Today's Outline

Lecture 5

Jan 31st, 2012

I Last Lecture

II The Equivalence Principle

III Gravitational Time Dilation

IV Grav. Time Dilation as Geometry

Doppler Effect:

$$\omega' = \omega \frac{\sqrt{1-V^2}}{1-V\cos\alpha'}$$

Check out relativistic beaming in text.

II The Equivalence Principle

Recall the parable of Galileo and the leaning tower of Pisa.

I Last Lecture

- Discussed S.R. kinematics & dynamics
- Touched on light

Light:

$$E = \hbar\omega = |\vec{p}|$$

Write,

$$\vec{p} = \hbar\vec{k} \quad \text{w/} \quad |\vec{k}| = \omega$$

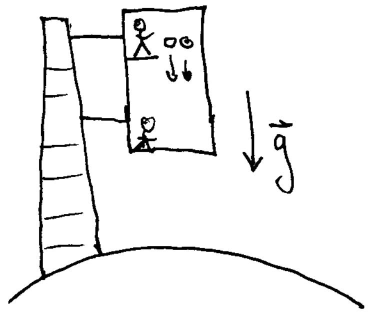
then

$$p^\alpha = (E, \vec{p}) = \hbar(\omega, \vec{k}) = \hbar k^\alpha$$

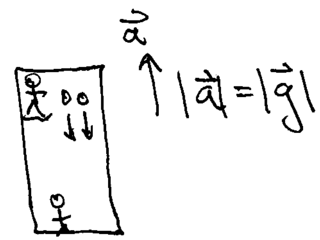
The importance of this story:

$$\boxed{m_I = m_G}$$

This lead to Einstein's "happiest thought", a free fall observer has no idea there's a gravitational field. Conversely, an accelerated observer's experiments are identical to those of an observer immersed in a gravitational field pointing in the opposite direction:

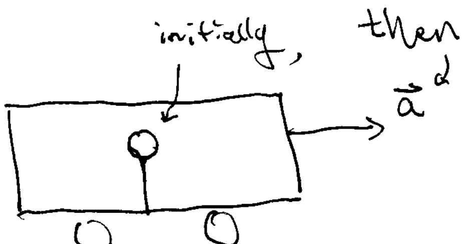


same results as

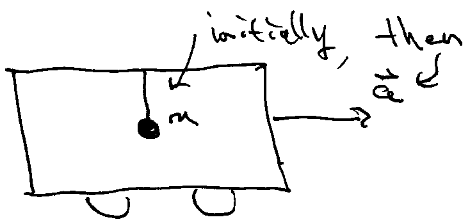


Only an outside, inertial observer would say that these results happen for different reasons: gravitational attraction and rocket thruster respectively.

<u>Example</u>: Balloon in an accelerated train car.



Which direction does the balloon deflect in? How about a pendulum?

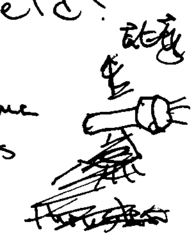


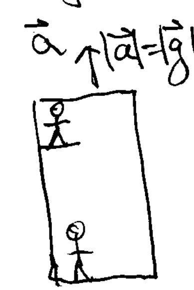
Easy to answer when you think of the gravitational analogs.

Works both ways:

Example: What happens to light rays in a grav. Field?

same as

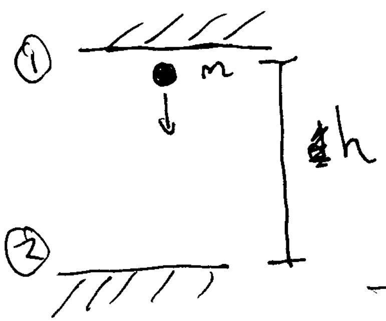




They're bent!

III Gravitational Time Dilation

"Naive" approach: Drop a rock down an elevator shaft: Its kinetic energy is greater at the bottom,



$$E_2 = E_1 + mgh$$

What if I drop a photon? Photons have $E = \hbar\omega$, so

$$\hbar\omega_2 = \hbar\omega_1 + \frac{\hbar\omega_1}{c^2} gh,$$

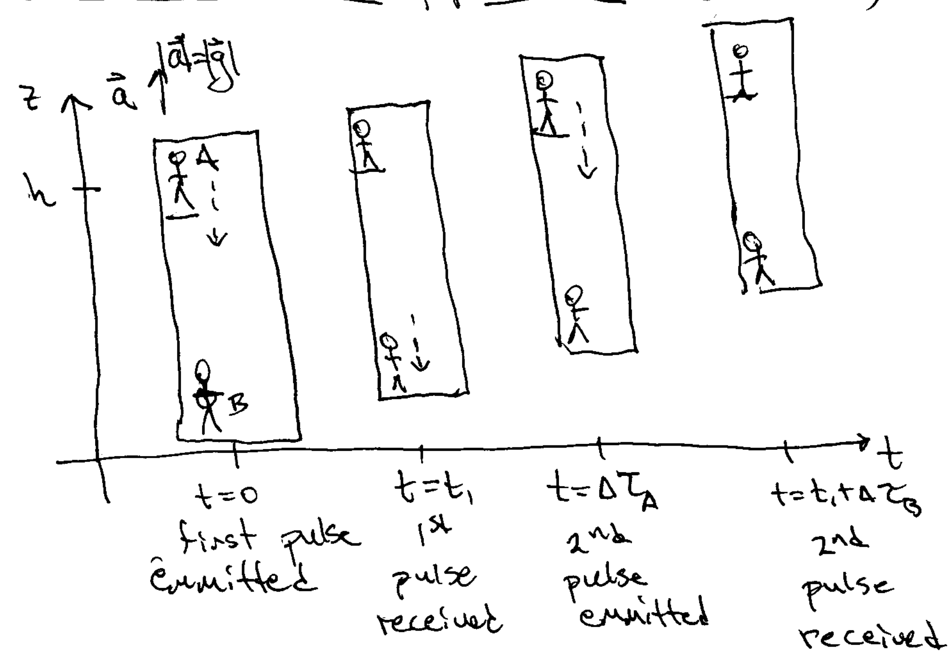
where I've used $m \to E/c^2 = \frac{\hbar\omega}{c^2}$. So,

$$\boxed{\omega_2 = \omega_1\left(1 + \frac{gh}{c^2}\right)}$$

The photon is blue-shifted by the fall. Confirmed experimentally by Pound & Rebka (1960). Tiny effect,

$$\frac{gh}{c^2} \simeq \frac{10(20)}{9\times10^{16}} \simeq 2\times10^{-15}.$$

Quantitative approach: (Hartle)



Neglect $\left(\frac{v}{c}\right)^2$, $\left(\frac{gh}{c^2}\right)^2$ and $g\left(\frac{gh}{c^2}\right)$ but keep $\frac{v}{c}$ and $\frac{gh}{c^2}$.

$$z_B(t) = \frac{1}{2}gt^2$$

$$z_A(t) = h + \frac{1}{2}gt^2$$

Distance travelled by 1st pulse,

$$z_A(0) - z_B(t_1) = ct_1$$

by 2nd pulse,

$$z_A(\Delta\tau_A) - z_B(t_1 + \Delta\tau_B) = c\left(t_1 + \Delta\tau_B - \Delta\tau_A\right)$$

If $\Delta\tau_A$ (hence $\Delta\tau_B$) is small then,

$$h - \tfrac{1}{2}g t_1^2 = c t_1 \qquad (1)$$

$$h - \tfrac{1}{2}g(t_1 + \Delta\tau_B)^2 \simeq h - \tfrac{1}{2}g t_1^2 - g t_1 \Delta\tau_B$$
$$= c(t_1 + \Delta\tau_B - \Delta\tau_A) \qquad (2)$$

And Eq (1) - Eq (2) is,

$$g t_1 \Delta\tau_B = c(\Delta\tau_A - \Delta\tau_B)$$

$$\Rightarrow \Delta\tau_B = \frac{\Delta\tau_A}{(1 + g t_1/c)}$$

Reexpress in terms of gravitational potential (per unit mass, e.g. $\Phi = -\frac{GM}{r}$ not $U = -\frac{GMm}{r}$ and units of $[\Phi] = \frac{\text{Energy}}{\text{Mass}}$) Φ,

$$\Phi_A - \Phi_B = gh$$

and rates of reception $\text{rate}_A = \frac{1}{\Delta\tau_A}$, $\text{rate}_B = \frac{1}{\Delta\tau_B}$, then,

$$\text{rate}_B = \text{rate}_A\left(1 + \frac{gh}{c^2}\right) = \text{rate}_A\left(1 + \frac{\Phi_A - \Phi_B}{c^2}\right)$$

But from Eq. (1),

$$\tfrac{1}{2}g t_1^2 + c t_1 - h = 0$$

$$\Rightarrow t_1 = \frac{-c \pm \sqrt{c^2 + 2gh}}{g}$$
$$= -\frac{c}{g} \pm \frac{c}{g}\left(1 + \frac{gh}{c^2} + \dots\right)$$
$$= \frac{h}{c} + \dots$$

Thus, $\Delta\tau_B = \Delta\tau_A\left(1 - \frac{gh}{c^2}\right)$

Can summarize as:

"Slouching clocks run slow"

IV Gravitational Time Dilation as Geometry

We begin our program in earnest: I hand you a metric, say a little about it and then you explore it.

Our First curved Spacetime metric:

$$ds^2 = -\left(1 + \frac{2\Phi(x^i)}{c^2}\right)(c\,dt)^2 + \left(1 - \frac{2\Phi(x^i)}{c^2}\right)(dx^2 + dy^2 + dz^2),$$

here Φ is the gravitational potential in the Newtonian sense and is a

So this is the static weak field metric.

function of position alone,

$$\Phi = \Phi(x^i) \quad i = 1, 2, 3$$

we call this a static potential. This line element is only valid for small curvatures and weak sources, which is known as a weak field.