szekeres coord.s

Last time: Coord.s (V, U, θ, ϕ)

$$\left.\begin{aligned} U &= \left(\frac{r}{2M}-1\right)^{1/2} e^{r/4M} \cosh\left(\frac{t}{4M}\right) \\ V &= \left(\frac{r}{2M}-1\right)^{1/2} e^{r/4M} \sinh\left(\frac{t}{4M}\right) \end{aligned}\right\} r>2M$$

$$\left.\begin{aligned} U &= \left(1-\frac{r}{2M}\right)^{1/2} e^{r/4M} \sinh\left(\frac{t}{4M}\right) \\ V &= \left(1-\frac{r}{2M}\right)^{1/2} e^{r/4M} \cosh\left(\frac{t}{4M}\right) \end{aligned}\right\} r<2M$$

lead to the metric,

$$ds^2 = \frac{32M^3}{r} e^{-r/2M}\left(-dV^2+dU^2\right)+r^2 d\Omega^2$$

with $r = r(V, U)$ given by,

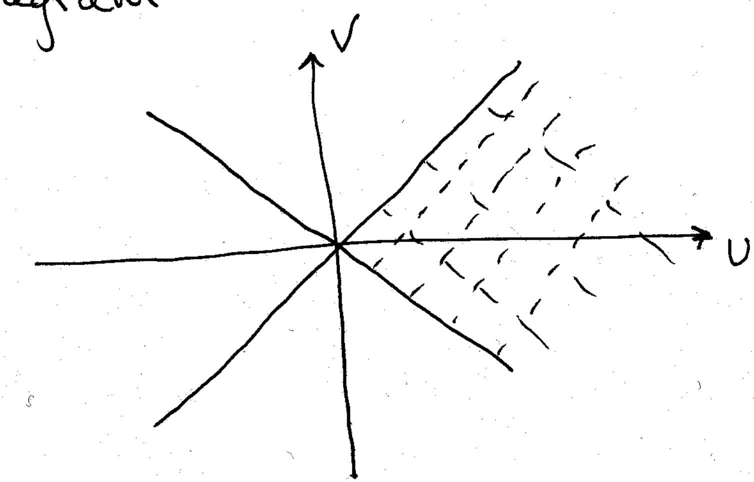
$$\left(\frac{r}{2M}-1\right)e^{r/2M} = U^2 - V^2$$

The raison d'etre of these coord.s is the simple behavior of (radial) light rays:

$$ds^2 = 0 = f(r)\left(-dV^2+dU^2\right)$$

$$\Rightarrow dV = \pm dU \Rightarrow V = \pm U + \text{const.}$$

Light rays are 45° lines in the (V, U) plane — called the Kruskal diagram



Now, constant r curves,

$$\text{const.} = U^2 - V^2$$

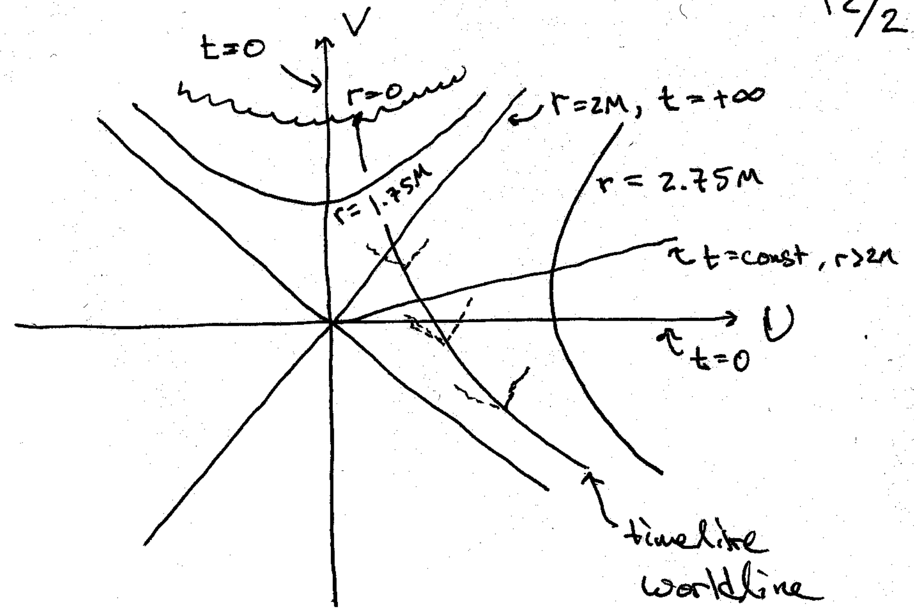
are hyperbolae with $r = 2M$ corresponding to

$$0 = U^2 - V^2.$$

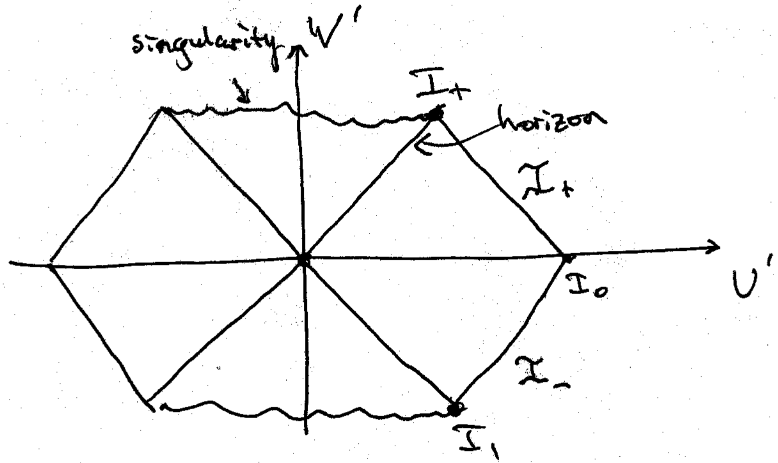
Constant t curves are straight lines

$$\tanh\left(\frac{t}{4M}\right) = \frac{V}{U} \qquad r > 2M$$

$$\tanh\left(\frac{t}{4M}\right) = \frac{U}{V} \qquad r < 2M$$



Short jump to Penrose diagram



See p. 274 of Hartle for defns of U', V'.

II Astrophysical Black Holes

Mass (in $M_\odot$)

- 10^9 – 10^6: Supermassive BHs } Important for Galaxy formation
- 10^4 – 10^2: Intermediate Mass BHs } Experimental observation controversial; possible seeds of galaxy formation
- 10 – 3: BHs in X-ray Binaries } Common end state of stellar collapse
- 10^{-7} – 10^{-16}: Primordial BHs] In this range – possible dark matter candidate (speculative)
- 10^{-19}] In this range – possible source of Hawking radiation / constrains early cosmology.

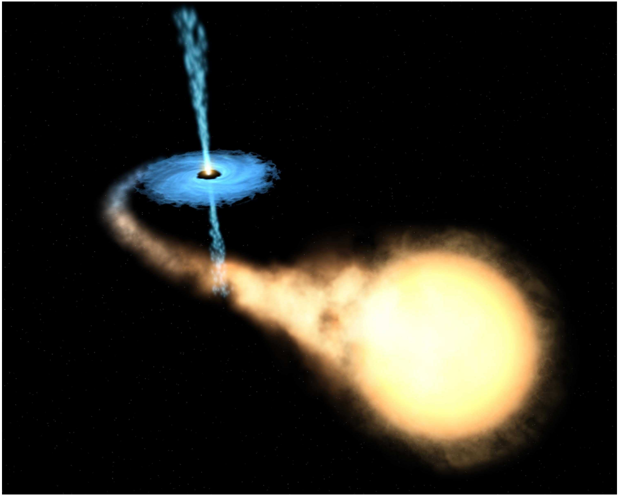
Figures are reproduce for educational purposes only. They are from:

- Wikipedia
- Paredes, "Black Holes in the Galaxy"
- Genzel et al, "The Galactic Center Massive Black Hole and Nuclear Star Cluster"

Three key elements of Black Hole observation:

1. Black holes have an event horizon — no causal signals leave the vicinity of black hole.
2. The presence of a black hole must be inferred from its effects on nearby observable bodies.
3. The first two rules will be subverted if Hawking radiation is directly observed.

X-ray Binary



Approximately 2/3 of all stars are members of a binary pairs.

If one star collapses and an accretion disk forms, the in-falling matter is heated and emits x-rays. These are the most luminous x-ray sources in the sky.

By observing the luminous partner the mass of the accreting compact object can be estimated. If this mass is greater than the maximum mass of a neutron star then we conclude that the compact object is a black hole.

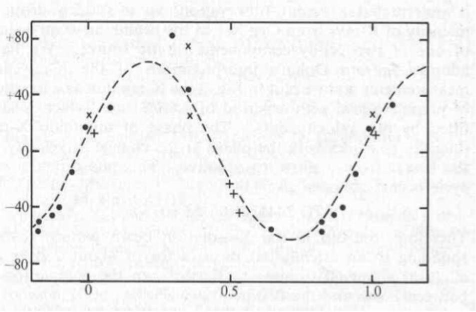


Figure 4. Radial velocity curve of HD 226868, the O9.7Iab companion star in the HMXB Cyg X-1, folded on the 5.6 day orbital period. Figure reproduced from Webster & Murdin (1972).

Compact object estimated to have a mass in the range 4 to 13 $M_\odot$.

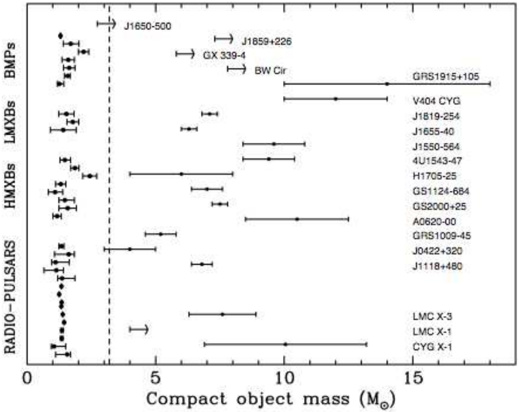


Figure 6. Mass distribution of compact objects in X-ray binaries. Arrows indicate lower limits to BH masses. Figure reproduced from Casares (2007).

Supermassive Black Holes

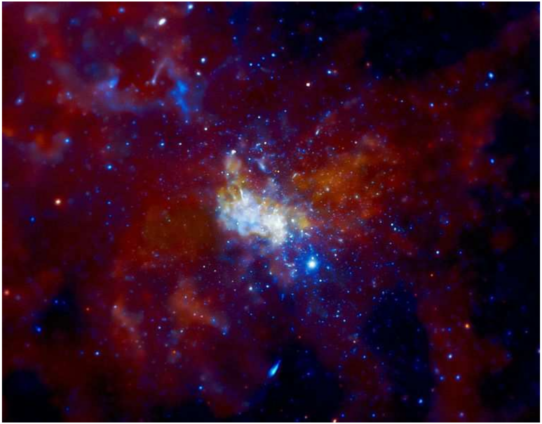


Figure: The supermassive black hole at the center of our galaxy, Sgr A*.

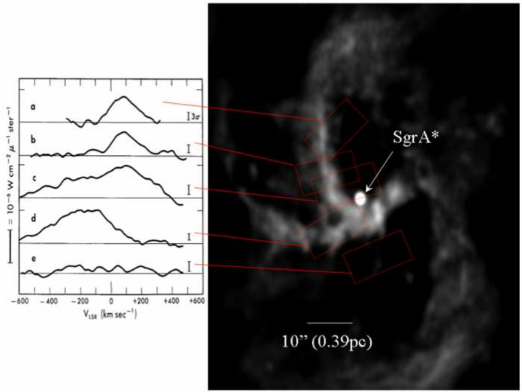


Figure: First evidence came from orbiting gas. Gas has radial velocities up to a few hundred km/s. Suggested a central mass of a few times $10^6 M_\odot$.

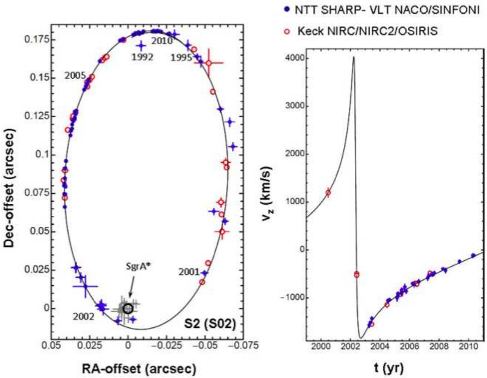


Figure: Began tracking orbiting stars. The star S2, with an eccentricity of $\epsilon = 0.88$ became an important signature. Put central mass at $4 \times 10^6 M_\odot$.

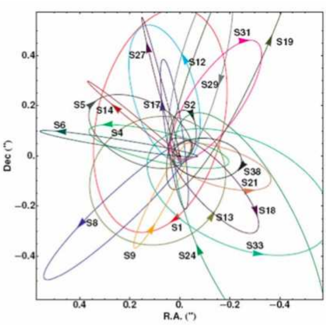


Figure 4.3.2. A summary of 20 of the ~ 30 S-star orbits delineated by the most recent orbital analysis of Gillessen et al. (2009b) [5].

Figure: Continued tracking orbiting stars. Accurately put mass at $4.3 \times 10^6 M_\odot$.

From Genzel et al: "In summary, from the stellar orbits it is now established that the Galactic Center contains a highly concentrated mass of ~ 4 million solar masses within the peri-center of S2, i.e. within 125 AU. This requires a minimum density of $5 \times 10^{15} M_{\odot} pc^{-3}$. The mass centroid lies within ± 2 mas at the position of the compact radio source Sgr A*, which itself has an apparent size of < 1 AU only (Shen et al. 2005, Bower et al. 2006, Doeleman et al. 2008). Taken together, this makes the Galactic Center Black Hole the currently best case for the existence of astrophysical black holes. Further support for this conclusion comes from the fact that near-infrared and X-ray flares are observed from the same position, which naturally can be ascribed to variations in the accretion flow onto the massive black hole."

There are even observational consequences of the event horizon!