Lecture 20

April 3rd, 2012

Today's Outline

I Last Lecture

II Gyros & Rotating Sources

III Survey

IV "No-Hair" & "Censorship" (?)

V The Kerr Geometry and its Event Horizon

I Last Lecture

- Why gyros
  (i) operationally determine $\underset{\sim}{e}_{\hat{i}}$ $(i = 1,2,3)$
  (ii) Experimental tests of G.R.: Gravity probe B
  (iii) Gravitomagnetism

- Gyroscope Equation:

$$\frac{ds^\alpha}{d\tau} + \Gamma^\alpha_{\beta\gamma} s^\beta u^\gamma = 0$$

- Geodetic precession

"Like spin-orbit coupling"

Has both geometrical and dynamical origins.

$$\Delta\phi_{geodetic} = 2\pi\left[1 - \left(1 - \frac{3M}{R}\right)^{1/2}\right]$$

Earth's radius $\approx$ 6,378 km

$$\approx 6.5 \times 10^{-8}\left(\frac{R_\oplus}{R}\right) \text{rad}$$

$$\rightarrow \quad 8.4''\left(\frac{R_\oplus}{R}\right)^{5/2} \text{ per year}$$

We didn't get to:

II Gyros & Rotating Sources

For the geodetic effect the source mass, M, was not rotating. If the massive source is rotating it gives rise to a different (from Schwarzschild) spacetime geometry:

$$ds^2 = ds^2_{schw} - \frac{4GJ}{c^3 r^2}\sin^2\theta\,(r\,d\phi)(c\,dt) + \left(\begin{matrix}\text{terms quadratic and higher}\\ \text{order in } J\end{matrix}\right),$$

where J is the magnitude of the source's angular momentum. This new term determines the scale of gravitomagnetic effects (see table at right). We can estimate:

$$J \sim I\Omega \sim MR^2\Omega \sim MRV$$

So,

$$\frac{GJ}{R^2c^3} \sim \underbrace{\frac{GM}{Rc^2}}_{\text{G.R. effect}} \cdot \underbrace{\frac{V}{c}}_{\text{moving}}$$

} weaker i.e. higher order effect

Gravitomagnetism more precisely:	
S.R. Scale	V/c
G.R. Scale	$\frac{GM}{Rc^2}$
Gravitomag. Scale	$\frac{GJ}{R^2c^3} \sim \frac{GM}{Rc^2} \cdot \frac{V}{c}$

this is analagous to the spin-spin coupling. See the book for the full derivation:

$$\Omega_{LT} = \frac{2GJ}{c^2 z^3}$$

height above source along spin axis

↑ Lense-Thirring precession

A.K.A. "frame dragging"!

These two effects have recently been measured by Gravity Probe B:

This metric causes a precession independent of the geodetic effect;

	Geodetic	Frame dragging
G.R.	$-6{,}606.1\ \frac{mas}{yr}$	$-39.2\ \frac{mas}{yr}$
GPB	$-6{,}601.8 \pm 18.3\ \frac{mas}{yr}$	$-37.2 \pm 7.2\ \frac{mas}{yr}$

III Survey

Q1: What percentage of the time do you feel you've understood the main points of a lecture when its ended?

Q2: Is the lecture moving too fast, too slow, or at the right pace?

Q3: What would you change were you teaching the course? What's going well?

Q4: Current plan: 1 week Rotating BHs
1 week GR waves
1 week math tools, 1 week Einstein Eqns.

2nd option: 1 week rotating BHs
3 weeks math tools & Einstein Eqns

Would you prefer Current plan or 2nd option?

Q5: Are you in love with G.R. yet?

IV "No-Hair" & "Censorship"

There are two important conjectures, with quasi-theorem status, in black hole physics:

- No-Hair: Black hole solutions are completely characterized by three numbers: Mass, Angular Momentum and (if we allow E&M) Charge.

In this sense black holes are remarkably simple solutions of the Einstein equations. The name comes from thinking of all of the details of the in-fallen matter as "hair."

- Cosmic Censorship: The singularities that arise from gravitational collapse are never naked — that is they always appear behind an event horizon.

Both claims are conjectures and exotic counter examples have been found, nothing I've found particularly compelling.

V The Kerr Geometry and its Event Horizon

The Kerr metric is,

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4Mar\sin^2\theta}{\rho^2} d\phi\, dt$$

$$+ \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta\, d\phi^2$$

where, "Kerr parameter"

$$a \equiv \frac{J}{M}, \quad \rho^2 \equiv r^2 + a^2\cos^2\theta, \quad \Delta \equiv r^2 - 2Mr + a^2$$

(t, r, θ, ϕ) are Boyer-Lindquist coord.s and in the limit $a \to 0$ we recover (t, r, θ, ϕ) of the Schw. coord.s.

You are now G.R. and B.H. experts, what do you see?

- metric is independent of ϕ, t, $\xi^\alpha = (1, 0, 0, 0)$ and $\eta^\alpha = (0, 0, 0, 1)$

are Killing vectors.

- Singularities at $\rho = 0$ and $\Delta = 0$

Guess that $\rho = 0$ is a true singularity ✓ (correct)

$$\Delta = 0 \Rightarrow r^2 - 2Mr + a^2 = 0$$

$$\Rightarrow r_\pm = \frac{2M \pm \sqrt{4M^2 - 4a^2}}{2} = M \pm \sqrt{M^2 - a^2}$$

Guess that r_+ is event horizon ✓ (Correct)

Note that for r_+ to be real $M \geq a$.

- For $r \gg M$ and $r \gg a$

$$ds^2 = ds^2_{Schw.} - \frac{4J}{r^2}\sin^2\theta\,(r\,d\phi)\,dt + \cdots$$

which produces previously mentioned gravitomag. metric and shows that Kerr is asymptotically flat (since Schw. is).

- The $M \to 0$ limit is an interesting puzzle. What should happen in this limit? What does?